Simulation of Consensus Formation Models based on Structural Modeling

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***Abstract*—Consensus formation models are mathematical models for obtaining potential candidates for a group when all group members pursue their individual preferences. In such models, the members often change their preferences based on the group's global preference which summarizes the overall individual preferences. Based on the theory of partially-filled reachability matrices, the Flexible Interpretive Structural Modeling (FISM) provides a method for developing structural models of complex systems and is expected to be applied to decision-making support systems. In this paper, we propose a method for simulating consensus formation in FISM and study its effectiveness through the experiments with several consensus strategies.**

***Index Terms*—Consensus formation, Structural Modeling**

I. Introduction

In the process of consensus formation, the agent accepts macro–information and adapts its preference to the overall group preference. A technique that uses computer-based, group decision-making support systems (GDSS) is an important approach to solving this problem. Analytic Hierarchy Process (AHP) and Interpretive Structural Modeling (ISM)[1] were particularly used for solving the problem.

Flexible ISM (FISM)[2], [3], [4] is a method for developing structural models of complex systems. To perform FISM logically and effectively, a partially filled reachability matrix and implication rules are used. The partially filled reachability matrix is an extension of a reachability matrix and has great utility in the process of developing a new reachability matrix, and the implication rules guarantees that the matrix updating operation can construct the partially filled reachability matrix.

In this paper, we propose consensus formation models based on FISM with a comparison matrix. Two agent's strategies for consensus formation with an associated matrix are proposed, and some quantitative measures of the difference between the individual preferences and the consensus are analyzed.

II. Group Decision Making Support System

Group decision-making is performed after having repeated discussions in which an organization takes in opinions from different sources. This paper examines the problem of how to perform this process efficiently by computer-assisted group decision making. Consensus formation is one of the capabilities demanded for group decision-making support.

A. Basic models

In this section, a basic model of consensus formation is described.

An individual k has a matrix $M^{(k)}(k = 1, 2, \ldots, K)$ of the set of alternatives as his own mental model.

$$W = \{O_1, O_2, \ldots, O_N\} \quad (1)$$

$$M = \{M^{(k)} \mid k = 1, 2, \ldots, K\} \quad (2)$$

$$M^{(k)} = [m_{ij}^k](i \in W, j \in W) \quad (3)$$

$$= \begin{cases} 1 & O_i \succeq^{(k)} O_j \\ 0 & otherwise \end{cases}$$

W is a set of alternatives. M is a relational matrix whose (i, j)th element is represented as m_{ij}. The partial order $O_x \succ^{(k)} O_y$ means that an agent A_k prefers O_x over O_y. $M^{(k)}$ also is antisymmetric, reflexivity and transitivity. Therefore the matrix M is a reachability matrix which is satisfied the following conditions.

A reachability matrix is a square, reflexive, transitive, binary matrix M. Such a matrix M satisfies the following two conditions:

$$m_{ii} = 1 \quad (4)$$

$$m_{ij} \geq m_{ik} m_{kj} \quad (5)$$

In the process of consensus formation, group members compare their matrices, clarify areas of disagreement, and discuss important points, and then construct one consensus matrix. Therefore, consensus formation may be defined in the form of the following problems. "For a certain group pruducing possible arrival matrixes $M^{(1)}, M^{(2)}, \ldots, M^{(K)}$, group opinion reaches a consensus if it is $M^{(1)} = M^{(2)} = \cdots = M^{(K)}$", there is a difference of opinion in the group. The process is aimed at constructing one reachability matrix C under the consensus of a group".

B. Evaluation of a consensus

A matrix C provided by a consensus formation process becomes a reachability matrix, that is, the consensus model for the group. To evaluate the result of consensus negotiations

is to calculate the difference between the consensus model C and group matrices $M^{(1)}$, $M^{(2)}$, ..., $M^{(K)}$. As a result of consensus, the consensus degree FI and group consensus degree AFI are defined as demonstrating how well each person agrees with the group.

$$\begin{aligned} DI^{(k)} &= [di_{ij}^k] \quad (6) \\ di_{ij}^k &= m_{ij}^k - c_{ij} \\ FI^{(k)} &= 1 - \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} |di_{ij}^k| \quad (7) \end{aligned}$$

$$AFI(M, C) = \frac{1}{K} \sum_{k=1}^{K} FI^{(k)} \quad (8)$$

In addition, the number of iterations required for matrix C to known matrix is defined as the update count. The consensus arrival rate is defined as the update count divided by $N(N-1)$, which is the number of elements in the matrix excluding diagonal elements. Good consensus formation has a high group consensus degree AFI and a low update count.

