Longitudinal and transverse exciton-spin relaxation in a single InAsP quantum dot embedded inside a standing InP nanowire using photoluminescence spectroscopy


Physical Review B, 85(7): 075324

2012-02-15

http://hdl.handle.net/2115/48661

©2012 American Physical Society

article

PRB85-7_075324.pdf
Longitudinal and transverse exciton-spin relaxation in a single InAsP quantum dot embedded inside a standing InP nanowire using photoluminescence spectroscopy


1Research Institute for Electronic Science, Hokkaido University, Sapporo 001-0021, Japan
2Kavli Institute of Nanoscience, Delft University of Technology, NL-2628 CJ Delft, The Netherlands
3Graduate School of Information Science Technology, Hokkaido University, Sapporo 060-0814, Japan
4Research Center for Integrated Quantum Electronics (RCIQE), Hokkaido University, Sapporo 060-8628, Japan

We have investigated the optical properties of a single InAsP quantum dot embedded in a standing InP nanowire. Elongation of the transverse exciton-spin relaxation time of the exciton state with decreasing excitation power was observed by first-order photon correlation measurements. This behavior is well explained by the motional narrowing mechanism induced by Gaussian fluctuations of environmental charges in the nanowire. The longitudinal exciton-spin relaxation time is evaluated by the degree of the random polarization of emission originating from exciton states confined in a single-nanowire quantum dot by using Mueller calculus based on Stokes parameters representation. The reduction in the random polarization component with decreasing excitation power is caused by suppression of the exchange interaction of electron and hole due to an optically induced internal electric field by the dipoles at the wurtzite and zinc-blende heterointerfaces in the InP nanowire.

DOI: 10.1103/PhysRevB.85.075324 PACS number(s): 78.67.Uh, 71.70.Ej, 78.55.–m, 78.67.Hc

I. INTRODUCTION

Semiconductor quantum dots (QDs) are called artificial atoms because of their atom-like discrete electronic structure due to quantum confinement. Exciton spins confined in a QD can relax via a wide variety of interaction and/or scattering, specific examples are valence band mixing, Coulomb interaction with environmental charge fluctuation, and nuclear spin fluctuation. QDs embedded in semiconductor nanowires (NWs) (hereafter called NW-QDs), i.e., combined zero-dimensional and one-dimensional structures, are among the most promising candidates for realizing single-photon detectors and on-demand single-photon sources with high coherences and long dephasing times. Fast dephasing of carriers in NW structures fabricated by top-down techniques such as photo/electron-beam lithography and reactive ion/plasma etching is improved by the reduction of surface traps induced by defects and/or disorders within epitaxially grown bottom-up single or multi core-(shelled) NW structures. Emitter dephasing and polarization instability set up bit errors and degrade the communication distance limit when implementing phase and polarization encoding quantum key distribution protocols. In this paper we investigate the transverse and longitudinal exciton-spin relaxation times. The transverse exciton-spin relaxation time of our NW-QD, caused by fluctuation in surrounding excess charges in the NW, is suppressed with decreasing excitation power, which was observed by first-order photon correlation measurements under nonresonant excitation condition. The longitudinal exciton-spin relaxation time was evaluated by the degree of random polarization in the time-integrated photoluminescence. The degree of the random polarization component is obtained by Mueller analysis of the experimental results of a full polarization measurement based on Stokes parameters.

II. SAMPLE PREPARATION AND MEASUREMENT SETUP

Arrays of InAsP QDs embedded in InP NWs were synthesized by selective area metal-organic vapor phase epitaxy. An InP (111)A wafer was covered with 30 nm of SiO2. By electron beam lithography and wet-etching, 40- to 60-nm-diameter openings were created to form NW nucleation sites (Fig. 1). At a growth rate of 3 nm/s, the first 1-μm-long segment of InP was grown by adding trimethylindium and tertiarybutylphosphine (TBP) to the MOVPE reactor at 640 °C. To form the QDs the temperature was lowered to 580 °C and arsine (AsH3) was added to the reactor (V/III ratio 340; partial pressure, TBP:AsH3 3:1). At a growth rate of 3 nm/s an 8- to 10-nm layer of InAsP was created, forming the QDs. To cover the QDs with an InP shell, InP growth was first performed at 580 °C. To finalize the NW-QDs, a second 1-μm segment of InP was grown at 640 °C.

