<table>
<thead>
<tr>
<th>Title</th>
<th>Design and analysis of large-effective-area heterogeneous trench-assisted multi-core fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Tu, Jiajing; Saitoh, Kunimasa; Koshiba, Masanori; Takenaga, Katsuhiro; Matsuo, Shoichiro</td>
</tr>
<tr>
<td>Citation</td>
<td>Optics Express, 20(14): 15157-15170</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2012-07-02</td>
</tr>
<tr>
<td>Doc URL</td>
<td><a href="http://hdl.handle.net/2115/49740">http://hdl.handle.net/2115/49740</a></td>
</tr>
<tr>
<td>Rights</td>
<td>©2012 Optical Society of America</td>
</tr>
<tr>
<td>Type</td>
<td>article</td>
</tr>
<tr>
<td>File Information</td>
<td>OE20-14_15157-15170.pdf</td>
</tr>
</tbody>
</table>

Hokkaido University Collection of Scholarly and Academic Papers: HUSCAP
Design and analysis of large-effective-area heterogeneous trench-assisted multi-core fiber

Jiajing Tu, Kunimasa Saitoh, Masanori Koshiba, Katsushi Takenaga, and Shoichiro Matsuo

1Division of Media and Network Technologies, Hokkaido University, Sapporo 060-0814, Japan
2Optics and Electronics Laboratory, Fujikura Ltd., Sakura 285-8550, Japan

*Corresponding author:
tujiajing@icp.ist.hokudai.ac.jp

Abstract: Based on the overlap integral of electromagnetic fields in neighboring cores, a calculating method is proposed for obtaining the coupling coefficient between two adjacent trench-assisted non-identical cores. And a kind of heterogeneous trench-assisted multi-core fiber (Hetero-TA-MCF) with 12 cores is proposed to achieve large effective area ($A_{\text{eff}}$) and high density of cores. As bending radius becomes larger than 50 mm, the crosstalk value at 1550-nm wavelength of the Hetero-TA-MCF is about $-42$ dB after 100-km propagation and the $A_{\text{eff}}$ of this Hetero-TA-MCF can reach 100 $\mu$m$^2$.

OCIS codes: (060.2270) Fiber characterization; (060.2280) Fiber design and fabrication.

References and links

1. Introduction

How to cope with the exponentially increasing demand for transmission capacity per fiber is a hot topic nowadays. As an approach to achieve space division multiplexing (SDM), multi-core fiber (MCF) has been proposed to solve the issue related above [1].

Recently, several kinds of homogeneous MCFs (Homo-MCFs) in which all the cores are identical to each other have been designed and fabricated in order to realize long-haul transmission with low crosstalk [2, 3]. Furthermore, a type of optical fiber called heterogeneous MCF (Hetero-MCF) has been proposed to obtain much lower crosstalk, in which there are not only identical cores but also non-identical cores and the cores are more closely packed in a definite space [4]. On the other hand, a trench-assisted MCF (TA-MCF) that realizes much smaller crosstalk and larger effective area ($A_{eff}$) comparing to MCF with step-index profile also has been proposed [5]. It has been proved that the crosstalk between the identical cores will become larger and larger as bending radius ($R$) increases [6]. However the Hetero-MCF is insensitive to the bending radius after the $R$ reaching a threshold value which was called $R_{pk}$ in [7]. Moreover, if the cores have slight differences in their core radii and core refractive indices, the maximum power transferred between the cores goes down drastically [4]. Therefore, Hetero-MCF is a good candidate for the research of fiber under the bending condition. In addition, if we want to accommodate more cores inside the fiber, the core pitch between each pair of cores needs to be reduced. But small core pitch will result in a large crosstalk between cores. So in order to lower the crosstalk and meanwhile increase the core number in the fiber, a Hetero-MCF which has an index trench structure around each core (Hetero-TA-MCF) can be a solution.