III. A Consensus Process based on FISM

This section describes a consensus process based on FISM.

A. FISM

FISM is an expansion of ISM proposed by J. N. Warfield [1]. FISM is based on the theory of partially filled reachability matrix models. The partially filled reachability matrix is the expansion of the binary reachability matrix and has unknown elements. In this paper, this model is applied to consensus formation.

Relevant notations and definitions are given below.

B. Partially filled reachability matrix

A partially filled reflexive binary matrix is a matrix M whose elements m_{ij} contain either 1, 0, or an unknown value x_{ij}; all diagonal elements have the value 1.

To simplify the notation, we use "$m_{ij} = x$" for "m_{ij} is unknown".

If the partially filled reflexive binary matrix M is required to be a reachability matrix, it necessarily satisfies the following conditions:

Consistency Property: There exists no index triplet (i, j, k) such that

$$m_{ij} = 0, m_{ik} = 1, m_{kj} = 1. \quad (9)$$

Maximality Property: There exists no index triplet (i, j, k) such that

$$m_{ij} = x, m_{ik} = 1, m_{kj} = 1, \quad (10)$$
$$m_{ik} = 0, m_{ij} = x, m_{jk} = 1, \quad (11)$$
$$m_{kj} = 0, m_{ki} = 1, m_{ij} = x. \quad (12)$$

In other words, M is consistent if and only if(iff) it has no values that contradict transitivity, and M is maximal iff it has no unknowns that can be implied from transitivity.

In sum, a partially filled reachability matrix is a partially filled reflexive binary matrix that satisfies these consistency and maximality conditions.

C. Implication rules

Suppose that a value for an unknown element of the partially filled reachability matrix is updated. The value of some unknown elements can be implied from supplied values by applying the implication rules. There are three implication rules:

$1 \Rightarrow 1$ implication rule: Unknown element x_{lm} is implied to be 1 when x_{ij} is 1 and satisfies the following condition

$$m_{li} = m_{jm} = 1 \quad (13)$$

$0 \Rightarrow 0$ implication rule: Unknown element x_{lm} is implied to be 0 when x_{ij} is 0 and satisfies the following condition

$$m_{il} = m_{mj} = 1 \quad (14)$$

$1 \Rightarrow 0$implication rule(1) : Unknown element x_{lm} is implied to be 0 when x_{ij} is 1 and satisfies the following condition

$$m_{lj} = 0, m_{mi} = 1 \quad (15)$$

$1 \Rightarrow 0$implication rule(2) : Unknown element x_{lm} is implied to be 0 when x_{ij} is 1 and satisfies the following condition

$$m_{im} = 0, m_{jl} = 1 \quad (16)$$

D. Associated implication matrix model

An associated implication matrix Ψ shows a dependence expressed by connotation relations between each unknown element x_{ij} based on the partially filled reachability matrix M.

$$\begin{aligned} \Psi(x_{ij}) &= \{\psi_{11}(x_{ij}), \psi_{10}(x_{ij}), \psi_{00}(x_{ij})\} \quad (17) \\ \psi_{11}(x_{ij}) &= \{(p, q) \mid m_{pi} m_{jq} = 1\} \\ \psi_{10}(x_{ij}) &= \{(p, q) \mid \bar{m}_{iq} m_{jp} + m_{qi} \bar{m}_{pj} = 1\} \\ \psi_{00}(x_{ij}) &= \{(p, q) \mid m_{ip} m_{qj} = 1\} \end{aligned}$$

It is possible to show how an unknown element can imply the values of other unknown elements using this associated implication matrix. When an element (p, q) belonging to $\Psi(x_{ij})$ in all unknown elements x_{ij} is decided with a value of the most unknown elements, in partially filled reachability matrix M, the total number of unknown elements will decrease.

E. A consensus formation process

A consensus formation process using FISM is proposed as follows.