Single NW-QD photoluminescence (PL) spectroscopy was performed by standard micro-PL measurements in the far field using a microscope objective as shown in Fig. 1. The sample was cooled to 5 K in a 4He flow cryostat. The excitation laser beam traveling along the NW growth direction was focused on the sample surface with a microscope objective (NA = 0.42). The NW-QD emission was collected by the same objective lens, dispersed with a double (f = 1.0 m) grating spectrometer under zero magnetic field and a triple (f = 0.64 m) grating spectrometer under a magnetic field and was detected with a liquid-nitrogen-cooled InGaAs photodiode array. The typical exposure time to obtain a spectrum with a high signal-to-noise ratio was 1 s.

III. BASIC OPTICAL PROPERTIES

Figure 2(a) shows the excitation power dependence of the PL peak intensities of X0 and XX0. Excitation was carried out...
with a pulsed Ti:sapphire laser at a wavelength of $\sim 800$ nm (1.55 eV) and a repetition rate of 76 MHz. At a low excitation intensity, $X^0$ is centered at $\sim 1.023282$ eV, with a 46-μeV FWHM. With increasing excitation power, the PL intensity of $X^0$ increases linearly and saturates at an excitation power of $\sim 2$ μW, while an additional line, labeled $XX^0$, appears, with its PL intensity increasing superlinearly. This indicates the origin of the PL peaks from neutral exciton and biexciton recombination. Figure 2(b) shows time-integrated PL spectra of a single NW-QD using a two-color excitation method.\textsuperscript{20} We used a continuous-wave (cw) diode laser (2.33 eV, 532 nm), which is above the bandgap energy of InP, and a cw Ti:sapphire laser (1.35 eV, 920 nm) below the bandgap energy, for two-color excitation. The excitation energy of the 1.35-eV laser is still above the discrete levels in InAsP NW-QDs and creates electron-hole pairs in continuum states;\textsuperscript{21–24} it is estimated by PL excitation measurements (not shown). Although $X^*$ PL (charged exciton) is dominant under excitation condition (II; 2.33 eV), $X_0$ appears under two-color excitation condition (II) + (I; 1.35 eV), which corresponds to the PL spectra under high-power excitation of (I). Note that the power of the 2.33-eV laser (I) is sufficiently low so as not to allow observation of any PL emission, which suggests that the created carriers are almost entirely used for the screening of the internal electric field induced by the dipoles at the wurtzite and zinc-blende heterointerfaces\textsuperscript{25} in the InP NW. Although InP NWs with a wurtzite crystal structure grown on InP (111)A substrate show type II optical transitions with lifetimes of the order of 100 ns,\textsuperscript{26,27} the decay time constant of the specific InP NWs presented in this paper is as short as $\sim 13$ ns (not shown). This indicates that this type of InP NW exhibits a mixed wurtzite and zinc-blende crystal structure.\textsuperscript{25} In Fig. 2(c), the Zeeman energy ($\Delta E_{\text{ze}}$) and diamagnetic shift ($\Delta E_{\text{diam}}$) of a single NW-QD are plotted against the external magnetic field along the growth direction (Faraday geometry). By fitting with $g_{\text{ex}}\mu_B B_z$ and $\gamma_2 B_z^2$,\textsuperscript{29} where $\mu_B$ is the Bohr magneton, we obtained an exciton $g$ factor of $|g_{\text{ex}}| = 0.4$ and a diamagnetic coefficient of $\gamma_2 = 23.2 \pm 0.3$ μeV/T². $\gamma_2$ is a good indicator of the degree of the lateral confinement potential. Compared to reports on small $\gamma_2$ values ($\ll 10$ μeV) in type I InAs/GaAs QDs\textsuperscript{30,31} and InP/GaAs QDs,\textsuperscript{32,33} our observed $\gamma_2$ is larger than for typical type I QDs but smaller than for type II InP/GaAs QDs.\textsuperscript{34} The Bohr radius is deduced to be $\sim 7$ nm using the effective masses in InP. The $X^0$ PL decays exponentially with an evaluated decay time constant $\tau_r \sim 2$ ns, obtained by the fitting [solid line in Fig. 2(d)], which suggests that the investigated InAsP QD exhibits a type I transition and thus a zinc-blende-like crystal structure. These observed results and the quantum-disk-like structure (height, $\sim 9$ nm; diameter, $\sim 100$ nm)\textsuperscript{19} indicate that the $X^0$ initial state is localized in a type I confinement potential as well as a relatively weak lateral confinement of the InAsP QD embedded in the InP NW. In what follows we focus on $X^0$, which generates the single-photon state estimated by the second-order photon correlation measurements presented in Ref. 19.