In this paper, we propose a Hetero-TA-MCF with high density of cores to realize a large-scale SDM transmission. Besides increasing the number of cores, we also aim at achieving low crosstalk and enlarging the $A_{eff}$ as much as possible. Additionally, when we analyze the crosstalk characteristics between a couple of non-identical cores with trench regions, we should know the coupling coefficient ($\kappa$) between them. However, to the best of our knowledge, an analytical method which can be used to figure out the value of $\kappa$ between two trench-assisted non-identical cores has not been reported yet. Therefore, before proposing a new type of Hetero-TA-MCF and analyzing the characteristics of it, we will introduce an effective method for calculating the $\kappa$ between two trench-assisted non-identical cores by considering the overlap integral of electromagnetic fields in the neighboring cores. Then we will compare such method with finite element method (FEM) [8] by calculating the crosstalk of Homo-TA-MCF in order to estimate the accuracy degree of it. Based on this calculating method of $\kappa$, an optimal design condition for the Hetero-TA-MCF will be given.

2. Calculation of $\kappa$ between trench-assisted non-identical cores

2.1 Analysis of refractive-index distribution in the coupled region

Figure 1 shows the refractive index profile of two trench-assisted non-identical cores and the part outside the cores. The refractive-index distribution in the entire coupled region can be written as

$$N^2(r, \theta) = N_1^2(r, \theta) + N_2^2(r, \theta) - n^2(r, \theta),$$

where $N_1(r, \theta)$ and $N_2(r, \theta)$ represent the refractive-index distribution of each core with trench structure, and $n(r, \theta)$ means the refractive-index distribution outside the cores, which are shown in Fig. 1(a), Fig. 1(b), and Fig. 1(c), respectively. The expression of coupling coefficient between two waveguides is expressed as [9]
\[ \kappa_{pq} = \frac{\omega \varepsilon_0}{4P} \int_0^{2\pi} \int_0^{\alpha_{1p}} (n_p^2 - n_{cl}^2) \mathbf{E}_p \cdot \mathbf{E}_q \, rdrd\theta, \]  

(2)

where \( \omega \) is an angular frequency of the sinusoidally varying electromagnetic fields, \( \varepsilon_0 \) is the permittivity of the medium, and \( \mathbf{n}_r \) means the outward-directed unit vector. The pair of \( p \) and \( q \) is either \((1, 2)\) or \((2, 1)\). \( \mathbf{E} \) and \( \mathbf{H} \) represent the electric and magnetic fields respectively.

As shown in Fig. 2(a), \( N_2^2 - N_2^2 \) is zero except the region inside core 1, so the difference of the refractive-index distribution inside the core 1 is \( n_1^2 - n_{cl}^2 \), while according to Fig. 2(b), \( N_2^2 - N_1^2 \) is zero except the region inside core 2, so the difference of the refractive-index distribution inside the core 2 is \( n_2^2 - n_{cl}^2 \). Furthermore, the denominator of \( \kappa_{pq} \) equals \( 4P \), where \( P \) means the total power flow [9]. So the expression of \( \kappa_{pq} \) can be rewritten as

\[ \kappa_{pq} = \frac{\omega \varepsilon_0}{4P} \int_0^{2\pi} \int_0^{\alpha_{1p}} (n_p^2 - n_{cl}^2) \mathbf{E}_p \cdot \mathbf{E}_q \, rdrd\theta, \]  

(3)

where \( \mathbf{E}_p \) and \( \mathbf{E}_q \) represent the amplitude of electric field distribution of core \( p \) inside the range of core \( p \), and the amplitude of electric field distribution of core \( q \) inside the range of core \( p \), respectively. \( \alpha_{1p} \) denotes the radius of core \( p \).
2.2 Derivation of the expression of $\kappa$ between trench-assisted non-identical cores

The electric fields in optical fibers are expressed in cylindrical coordinates as

$$\tilde{E} = E(r, \theta)e^{j(\omega t - \beta z)}.$$  \hspace{1cm} (4)

Substituting Eq. (4) into Maxwell’s equation, we can obtain the wave equation as

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \theta^2} + [k^2 n(r, \theta)^2 - \beta^2] E_z = 0.$$  \hspace{1cm} (5)