1) Construct a matrix $M^{(k)}$ of preference relations of an individual in a group

$$M = \{M^{(1)}, M^{(2)}, ..., M^{(K)}\} \quad (18)$$

2) Construct the consensus matrix C

$$c = [c_{ij}] \tag{19}$$
$$c_{ij} = \begin{cases} m_{ij}^{(1)} & (if\ m_{ij}^{(1)} = m_{ij}^{(2)} = \cdots = m_{ij}^{(K)}) \\ x & otherwise \end{cases}$$

3) Repeat the following steps until C becomes a known matrix
   a) Select an unknown element ($c_{ij} = x$)
   b) Input a value of 0 or 1 for c_{ij}
   c) Update the matrix C using implication rules and the value of c_{ij}
4) As a result, the matrix C becomes a consensus reachability matrix

The steps in the consensus formation process are explained as follows.

A matrix $M^{(k)}$ of the preference relations of an individual in the group is constructed. In that case, the matrix can be constructed efficiently and interactively using the implication rules of FISM.

The consensus matrix C can invent a group personal difference. If all the members agree, c_{ij} is a known element ($c_{ij} = \{0, 1\}$). If not, c_{ij} is an unknown element ($c_{ij} = x$).

In the consensus process, a consensus matrix C is generated and the value of unknown elements of C is decided by discussion in the group. The partially filled reachability matrix automatically infers the value of an unknown element based on values of other unknown elements of C. The updated matrix C is guaranteed to be a partially filled reachability matrix again.

The consensus process repeats until all unknown elements of matrix C become known. As a result, the matrix C becomes a reachability matrix expressing the consensus group preference.

A consensus process repeats $n(n-1)$ times at most. The computational complexity of updating the matrix with implication rules is $O(n^2)$, because the value of an arbitrary (l, m) element is decided from the value of an (i, j) element.

IV. CONSENSUS STRATEGY SIMULATION

In the consensus formation process, there is a problem with how individuals come to agreement on topics on which opinion was divided. This section examines the strategies for deciding the value of elements group discussion during consensus formation.

The following strategies must be considered for effectively simulating consensus formation.

1) A strategy for selecting unknown elements (i, j) of matrix C
2) A strategy for inputting a value for c_{ij}

These strategies can involeve several methods, and a numerical agreement strategy based on the combination of methods can be considered. In this paper, the strategies of using an associated implication matrix and deciding at random are proposed for choosing an (i, j) element of matrix C. In addition, strategies of random selection and majority rule are proposed as ways of inputting a value for c_{ij}.

In the following subsection, the effectiveness of consensus formation by the structural modeling method is examined using simulations.

A. Experiment 1

The simulation was run ten times for each of the four methods and each of the three values of N. The number of individuals in the group, K, is 10, and the number of alternatives, N, is 10, 20, or 30. To examine this method of consensus formation, the results of the simulations were compared.

1) Experimental conditions:

- Method 0: Select (i, j) elements of the consensus matrix C at random. Input values for c_{ij} at random.
- Consensus formation by search($S = 100, 1000, 10000$)

In this study, S different reachability matrices were constructed for consensus formation by searching, and the AFI from the matrix with the highest degree of agreement was used. Because it was difficult to generate a large number of reachability matrices A, FISM was used to construct the matrices according to the following procedure.

1) Set $B = \emptyset$
2) Repeat the follow step until $|B| = S$
   a) Construct a reachability matrix A.
   b) If $A \notin B$ then
      $B = B + A$

The method of constructing the reachability matrix A is as follows.

1) Construct an initial matrix that satisfies reflexivity, and otherwise contains unknown elements.
2) Repeat the following steps until the number of unknown elements in A becomes 0.
   a) Select an unknown element ($a_{ij} = x$) at random
   b) Choose at random the value of 1 or 0 for a_{ij}
   c) Update the matrix A using implication rules and the value of a_{ij}

An agreement strategy suggested in this study is compared with the method described above.

2) Experiment result: The results of experiment 1 are compared (Fig. 1, Table I).

The random results and the high-AFI results from the consensus formation by search were chosen at random to fill unknown (i, j) elements of matrix C.

In consensus formation by search, we can predict that there is a cost associated with increasing the number of candidate solutions to a number sufficient for attaining a high degree of agreement. However, for an effective search, the time required to generate candidate solutions becomes a problem.