IV. TRANSVERSE EXCITON-SPIN RELAXATION

The Mach-Zehnder interferometer inserted in the optical path was used to perform first-order photon correlation measurements on single NW-QD $X^0$ emission. This is a type of time-domain spectroscopy called single-photon Fourier
FIG. 2. (Color online) (a) PL intensities of $X^0$ and $XX^0$ peaks as a function of excitation power. Red and blue lines are guides for the eye. (b) PL spectra of an InAsP NW-QD obtained under different excitation conditions. $X^0$ PL appears under two-color excitation (I) + (II), which corresponds to the PL spectra under laser (I) high-power excitation at 1 μW. Excitation conditions are as follows: (I) a green 2.33-eV laser creates electrons and holes in an InP NW; (II) an NIR 1.35-eV laser creates excitons in an InAsP NW-QD. Laser powers are 10 nW (I) and 25 μW (II). Inset: Schematics energy band diagram. Graded areas in the InAsP QD schematic indicate continuum states. (c) Zeeman energy and diamagnetic shift of $X^0$. Filled (black) circles and open (red) circles represent experimental data, the (black) line and (red) curve are fits to the data with $|g_α|μ_B B_z$ and $γ_2 B_z^2$, respectively. $|g_α| = 0.4$ and $γ_2 = 23.2 \pm 0.3 \mu eV/μT$. (d) Time-resolved PL of $X^0$ (filled (black) circles) under type (I; pulsed) excitation at 0.1 μW. The $X^0$ decay time of ∼2 ns is fitted with a single-exponential function [solid (black) line].

spectroscopy first demonstrated by Kummerer et al. The cw photoexcitation was carried out using a He-Ne laser for NW barrier excitation. Rotating a thin glass plate in one of the interferometer arms enables fine-tuning of the relative phase between the two arms.

Short-period fringe evolution of single-NW-QD $X^0$ exciton PL is shown in the inset in Fig. 3(a) at $τ \sim 10$ ps. The fringe evolution as a function of delay $τ$ is given by $I(τ) = I_0[1 + V(τ) \cos(E_0/\hbar + θ)]$, where $E_0$, $I_0$, $V(τ)$, and $θ$ are the central detection energy (∼1.023 282 eV for $X^0$), the averaged signal intensity, the visibility contrast given by the modulus of the Fourier transform of the intensity spectrum, and the phase, respectively. By varying the time delay $τ$, interference fringes of single-photon events are recorded. The visibility contrast $V(τ)$ decays with increasing delay time $τ$ between the two arms, and the coherence time is measured from the decay contrast of $V(τ)$. The visibility curves shown in Fig. 3(a) decay following almost-single-exponential functions in the investigated range of excitation power, suggesting that the spectral diffusion phenomenon is in the unconventional motional narrowing regime, $Σ τ_Γ/\hbar < 1$. $Σ = 2 ΔE_s τ_i/τ_{esc cap}$ is the modulation amplitude caused by an individual point charge and $ΔE_s$ is the energy saturation value. The rate of correlation between the confined exciton and the point charges in the NW is expressed by the combination of capture and escape rates, $1/τ_i = 1/τ_{esc cap} + 1/τ_{esc}$. In this regime the observed transverse exciton-spin relaxation time $T_2$ is given by $1/T_2 = (Σ/ℏ)^2 τ_i$ and the line shape remains Lorentzian under Gaussian fluctuations.