Here, we define the wave number in the core $m$ ($m = 1, 2$), cladding and trench along the transversal direction as follows:

$$\xi_m = \sqrt{n_m^2 k^2 - \beta_m^2},$$  \hspace{1cm} (6)

$$\sigma_m = \sqrt{\beta_m^2 - n_\text{cl}^2 k^2},$$  \hspace{1cm} (7)

$$\gamma_m = \sqrt{\beta_m^2 - n_\text{t}^2 k^2},$$  \hspace{1cm} (8)

where $\beta$ is the propagation constant and $k$ is the wavenumber in a vacuum. The normalized frequency ($V_{1,m}$, $V_{2,m}$), the normalized transverse wave number in core $m$ ($U_{1,m}$), that in cladding ($W_{1,m}$) and that in trench ($W_{2,m}$) can be expressed as follows:

$$V_{1,m} = a_{1,m} k \sqrt{n_m^2 - n_\text{cl}^2},$$  \hspace{1cm} (9)

$$V_{2,m} = a_{1,m} k \sqrt{n_\text{cl}^2 - n_\text{t}^2},$$  \hspace{1cm} (10)

$$W_{1,m} = a_{1,m} \sigma_m = 1.1428 V_{1,m} - 0.996,$$  \hspace{1cm} (11)

$$U_{1,m} = a_{1,m} \xi_m = \sqrt{V_{1,m}^2 - W_{1,m}^2},$$  \hspace{1cm} (12)
\[ W_{2-m} = a_{1-m} \gamma_m = \sqrt{W_{2-m}^2 + W_{1-m}^2}, \]  

where \( a_{1-m} \) is the radius of core \( m \) and Eq. (11) is the approximation which is in error by less than 0.2 percent for \( 1.5 \leq V_{1-m} \leq 2.5 \) [10].

In the core region of fiber, the solutions for Eq. (5) of TM modes are the 0th-order Bessel function \( J_0(\xi m r_m) \) and Eq. (11) is the approximation which is in error by less than 0.2 percent for \( 1.5 \leq V_{1-m} \leq 2.5 \) [10].

In the core region of fiber, the solutions for Eq. (5) of TM modes are the 0th-order Bessel function \( J_0(\xi m r_m) \) and the 0th-order Neumann function \( N_0(\xi m r_m) \) [9], respectively. However, \( N_0(\xi m r_m) \) diverges infinitely at \( r = 0 \). Therefore \( J_0(\xi m r_m) \) is the proper solution for the field in the core. In the cladding region of fiber, the solutions for Eq. (5) of TM modes are the modified Bessel function of the first kind \( I_0(\sigma m r_m) \) and modified Bessel functions of the second kind \( K_0(\sigma m r_m) \), respectively. However, \( I_0(\sigma m r_m) \) diverges infinitely at \( r = \infty \). Therefore \( K_0(\sigma m r_m) \) is the proper solution for the field in the cladding. In the same principle, \( K_0(\sigma m r_m) \) is the proper solution for the field in the trench. Nevertheless, in hybrid modes, the solutions for Eq. (5) are given by the product of the \( n \)th-order Bessel functions and \( \cos(n \theta + \psi) \). Thus, the z-components of the electric field can be obtained as

\[
E_{z,i} = \begin{cases} 
A_n J_n(\xi m r_m) \cos(n \theta + \psi) & \text{in Co } m \\
B_n K_n(\sigma m r_m) \cos(n \theta + \psi) & \text{in IC } m \\
C_n K_n(\gamma m r_m) \cos(n \theta + \psi) & \text{in Tr } m \\
D_n K_n(\sigma m r_m) \cos(n \theta + \psi) & \text{in OC} \\
E_n K_n(\gamma m r_m) \cos(n \theta + \psi) & \text{in Tr } m' \\
F_n K_n(\sigma m r_m) \cos(n \theta + \psi) & \text{in IC } m' 
\end{cases}
\]

(14)

the azimuthal dependency of the electric fields in axially symmetric fibers is expressed by \( \cos(n \theta + \psi) \), where \( n \) is an integer and \( \psi \) denotes the phase. As shown in Fig. 3, core \( m \), inner cladding between core \( m \) and trench \( m \), trench \( m \), outer cladding outside trench \( m \), trench \( m' \) and inner cladding inside trench \( m' \) are abbreviated as Co \( m \), IC \( m \), Tr \( m \), OC, Tr \( m' \), and IC \( m' \). \( D \) is the core pitch, \( a_{1-m} \) is the radius of core \( m \), \( a_{2-m} \) is the distance from the center of core \( m \) to the inner circumference of trench \( m \), and \( a_{3-m} \) is the distance from the center of core \( m \) to the outer circumference of trench \( m \). \( R, R_1 \) and \( R_2 \) denote the distance between the center of core \( m \) to the objective point in IC \( m' \), the distance from the center of core \( m \) to the outer circumference of trench \( m \) and the distance from the center of core \( m \) to the inner circumference of trench \( m' \), respectively.