B. Experiment 2

The simulation was run ten times for each of the four methods and each of the three values of N. The number of individuals in the group K, is 10, and the number of

TABLE I
RESULT 1(AFI)

	$N = 10$	$N = 20$	$N = 30$
	Ave. ± S. D.	Ave. ± S. D.	Ave. ± S. D.
Method 0	0.50 ± 0.0208	0.50 ± 0.0092	0.50 ± 0.0096
$S = 100$	0.62 ± 0.0113	0.62 ± 0.0089	0.63 ± 0.0086
$S = 1000$	0.64 ± 0.0163	0.64 ± 0.0065	0.64 ± 0.0062
$S = 10000$	0.65 ± 0.0139	0.64 ± 0.0062	0.65 ± 0.0061

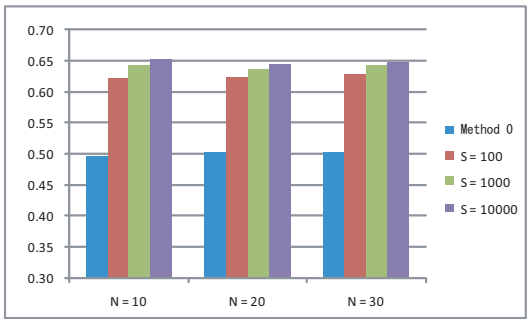

Fig. 1. Result1(AFI)

alternatives, N, is 10, 20, or 30. I compared the following three methods of consensus formation.

1) Experimental conditions:

1) Method 1: The opinion of one person, with many others in agreement, is assumed to become the dominant value of c_{ij} in a group.
2) Method 2: A probability is calculated by dividing the number of people in agreement with an opinion by the total number of people. The value of c_{ij} is decided based on the prevalence of the opinion.
3) Method 3: A value representing a minority opinion is assumed to be the value of c_{ij}, in contrast with method 1.

In addition, the location in matrix C of unknown elements (i, j) were chosen at random.

2) Experiment result: The resulting agreement degree AFI and consensus arrival rate for experiment 2 are shown (Table III, Fig. 2).

TABLE II
RESULT 2

	$N = 10$	$N = 20$	$N = 30$
	Ave. ± S. D.	Ave. ± S. D.	Ave. ± S. D.
Method 1	0.68 ± 0.0175	0.70 ± 0.0093	0.72 ± 0.0071
Method 2	0.62 ± 0.0149	0.65 ± 0.0138	0.66 ± 0.0113
Method 3	0.41 ± 0.0490	0.48 ± 0.0516	0.53 ± 0.0331
S = 10000	0.65 ± 0.0139	0.64 ± 0.0062	0.65 ± 0.0061

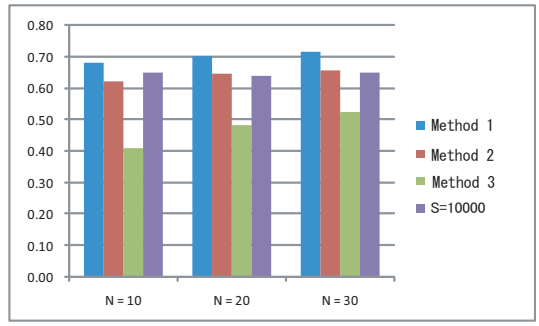

Fig. 2. Result 2(AFI)

Consensus formation by search ($S = 10,000$) yielded a better result than method 2 at $N = 10$, but method 2 performs better than the search at $N = 20$ and $N = 30$. This is because of, for the search, the search space increases as the number of choices increases, but the range of possible solutions does not change.

Also, method 1 achieved a higher score than any of the other methods or the search ($S = 10000$). Regarding the analysis model used in this article, it is thought that consensus formation by method 1 got the best result out of all the methods that showed good results so that the opinion of one person becomes the consensus. In addition, the number of points on which individuals must agree increases with the number of choices, which may be in evidence here (Table I).