With decreasing excitation power, the measured $T_2$ increases and saturates at ∼45 ps as shown in Fig. 3(b). This indicates that the environmental charge fluctuations were sufficiently suppressed under weak excitation and that the observed $T_2$ is limited by the relaxation processes in the QD. The observed low-power (lp) saturation value of $T_2^{(lp)} ∼ 45$ ps is about 10 times shorter than that measured under resonant excitation using various different experimental methods and materials. With increasing excitation power, $T_2$ decreases gradually and saturates to $T_2^{(hp)} ∼ 18$ ps above the excitation power of ∼0.4 μW. The prolonged transverse exciton-spin relaxation time corresponds to the red-shift, $ΔE$ of $X^0$, as shown in Fig. 3(b) (right axis). This energy shift is attributed to the Stark effect due to the internal electric field induced by dipoles at the wurtzite and zinc-blende heterointerfaces in the NW and to the screening of the internal field by the photoexcited carriers in the InP NW barrier.
For simplicity, we adopt the approach of Favero et al., where the acoustic and optical phonon-assisted capture and escape rates are expressed as

\[
\frac{1}{\tau_{\text{cap}}} = \frac{1}{\tau_1} \left[ 1 + n_1(T) \right] + \frac{1}{\tau_2} \left[ 1 + n_2(T) \right],
\]

\[
\frac{1}{\tau_{\text{esc}}} = \frac{1}{\tau_1} n_1(T) + \frac{1}{\tau_2} n_2(T) + \frac{1}{\tau_3} P^2 + \left( \sqrt{3} P_0 \right)^2.
\]

\( n_{1(2)} \), \( \tau_{1(2)} \), \( \tau_3 \), \( P \), and \( P_0 \) are the Bose-Einstein occupation factors of acoustic (optical) phonons given by \( 1 / [\exp(E_{1(2)}/k_B T) - 1] \), the correlation times between point charge in the InP NW and acoustic (optical) phonon, the correlation time of Auger processes, the excitation power, and the characteristic excitation power of the appearance of Auger processes, which gives the inflection point of the calculated \( T_2 \) value, respectively. \( E_1 \approx 1 \mu \text{eV} \) and \( E_2 = 42.4 \mu \text{eV} \) are the acoustic phonon mean energy and the optical phonon energy of bulk InP, which indicates that optical phonon-assisted escape processes are negligibly small at low temperatures and which allows for the following simplification: \( n_2(5 \text{ K}) \approx 0 \). Substituting both excitation power limits, i.e., \( P \gg P_0 \) and \( P \ll P_0 \), into Eq. (1), we can derive empirical relations among the three correlation times: \( \tau_2 = (0.875 \pm 0.05) \times P_0^{1/2} \tau_1^{1/2} \) and \( \tau_3 = (6.6 \pm 1.0) \times \tau_1 \) [see Eqs. (A1) and (A2) in the Appendix]. These relations suggest that the optical phonon-assisted capture process and acoustic phonon-assisted escape process are dominant under the present experimental conditions. Both trends are attributed to the strong coupling between optical phonon and point charges and to the low activation energy of point charges. We can thus simulate the approximated excitation power dependence of \( T_2 \) by using the above relations, together with the empirical parameter deduced from experimental results, \( \Delta E_1 = 180 \mu \text{eV} \), as shown in Fig. 3(b), and one fitting parameter, \( P_0 = 0.12 \mu \text{W} \). As displayed in Fig. 3(b), we observe good agreements between the experimental \( T_2 \) values and the calculations [solid (red) curve] using a correlation time \( \tau_1 \approx 250 \text{ ps} \), which is a value similar to those reported in Refs. 9 and 10.

V. LONGITUDINAL EXCITON-SPIN RELAXATION

In order to investigate the longitudinal exciton-spin relaxation time of a single NW-QD, a quarter-wave plate (QWP), half-wave plate (HWP), and linear polarizer (LP) were placed in front of the spectrometer. The transmission axis of the LP was set horizontal in the laboratory frame. \( \theta \) and \( \rho \) are the relative angles of the QWP’s and HWP’s fast axes with respect to the transmission axis of the LP. A He-Ne laser was used as a quasirandomly polarized excitation source and was focused on the sample surface by the microscope objective. Figure 4(a) shows a two-dimensional (2D) plot of normalized PL intensities \( I_{\theta,\rho}(\theta,\rho)/[2(I_{\theta,\rho}(\theta,\rho)] \) as a function of \( \rho \) and \( \theta \). This checkered-flag-like pattern signifies that the observed photon state contains random polarization components.

Although the amplitude of the oscillation reaches a maximum at \( \theta \approx 59^\circ \), which is the cancellation condition of circular polarization components by the QWP, the observed amplitude is \(<1 \) [Fig. 4(c)], suggesting that the observed photon state includes random polarization components.