![Fig. 3. The profile of core m with trench structures.](image)

Without any doubt, the electric field should be continuous in each boundary, therefore the solutions in these six parts have the relationship which is shown as follows:
Based on the boundary condition related above, the expression of $D_m$ and $F_m$ can be obtained, which are shown as follows:

$$D_m = L_m A_m,$$

$$F_m = Q_m A_m,$$

where

$$L_m = \frac{J_n(U_{1-m})K_n(W_{1-m})}{a_{1-m}} \frac{\alpha_{2-m}}{a_{1-m}} \frac{\alpha_{3-m}}{a_{1-m}},$$

$$Q_m = \frac{\alpha_{R_1}}{a_{1-m}} \frac{\alpha_{R_2}}{a_{1-m}}.$$
Here

\[
E_{q} = \begin{cases} 
E_{q_1} = -jQ_q A_q A_{q_1} \beta_q \alpha_{1-q} K_q (W_{1-q} \frac{R}{a_{1-q}}) \cos \psi \\
E_{q_2} = jQ_q A_q A_{q_1} \beta_q \alpha_{1-q} K_q (W_{1-q} \frac{R}{a_{1-q}}) \sin \psi \\
E_{q_3} = Q_q A_q A_{q_1} K_q (W_{1-q} \frac{R}{a_{1-q}}) \cos (\Theta + \psi) 
\end{cases} \quad (22)
\]

where

\[
R = \sqrt{D^2 + r^2 - 2Dr \cos \theta} \approx D - r \cos \theta ,
\]

\[
a_{1-p} = \sqrt{D^2 + R_1^2 - 2DR \cos (\pi - \Theta)} \approx D - R_1 \cos (\pi - \Theta) ,
\]

\[
r = \sqrt{D^2 + R^2 - 2DR \cos (\pi - \Theta)} \approx D - R \cos (\pi - \Theta) ,
\]

\[
\Rightarrow R_1 \approx \frac{(D - a_{1-p})(D - r \cos \theta)}{D - r} .
\]

In the same principle,

\[
R_2 \approx \frac{(D - a_{2-p})(D - r \cos \theta)}{D - r} .
\]

Inside Eq. (3), \( \mathbf{E}_p \cdot \mathbf{E}_q \) can be expressed as

\[
\mathbf{E}_p \cdot \mathbf{E}_q = Q_q A_q A_{q_1} \beta_q \alpha_{1-q} J_q (U_{1-p} \frac{r}{a_{1-p}})K_q (W_{1-q} \frac{R}{a_{1-q}}) \\
+ Q_q A_q A_{q_1} J_1 (U_{1-p} \frac{r}{a_{1-p}})K_q (W_{1-q} \frac{R}{a_{1-q}}) \cos (\Theta + \psi) \cos (\Theta + \psi) .
\]

Inside Eq. (3), \( \mathbf{E}_p \cdot \mathbf{E}_q \) can be expressed as

\[
\mathbf{E}_p \cdot \mathbf{E}_q = Q_q A_q A_{q_1} \beta_q \alpha_{1-q} J_q (U_{1-p} \frac{r}{a_{1-p}})K_q (W_{1-q} \frac{R}{a_{1-q}}) \\
+ Q_q A_q A_{q_1} J_1 (U_{1-p} \frac{r}{a_{1-p}})K_q (W_{1-q} \frac{R}{a_{1-q}}) \cos (\Theta + \psi) \cos (\Theta + \psi) .
\]
Since the second term of the right-hand side of the equation above is sufficiently smaller than the first term, the integration of the first part in the square brackets of Eq. (3) becomes:

\[
S_1 = \int_0^{2\pi} \int_0^{z_{eq}} (n_p^2 - n_{cl}^2) E_p \cdot E_r dr \theta = (n_p^2 - n_{cl}^2) L_q A_p A_q \beta_p \beta_q \frac{a_{1-p}^2 a_{1-q}^2}{U_{1-p} U_{1-q}}
\]