TABLE III
RESULT 2 (CONSENSUS ARRIVAL RATE)

	$N = 10$	$N = 20$	$N = 30$
	Ave. ± S. D.	Ave. ± S. D.	Ave. ± S. D.
Method 1	0.63 ± 0.0857	0.50 ± 0.0407	0.46 ± 0.0269
Method 2	0.46 ± 0.0828	0.31 ± 0.0530	0.26 ± 0.0320
Method 3	0.21 ± 0.0323	0.13 ± 0.0177	0.09 ± 0.0095

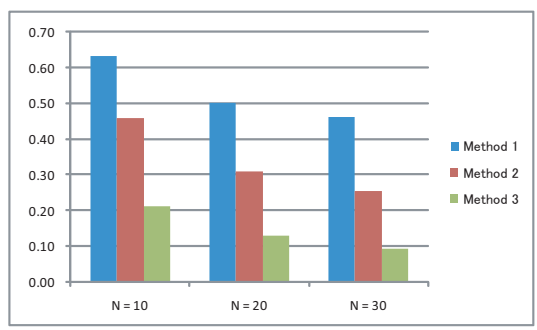

Fig. 3. Result 2 (Consensus arrival rate)

It is the value that broke the agreement number of times that the consensus arrival rate was provided with the number that the number of opposite angle ingredients is subtracted from square, and was found of choices. An agreement result will be arise early; therefore, the value of the consensus arrival rate, which reflects how long it takes to reach consensus, is small.

TABLE IV
RESULT 3(AFI, $n = 10$)

	Strategy A Ave. ± S.D.	Strategy B Ave. ± S.D.	Strategy C Ave. ± S.D.
100%(Random)	0.54 ± 0.04	0.61 ± 0.03	0.69 ± 0.02
90%	0.54 ± 0.04	0.60 ± 0.03	0.68 ± 0.02
80%	0.54 ± 0.04	0.60 ± 0.03	0.68 ± 0.02
70%	0.55 ± 0.04	0.60 ± 0.03	0.68 ± 0.02
60%	0.55 ± 0.04	0.61 ± 0.03	0.68 ± 0.02
50%	0.55 ± 0.04	0.61 ± 0.03	0.68 ± 0.02
40%	0.55 ± 0.04	0.61 ± 0.03	0.68 ± 0.02
30%	0.56 ± 0.04	0.62 ± 0.03	0.68 ± 0.02
20%	0.56 ± 0.04	0.62 ± 0.03	0.69 ± 0.02
10%	0.56 ± 0.04	0.62 ± 0.03	0.68 ± 0.02
0%	0.56 ± 0.04	0.62 ± 0.03	0.69 ± 0.02
Variance(Descending)	0.61 ± 0.04	0.63 ± 0.03	0.67 ± 0.04
variance(Ascending)	0.55 ± 0.03	0.62 ± 0.02	0.69 ± 0.02

The consensus arrival rate results were the opposite of agreement degree AFI results. Method 3 provided a better result than any of the other methods (Table II, Fig. 3). This may indicate that the values that an consensus is fixed at by progress, one consensus increase without an opinion of the majority being reflected. When the number of choices increases, the consensus arrival rate decreases. This may be due to an increase in strong connections between choices when the total number of choices increases; as a result, a higher number of values can be determined by implication at a single time.

C. Experiment 3

This experiment performs a comparison experiment by a choice strategy of a (i, j) element of line C and examines the effectiveness of a consensus formation model.

I assumed it number of population $K = 10$, the choices $N = 10$ in a group and tested it repeatedly in a $10,000$ time and compared inspection of a method of consensus formation equal to or less than it.

1) Experimental Conditions: A choice strategy of a (i, j) element of C is as follows.

1) A strategy that the values of a (i, j) a matrix C are decided by random numberin the first half, and decided by an associated implication matrix by the latter half.
2) Calculate variance of each one m_{ij}^k and choice a (i, j) element by descending order.
3) Calculate variance of each one m_{ij}^k and choice a (i, j) element by ascending order.

A strategy of deciding values of c_{ij} is as follows.

1) Strategy A: decide c_{ij} with a random number(Random)
2) Strategy B: decide c_{ij} stochastically with the distribution of m_{ij}^k(Probability)
3) Strategy C: decide c_{ij} as the most value with the distribution of m_{ij}^k(Majority Decision)

2) Experiment Result: A result of comparison in an experiment is shown(Table IV $\sim$ XII). A relation of AFI and consensus arrival rate is shown(Fig. 4$\sim$ 6).