To analyze the obtained results for \( X_0 \), we introduce normalized Stokes parameters \( 0,1,2 \) with a random polarization...
FIG. 4. (Color online) (a) 2D plot of normalized PL intensities of $X^0$ as a function of $\rho$ and $\theta$. (b) Mueller calculus’ results by using the components $\{1.0, 0.32, -0.6, 0.14\}$. (c) Polar plot of the PL intensity of $X^0$. The detection angle $(2\rho)$ dependence of $I_{PL}(59^\circ, \rho)/\{2\langle I_{PL}(\rho) \rangle\}$ (open (red) circles). Solid (red) and dashed (blue) lines are Mueller calculus’ results for $\alpha = 0.3, \{1.0, 0.32, -0.6, 0.14\}$ and $\alpha = 0, \{1.0, 32/0.7, -0.6/0.7, 0.14/0.7\}$, with the latter being perfectly coherent light with a fixed ratio of the three polarization components. All angles are measured in the same laboratory frame. (d) $\Delta E$-dependence of the DRP $\alpha$ under fixed ratios of Stokes parameters of the polarization components $\{S_1, S_2, S_3\}$. The linear polarization components after the light has passed through the QWP($\theta$); QWP($\theta$)$\{S_1, S_2, S_3\} \rightarrow \{S'_1, S'_2, S'_3\}$. Under the cancellation condition of circular polarization components by the QWP, $\theta = 0.5\arctan(S_2/S_1)$, the amplitude of oscillation reaches the maximum. Substituting $\theta = 0.5\arctan(S_2/S_1)$ and $\alpha = 0$ into Eq. (2), the amplitude becomes exactly 1. Note that this is suitable only for the limiting case where the exciton-spin-flip time, i.e., the longitudinal exciton-spin relaxation time $T_1$, is much longer than the decay time $\tau_r$. Figure 4(c) shows the polar plot at $\theta = 59^\circ$ of the observed $I_{PL}(59^\circ, \rho)/\{2\langle I_{PL}(\rho) \rangle\}$. The maximum amplitude, $0.85 = 1 - \alpha/2$, is <1. By using $\alpha = 0.3$ and $\theta = 0.5\arctan(S_2/S_1) = 59^\circ$, we can deduce...
the Stokes parameters. Assuming the Stokes parameters 
\{1,0.32,−0.6,0.14\}, we can reproduce not only the amplitude
of \(I_{PL}(θ, ρ)/2[I_{PL}(ρ)]\), but also its pattern, very well, as
shown in Figs. 4(b) and 4(c). The observed DRP is evaluated as
0.3 \([-1−\sqrt{S_1(\alpha)^2+S_2(\alpha)^2+S_3(\alpha)^2}]\), suggesting that
\(T_1 \sim 0.7 \times τ_e = 1.4\) ns [Fig. 2(d)]. Note that the random
polarization component can be observed in the time-integrated
PL as a case of \(T_1 \leq τ_r\). The ratio between \(T_1/τ_e = \int dt e^{-i/τ_e}/\int dt e^{-i/τ_r} = 1 - α\) within
the relaxation time approximation. Figure 4(d) shows the
ΔE dependence of \(α\) under the assumption of a fixed ratio
of the three polarization components. \(α\) decays slightly with decreasing ΔE, implying that the overlap integral of electron
and hole envelope wave functions, |⟨ϕ_e|ϕ_h⟩|^2, decreases due to
the internal field. In general, the exciton spin in a quantum
well at low temperatures can mainly flip via short- and long-
range exchange interactions, i.e., via the Maialle-Andrada-
Sham mechanism. In the motional narrowing type, \(T_1\) is given
by the fluctuation of an effective magnetic field with an angular
precession frequency, \(Ω\), originating from exchange interaction,
\(Δ\), between electron and hole and the scattering time,
\(τ_s\), of center-of-mass motion of the exciton, \(1/T_1 ≈ (Ω^2 × τ_e × \Delta)^{1/2}\). \(Δ\) and \(1/τ_s\) are approximately proportional to |⟨ϕ_e|ϕ_h⟩|^2, indicating \(α \propto 1 - |⟨ϕ_e|ϕ_h⟩|^2\). Both observed
results, the ΔE dependence of \(α\) and the high degree of linear
polarization, 0.98 (=\(\sqrt{S_1^2 + S_2^2}\)), suggest that the observed
\(T_1\) time is dominated by short-range exchange interactions
corresponding to strong light-hole/heavy-hole mixing.