\[
\times \int_0^{2\pi} \int_0^{z_{eq}} J_0(U_{1-p} \frac{r}{a_{1-p}}) K_0(W_{1-q} \frac{R}{a_{1-q}}) \frac{R}{a_{1-q}} K_1(P_1 D - r \cos \theta) K_1(Q_1 D - r \cos \theta) n \pi d \theta
\]

where \( P_1 = W_{1-q} (D - a_{1-p}) / a_{1-q} \), \( P_2 = W_{2-q} (D - a_{2-p}) / a_{2-q} \), \( Q_1 = W_{2-q} (D - a_{3-p}) / a_{3-q} \).

When the argument of the modified Bessel function \( K_n(z) \) in Eq. (29) is large, it can be approximated as

\[
K_n(z) \approx \sqrt{2 \pi z} \exp(-z).
\]

Substitution of Eq. (30) into Eq. (29) gives

\[
S_1 = (n_p^2 - n_{cl}^2) L_q A_p A_q \beta_p \beta_q \frac{a_{1-p}^2 a_{1-q}^2}{U_{1-p} U_{1-q}} \sqrt{\frac{\pi a_{1-q}}{2 W_{1-q} D}} \exp(-W_{1-q} D / a_{1-q})
\]

\[
\times \int_0^{2\pi} \int_0^{z_{eq}} J_0(U_{1-p} \frac{r}{a_{1-p}}) \exp(P_2 P_1 + Y_2 Y_1) \frac{D}{D - r} \int_0^{2\pi} \exp\left(\frac{W_{1-q}}{a_{1-q}} - P_2 - P_1 + Y_2 - Y_1\right) r \cos \theta dr d \theta.
\]

By using the integral formulas of the Bessel functions [11]:

\[
I_0(z) = \frac{\pi}{\theta} \exp(z \cos \theta) d \theta,
\]

the coupling coefficient \( \kappa_{pq} \) can be expressed as

\[
\kappa_{pq} = \left\{ n_p^2 - n_{cl}^2 \right\} L_q A_p A_q \beta_p \beta_q \frac{a_{1-p}^2 a_{1-q}^2}{U_{1-p} U_{1-q}} \sqrt{\frac{\pi a_{1-q}}{2 W_{1-q} D}} \exp(-W_{1-q} D / a_{1-q})
\]

\[
\times \int_0^{2\pi} \int_0^{z_{eq}} J_0(U_{1-p} \frac{r}{a_{1-p}}) \left[ \frac{W_{1-q}}{a_{1-q}} - P_2 - P_1 + Y_2 - Y_1\right] r \exp\left(\frac{W_{1-q}}{a_{1-q}} - P_2 - P_1 + Y_2 - Y_1\right) r dr d \theta.
\]

2.3 Comparison with finite element method (FEM)

In order to estimate the accuracy degree of this method, we compare it with FEM by calculating the coupling coefficient between adjacent cores in Homo-TA-MCF. Here we use the Homo-TA-7-core fiber as a model to do this comparison. Figure 5 shows index profile of a core with trench and the schematic of the Homo-TA-7-core model. The coiling diameter of the fiber was assumed to be 210 mm, which is the same with the value in [5]. The parameters which we used are summarized in Table 1.
Fig. 5. Schematic of a core with index trench and Homo-TA-7-core model.

Table 1. Structural Parameters for Calculation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Trench</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>[µm]</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>$r_2/r_1$</td>
<td>-</td>
<td>2.00</td>
<td>-</td>
</tr>
<tr>
<td>$r_3/r_1$</td>
<td>-</td>
<td>3.00</td>
<td>-</td>
</tr>
<tr>
<td>$W/r_1$</td>
<td>-</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta_1$</td>
<td>[%]</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td>$\Delta_2$</td>
<td>[%]</td>
<td>-0.65</td>
<td>-</td>
</tr>
<tr>
<td>$n_d$</td>
<td>-</td>
<td>1.45</td>
<td>1.45</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>[µm]</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$R$</td>
<td>[mm]</td>
<td>105</td>
<td>105</td>
</tr>
</tbody>
</table>

Fig. 6. Simulated crosstalk at 1550-nm wavelength as function of length.