I consider how alignment and agreement carry-over factor become it by a select method of a (i, j) factor of matrix C than experimental result. When one possible arrival matrix is generated than possible access line with little updating number of times in Structural Modeling method FISM for a

TABLE V
RESULT 3(AFI, $n = 20$)

	Strategy A Ave. ± S.D.	Strategy B Ave. ± S.D.	Strategy C Ave. ± S.D.
100%(Random)	0.59 ± 0.02	0.64 ± 0.02	0.70 ± 0.01
90%	0.59 ± 0.02	0.64 ± 0.02	0.70 ± 0.01
80%	0.59 ± 0.02	0.64 ± 0.02	0.70 ± 0.01
70%	0.59 ± 0.02	0.64 ± 0.02	0.70 ± 0.01
60%	0.58 ± 0.02	0.64 ± 0.02	0.70 ± 0.01
50%	0.58 ± 0.02	0.63 ± 0.02	0.70 ± 0.01
40%	0.58 ± 0.03	0.63 ± 0.02	0.70 ± 0.02
30%	0.58 ± 0.03	0.62 ± 0.02	0.70 ± 0.02
20%	0.57 ± 0.03	0.62 ± 0.02	0.70 ± 0.02
10%	0.57 ± 0.03	0.61 ± 0.02	0.70 ± 0.02
0%	0.56 ± 0.03	0.62 ± 0.02	0.70 ± 0.01
variance(Descending)	0.65 ± 0.02	0.67 ± 0.02	0.68 ± 0.03
variance(Ascending)	0.59 ± 0.02	0.65 ± 0.02	0.71 ± 0.01

TABLE VI
RESULT 3(AFI, $n = 30$)

	Strategy A Ave. ± S.D.	Strategy B Ave. ± S.D.	Strategy C Ave. ± S.D.
100%(Random)	0.61 ± 0.02	0.66 ± 0.01	0.72 ± 0.01
90%	0.61 ± 0.02	0.66 ± 0.01	0.72 ± 0.01
80%	0.61 ± 0.02	0.66 ± 0.01	0.71 ± 0.01
70%	0.61 ± 0.02	0.65 ± 0.01	0.71 ± 0.01
60%	0.61 ± 0.02	0.65 ± 0.02	0.71 ± 0.01
50%	0.60 ± 0.02	0.65 ± 0.02	0.71 ± 0.01
40%	0.60 ± 0.02	0.64 ± 0.02	0.71 ± 0.01
30%	0.60 ± 0.02	0.64 ± 0.02	0.71 ± 0.01
20%	0.59 ± 0.02	0.63 ± 0.02	0.71 ± 0.01
10%	0.59 ± 0.02	0.62 ± 0.02	0.71 ± 0.01
0%	0.58 ± 0.02	0.62 ± 0.02	0.71 ± 0.01
variance(Descending)	0.67 ± 0.01	0.68 ± 0.01	0.68 ± 0.03
variance(Ascending)	0.61 ± 0.01	0.67 ± 0.01	0.72 ± 0.01

TABLE VII
RESULT 3(PROCESSING TIME MS, $n = 10$)

	Strategy A	Strategy B	Strategy C
100%(Random)	3.594	3.658	3.784
90%	3.599	3.663	3.791
80%	3.627	3.706	3.864
70%	3.686	3.811	4.088
60%	3.788	3.977	4.478
50%	3.934	4.230	5.100
40%	4.141	4.589	6.013
30%	4.406	5.052	7.130
20%	4.966	5.977	8.995
10%	5.967	7.386	11.134
0%	8.050	9.492	12.741
variance(Descending)	3.697	3.714	3.722
variance(Ascending)	3.689	3.834	4.175

TABLE VIII
RESULT 3(PROCESSING TIME MS, $n = 20$)

	Strategy A	Strategy B	Strategy C
100%(Random)	33.445	34.225	36.074
90%	33.666	34.584	36.780
80%	34.648	36.347	41.680
70%	36.556	40.116	54.556
60%	39.361	46.024	79.014
50%	43.561	54.750	120.641
40%	49.814	67.831	187.731
30%	59.531	88.972	290.006
20%	77.561	125.225	441.758
10%	120.256	205.641	650.873
0%	239.933	355.306	841.924
variance(Descending)	35.075	35.180	35.275
variance(Ascending)	34.811	37.253	44.983

TABLE IX
RESULT 3(PROCESSING TIME MS, $n = 30$)