VI. SUMMARY

We investigated the optical properties of a single QD em-
bedded in a vertically standing NW structure and measured the
transverse and longitudinal exciton-spin relaxation times.
The transverse exciton-spin relaxation is induced by fluctuations
of environmental excess charges in the NW and can be reduced by
decreasing the excitation intensity. The experimentally
obtained transverse exciton-spin relaxation times, which were
evaluated as \(T_2 = 45–18\) ps, can be reproduced by the
unconventional motional narrowing theory. To determine the
longitudinal exciton-spin relaxation time, we performed full
polarization measurements and Mueller calculus. The longitudi-
nal exciton-spin relaxation times, \(T_1 = 1.4\) ns, evaluated by the
random polarization components are shorter than the decay
time. A decreasing random polarization component of emitted
photons from exciton states with decreasing emission energy
was observed due to the reduction of the overlap integral of electron
and hole envelope wave functions. We believe that the
obtained results can contribute to the understanding of effective
exciton-spin relaxation mechanisms in nanostructures.

ACKNOWLEDGMENTS

This work was supported in part by Grant-in-Aid for Young
Scientists (B) No. 20760002 and GCOE-GSIST, Hokkaido
University. M.K., N.A., and V.Z. acknowledge funding from
NWO; C.H. acknowledges financial support in the form of a
JSPS Fellowship for Foreign Researchers. H.S. acknowledges
Professor S. Muto, Professor S. Adachi and Dr. R. Kaji for
fruitful discussions.

APPENDIX: DERIVATION OF THE EMPirical
RELATION AMONG THREE CORRELATION TIMES

In the low- and high-power limits, the escape rate in Eq. (1)
can be transformed to

\[
\frac{1}{τ_{eq}} ≈ \begin{cases}
0.1/τ_1, & P \ll P_0,
0.1/(τ_1 + 1/τ_3), & P \gg P_0,
\end{cases}
\]

where the Bose-Einstein occupation factors of acoustic and
optical phonons are \(n_1(5 K) \approx 0.1\) and \(n_2(5 K) \approx 0.1\). The capture rate in Eq. (1) can be described as \(1/τ_{eq} \approx 1.1/τ_1 + 1/τ_2\). In the low-power limit, \(P \ll P_0\), the saturation value of the
transverse exciton-spin relaxation rate is given by

\[
\frac{1}{T_2^{(lp)}} \approx \frac{2ΔE_s}{\hbar} \frac{1}{τ_1^2 + 1/(2τ_2 + τ_1)^3} \frac{10}{45 \text{ ps}},
\]

where the values of \(T_2^{(lp)} \approx 45\) ps and \(ΔE_s = 180\) μeV are
deduced from Fig. 3(b). The calculation results are fitted with a
square root function,

\[
τ_2 = (0.87 ± 0.05) \text{ ps}^{1/2} × τ_1^{1/2},
\]

as shown in Fig. 5.

In the high-power limit, \(P \gg P_0\), by substituting \(τ_2 = 0.87 \text{ ps}^{1/2} × τ_1^{1/2}\) into the capture and escape rates of Eq. (1),
we obtain

\[
\frac{1}{T_2^{(hp)}} \approx \left(\frac{2ΔE_s}{\hbar}\right)^2 × \frac{a_1 τ_1 τ_3^2 (a_2 τ_1 τ_3^{3/2} + τ_1^2 + a_3 τ_1^{1/2} τ_3 + 0.1 τ_1 τ_3)}{(0.87 \text{ ps}^{1/2} τ_1^{3/2} + a_4 τ_1^{1/2} τ_3 + τ_1 τ_3)^3}
= \frac{1}{18 \text{ ps}}, \quad a_1 = 0.7569 \text{ ps}, \quad a_2 = 0.957 \text{ ps}^{1/2},
\]

\[
a_3 = 0.0957 \text{ ps}^{1/2}, \quad a_4 = 1.044 \text{ ps}^{1/2}.
\]

The value of \(T_2^{(hp)} \approx 18\) ps is deduced from Fig. 3(b). The
calculation results are fitted with a linear function,

\[
τ_3 = (6.6 ± 1.0) × τ_1,
\]

as shown in Fig. 5.