Figure 6 illustrates the length dependence of simulated crosstalk ($X_T$) of step-index MCF (S-MCF) and trench-assisted MCF (TA-MCF) at 1550-nm wavelength. The blue solid line relates the simulation result of S-MCF. On the other hand, the red solid line represents the result of TA-MCF which is calculated by using the above-mentioned method and the green solid line represents the result of TA-MCF that is obtained by using the FEM [8]. The error between crosstalk values which were calculated by these two methods is about $-0.4$ dB, a sufficiently small value, which proves the feasibility of this analytical method.

3. Model of Hetero-TA-MCF with high density of cores

3.1 Design of Hetero-TA-MCF

Figure 7 shows the schematic of Hetero-TA-12-core model. The index profile of the core with trench of this structure is the same with the one shown in Fig. 5. In the simulation of this work, we assumed two sorts of cores which were represented by two different colors — orange and green separately. The calculated wavelength was set to be 1550 nm, while relative
refractive-index difference between $n_{cl}$ and $n_{tr}$ ($\Delta n_2$) was assumed to be $-0.7\%$, which is the structural parameter used in the fabricated TA-MCFs [5]. And we define the distance between the outer circumferences of adjacent trenches ($D_{tr}$) as a value that is not smaller than 2 $\mu$m for making sure the trench not overlap to each other.

The reason why we design such Hetero-TA-MCF with a ring layout is that the cutoff wavelength of each core will not be very long in this case. As a result, we can ensure a wide wavelength band which is used for the single-mode transmission. The cutoff wavelength related here is defined as the wavelength at which the bending loss of LP$_{11}$ mode equals to 1 dB/m at the bending radius of 140 mm. If we arrange several layers inside the fiber, the cutoff wavelength of the cores at the center part will have a longer cutoff wavelength than that of the cores in the outer layer. This is because the tight confinement of the LP$_{11}$ mode of the cores at the center will result in a long cutoff wavelength when index trench structures are deployed [5].

In addition, if we expect the $A_{eff}$ to reach 110 $\mu$m$^2$ or 80 $\mu$m$^2$, the outer cladding thickness (OCT) needs to be at least 40 $\mu$m [12] or 30 $\mu$m [13], respectively, for reducing the micro-bending loss. And if we want to decrease the failure probability of a fiber in order to guarantee the mechanical reliability, the cladding diameter (CD) should not be larger than 200 $\mu$m [14]. So based on the linear relationship of $A_{eff}$ and OCT, the OCT should be at least 37 $\mu$m when the $A_{eff}$ equals 100 $\mu$m$^2$. Thus, if we set the maximum of the CD to be around 200 $\mu$m, the limit value of $A_{eff}$ equals 100 $\mu$m$^2$. In this case, the core number of 12 here is the limit value for this ring layout due to the required value of CD.

### 3.2 Crosstalk characteristics

In Hetero-MCFs, there is a threshold value of $R$ which was proposed as $R_{pk}$ [7]. The crosstalk is degraded at $R < R_{pk}$ due to the phase-matching between non-identical cores. In this phase-matching region, the bend perturbations are crucial. In the non-phase-matching region of $R > R_{pk}$, on the other hand, the crosstalk is dominated by the statistical properties [15]. Therefore, a large effective index difference ($\Delta n_{eff}$) between cores will be required for pushing the value of $R_{pk}$ toward sufficiently small range. Figure 8 shows the required $\Delta n_{eff}$ value between two non-identical cores as function of $\Lambda$ and $R_{pk}$. If we try to shift $R_{pk}$ to $R$ of smaller than 5 cm with $\Lambda$ being about 33 $\mu$m, $\Delta n_{eff}$ should be around 0.001 or larger than it between non-identical cores.
Figure 9 illustrates the effective index value ($n_{\text{eff}}$) of the fundamental mode at 1550-nm wavelength as function of core radius and core $\Delta_1$ in four conditions — (a) $r_2/r_1 = 2.0$, $W/r_1 = 1.0$, (b) $r_2/r_1 = 2.0$, $W/r_1 = 1.1$, (c) $r_2/r_1 = 2.0$, $W/r_1 = 1.2$, (d) $r_2/r_1 = 2.0$, $W/r_1 = 1.3$, where $n_{\text{eff}}$ was simulated by FEM. The black solid lines and the black dashed lines represent the values of $n_{\text{eff}}$ and $A_{\text{eff}}$, respectively. The couple of white solid lines correspond to the upper limit of bending loss ($BL$) of the higher-order mode (HOM) at 1530 nm when $R$ equals 140 mm and the lower limit of bending loss ($BL$) of the fundamental mode (FM) at 1625 nm when $R$ equals 30 mm. To define the single-mode operation, the bending loss of LP$_{11}$-like HOM should be $> 1$ dB/m at $R = 140$ mm [14] and we assume the limit value of the bending loss of FM to be 0.5 dB/100 turns at $R = 30$ mm, which is described in ITU-T recommendations G.655 and G.656. Therefore, in order to guarantee the transmission with single-mode operation and low bending loss from C-band to L-band, we only research the field that is surrounded by the couple of white solid lines.