	Strategy A	Strategy B	Strategy C
100%(Random)	136.469	139.567	148.834
90%	138.523	143.278	157.450
80%	146.316	159.120	213.695
70%	161.223	191.913	378.016
60%	186.517	251.697	749.248
50%	226.734	345.723	1464.527
40%	280.238	473.119	2474.205
30%	357.986	656.972	3977.302
20%	495.177	975.075	6019.963
10%	818.861	1645.766	8279.391
0%	1728.644	2864.052	10614.355
variance(Descending)	144.663	145.013	145.541
variance(Ascending)	142.995	155.270	201.028

TABLE X
RESULT 2 (CONSENSUS ARRIVAL RATE, $n = 10$)

	Strategy A Ave. $\pm$ S.D.	Strategy B Ave. $\pm$ S.D.	Strategy C Ave. $\pm$ S.D.
100%(Random)	0.34 ± 0.06	0.45 ± 0.07	0.66 ± 0.08
90%	0.29 ± 0.06	0.41 ± 0.08	0.69 ± 0.09
80%	0.29 ± 0.06	0.40 ± 0.08	0.69 ± 0.10
70%	0.30 ± 0.06	0.41 ± 0.08	0.68 ± 0.10
60%	0.30 ± 0.06	0.41 ± 0.08	0.67 ± 0.09
50%	0.31 ± 0.06	0.42 ± 0.08	0.65 ± 0.09
40%	0.32 ± 0.06	0.42 ± 0.07	0.64 ± 0.09
30%	0.33 ± 0.06	0.43 ± 0.07	0.63 ± 0.08
20%	0.34 ± 0.06	0.43 ± 0.07	0.62 ± 0.08
10%	0.35 ± 0.06	0.44 ± 0.07	0.61 ± 0.08
0%	0.35 ± 0.06	0.44 ± 0.07	0.61 ± 0.08
variance(Descending)	0.38 ± 0.05	0.39 ± 0.05	0.41 ± 0.07
variance(Ascending)	0.37 ± 0.05	0.52 ± 0.07	0.90 ± 0.04

TABLE XI
RESULT 2 (CONSENSUS ARRIVAL RATE, $n = 20$)

	Strategy A Ave. $\pm$ S.D.	Strategy B Ave. $\pm$ S.D.	Strategy C Ave. $\pm$ S.D.
100%(Random)	0.22 ± 0.03	0.31 ± 0.04	0.51 ± 0.05
90%	0.22 ± 0.03	0.30 ± 0.04	0.51 ± 0.05
80%	0.21 ± 0.03	0.29 ± 0.04	0.52 ± 0.05
70%	0.20 ± 0.03	0.28 ± 0.04	0.53 ± 0.05
60%	0.19 ± 0.03	0.28 ± 0.04	0.55 ± 0.06
50%	0.18 ± 0.02	0.27 ± 0.04	0.57 ± 0.06
40%	0.17 ± 0.02	0.26 ± 0.04	0.59 ± 0.07
30%	0.16 ± 0.02	0.25 ± 0.04	0.61 ± 0.07
20%	0.16 ± 0.02	0.25 ± 0.04	0.62 ± 0.07
10%	0.16 ± 0.03	0.25 ± 0.04	0.62 ± 0.07
0%	0.20 ± 0.03	0.30 ± 0.04	0.60 ± 0.06
variance(Descending)	0.26 ± 0.03	0.28 ± 0.03	0.29 ± 0.04
variance(Ascending)	0.59 ± 0.02	0.39 ± 0.04	0.89 ± 0.03

TABLE XII
RESULT 2 (CONSENSUS ARRIVAL RATE, $n = 30$)

	Strategy A Ave. $\pm$ S.D.	Strategy B Ave. $\pm$ S.D.	Strategy C Ave. $\pm$ S.D.
100%(Random)	0.17 ± 0.02	0.25 ± 0.03	0.46 ± 0.04
90%	0.17 ± 0.02	0.24 ± 0.03	0.47 ± 0.04
80%	0.15 ± 0.02	0.23 ± 0.03	0.48 ± 0.04
70%	0.14 ± 0.02	0.22 ± 0.03	0.49 ± 0.04
60%	0.14 ± 0.02	0.21 ± 0.03	0.51 ± 0.05
50%	0.13 ± 0.02	0.20 ± 0.03	0.53 ± 0.05
40%	0.12 ± 0.01	0.20 ± 0.03	0.55 ± 0.06
30%	0.12 ± 0.01	0.19 ± 0.03	0.58 ± 0.06
20%	0.11 ± 0.01	0.18 ± 0.03	0.59 ± 0.06
10%	0.11 ± 0.02	0.19 ± 0.03	0.59 ± 0.07
0%	0.14 ± 0.02	0.23 ± 0.03	0.58 ± 0.06
variance(Descending)	0.22 ± 0.02	0.23 ± 0.02	0.24 ± 0.03
variance(Ascending)	0.21 ± 0.02	0.34 ± 0.03	0.88 ± 0.02