Here, we fix $r_2/r_1$ to be 2.0 and change $W/r_1$ from 1.0 to 1.3 to investigate how the width of trench region influences the crosstalk between the neighboring cores. In the conditions (a), (b), (c), and (d), we do not set the target value of $A_{\text{eff}}$ to be 110 $\mu$m$^2$, because the core radii of cores are too large to ensure the required value of CD. So we select four pairs of cores with $A_{\text{eff}}$ equaling 100 $\mu$m$^2$ for the simulation in conditions (a), (b), (c), and (d). In Fig. 9(a), one of the cores has a radius of 5.34 $\mu$m and $\Delta_1 = 0.304\%$ and another core has a radius of 4.86 $\mu$m and $\Delta_1 = 0.242\%$. In Fig. 9(b), one of the cores has a radius of 5.27 $\mu$m and $\Delta_1 = 0.293\%$ and another core has a radius of 4.83 $\mu$m and $\Delta_1 = 0.239\%$. In Fig. 9(c), one of the cores has a radius of 5.18 $\mu$m and $\Delta_1 = 0.280\%$ and another core has a radius of 4.77 $\mu$m and $\Delta_1 = 0.234\%$. In Fig. 9(d), one of the cores has a radius of 5.03 $\mu$m and $\Delta_1 = 0.263\%$ and another core has a radius of 4.75 $\mu$m and $\Delta_1 = 0.240\%$. Furthermore, in order to make sure the trench not overlap to each other, we defined $D_t$ to be not smaller than 2 $\mu$m. Under this requirement, we set each required $\Lambda$. The optical properties of the cores in each condition are summarized in Table 2.
Fig. 9. Effective index value of the fundamental mode as function of core radius and core \( \Delta_1 \), where (a) \( r_2/r_1 = 2.0, W/r_1 = 1.0 \), (b) \( r_2/r_1 = 2.0, W/r_1 = 1.1 \), (c) \( r_2/r_1 = 2.0, W/r_1 = 1.2 \), and (d) \( r_2/r_1 = 2.0, W/r_1 = 1.3 \).

Table 2. Optical Properties of the Cores in Different Conditions (1550 nm)

<table>
<thead>
<tr>
<th>Item</th>
<th>( A_{\text{eff}} )</th>
<th>( \Delta n_{\text{eff}} )</th>
<th>( \Lambda )</th>
<th>( CD )</th>
<th>RCMF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit ( \mu m^2 )</td>
<td>- ( \mu m )</td>
<td>( \mu m )</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_2/r_1 = 2.0, W/r_1 = 1.0 )</td>
<td>100</td>
<td>0.001</td>
<td>33</td>
<td>201.5</td>
<td>5.8</td>
</tr>
<tr>
<td>( r_2/r_1 = 2.0, W/r_1 = 1.1 )</td>
<td>100</td>
<td>0.0009</td>
<td>34</td>
<td>205.4</td>
<td>5.6</td>
</tr>
<tr>
<td>( r_2/r_1 = 2.0, W/r_1 = 1.2 )</td>
<td>100</td>
<td>0.0007</td>
<td>34</td>
<td>205.4</td>
<td>5.6</td>
</tr>
<tr>
<td>( r_2/r_1 = 2.0, W/r_1 = 1.3 )</td>
<td>100</td>
<td>0.0005</td>
<td>35</td>
<td>209.2</td>
<td>5.4</td>
</tr>
</tbody>
</table>

In Table 2, the relative value of core multiplicity factor (RCMF) was used to compare the core density of fibers [12]. And the core multiplicity factor (CMF) is defined as

\[
CMF = \frac{N_{\text{core}}A_{\text{eff}}}{\pi (CD/2)^3},
\]

(34)
where $N_{\text{core}}$ is a number of core and $CD$ means a cladding diameter. And $RCMF$ is a ratio between $CMF$ of a MCF and a standard single core single mode fiber with $A_{\text{eff}} = 80 \text{ µm}^2$ at $1.55 \text{ µm}$ and $CD = 125 \text{ µm}$.