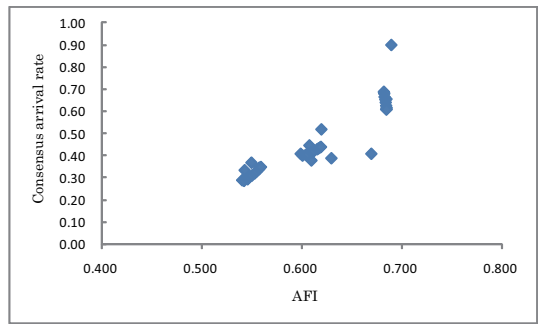

Fig. 4. Result 3 (Scatter Diagram, $n = 10$)

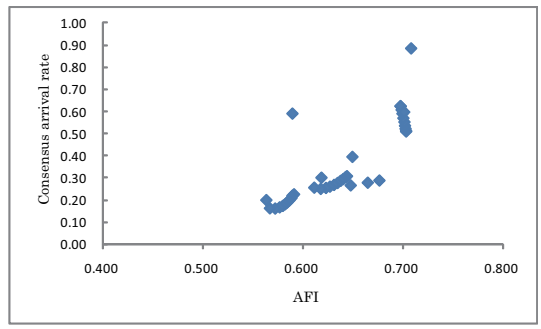

Fig. 5. Result 3 (Scatter Diagram, $n = 20$)

department, Method employing associated implication matrix is effective.

In Table VII,VIII,IX, About processing time, as for the Method which employs associated implication matrix more when only variance and a random number are used, processing time increases to generate associated implication matrix every post of matrix C.

According to Table X,XI,XII, the strategy how decision strategy of value used random strategy and probability for can be brought in consensus in 20% little update frequency for an unknown element number of initial condition. When use ab initio associated implication matrix, and perform consensus Process in the case of the strategy how used majority decision for, there is little it; can bring it in consensus in update frequency.

In Table IV,V,VI, AFI is not seen in difference by as-

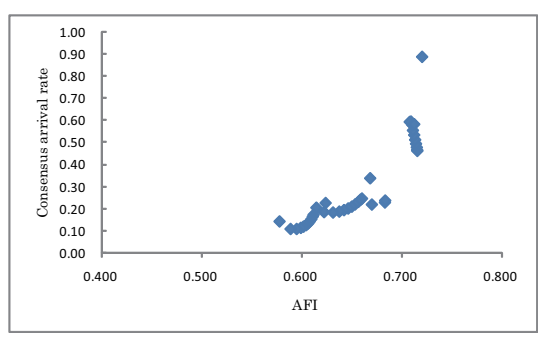

Fig. 6. Result 3 (Scatter Diagram, $n = 30$)

sociated implication matrix. Therefore, associated implication matrix is effective in reduction of update number of times.

In Fig. 4,5,6, AFI and update number of times are relationships of trade-off. Is the best, putting it together determine a select of a (i, j) element of comparing matrix C by descending of variance generally, and it can be inferred when it is Method performing Decision of c_{ij} in majority decision.

It is thought that application to the consensus formation support that used FISM than this experimental result rose. For example, it is thought that the suggestion which which Conflict part should have been discussed than can be exhibited so that agreement degree in group finally enhances rising possibility when a group performs consensus formation by consultation for direct interaction. In addition, group can exhibit various consensus results by the Simulation which various strategy was used for if each individual Preference relationship is decided beforehand. The consensus result is taken into account without group conferring directly, and it is thought that individual Preference-related flexible modification is possible. Disputation of an application method to real consensus formation will become learning activity in future.

V. Conclusion

In this article, we suggest a group decision-making support model that uses the structural modeling method, FISM. We performed simulations, and examined the relationship between individual preferences and the group consensus. Consensus formation by fuzzy models of choice good relations is suggested for future study.

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