According to the equation of $\kappa$ which was proposed above, the crosstalk can be calculated by using the novel power coupling coefficient (PCC) definition [15] and the coupled-power theory [3]. The PCC was proposed with correlation length ($d_c$) and $d_c$ of 0.05-m was proved to agree well with the measurement in [6]. Therefore, we assume $d_c$ to be 0.05 m in this simulation of crosstalk, and the twisting rate is defined to be 5 turns per 100 m.

Figure 10 shows the crosstalk of the Hetero-TA-12-core fiber at 100-km propagation as function of bending radius under the four kinds of conditions mentioned above. We can find that the crosstalk becomes smaller as the width of trench structure increases. But meanwhile the $\Delta n_{\text{eff}}$ between the adjacent cores decreases and $CD$ should be larger. Therefore, if we want the $R_{pk}$ of the designed fiber smaller than 50 mm, the $\Delta n_{\text{eff}}$ should be around 0.001. In this case, $r_2/r_1 = 2.0$ and $W/r_1 = 1.0$ is the optimal design condition for the Hetero-TA-MCF and under this condition, as $R$ becomes larger than 50 mm, the worst-case crosstalk is around $-42$ dB after 100-km propagation and it is insensitive to the bending radius.

Figure 11 illustrates the crosstalk of the Homo-TA-12-core fiber at 100-km propagation as function of bending radius under same four conditions. The Homo-TA-12-core fiber
mentioned here has the same ring structure with the Hetero-TA-12-core fiber and we choose four sorts of cores for the Homo-TA-12-core fiber under these four conditions. In order to compare the crosstalk characteristics of the Homo-TA-12-core fiber with that of the Hetero-TA-12-core fiber, we assume these four kinds of cores to have the same core parameters with the first kind of core in each condition of the Hetero-TA-12-core fiber that we described above. For the condition (a), $r_1 = 5.34 \, \mu m, \Delta_1 = 0.304\%$, and $\Lambda = 33 \, \mu m$. For condition (b), $r_1 = 5.27 \, \mu m, \Delta_1 = 0.293\%$, and $\Lambda = 34 \, \mu m$. For condition (c), $r_1 = 5.18 \, \mu m, \Delta_1 = 0.280\%$, and $\Lambda = 34 \, \mu m$. And for condition (d), $r_1 = 5.03 \, \mu m, \Delta_1 = 0.263\%$, and $\Lambda = 35 \, \mu m$. We can find obviously that the crosstalk of the Homo-TA-MCF become larger and larger as increasing the bending radius. Therefore, we can see the merit of Hetero-TA-MCF clearly from this comparison.

4. Conclusion

We proposed an analytical method to get the $\kappa$ value between two adjacent trench-assisted non-identical cores. And we designed a sort of Hetero-TA-12-core fiber with $A_{\text{eff}}$ of 100-$\mu m^2$ and showed that $R_{\text{CMF}}$ can reach 5.8. Based on the calculating method of $\kappa$, crosstalk characteristics of the Hetero-TA-MCF were analyzed. The parameters that $\Lambda = 33 \, \mu m, A_{\text{eff}} = 100 \, \mu m^2, r_2/r_1 = 2.0, \text{ and } W/r_1 = 1.0$ were proved to be the best design condition for this model. As $R$ becomes larger than 50 mm, the worst-case crosstalk at 1550-nm wavelength is about $-42 \, \text{dB}$ after 100-km propagation, which also demonstrates that the Hetero-TA-MCF with high density of cores is insensitive to the bending radius.

Acknowledgments

This work was partially supported by the National Institute of Information and Communication Technology (NICT), Japan under “Research on Innovative Optical Fiber Technology”.