I. INTRODUCTION

There are three typical energy scales, a neutrino mass scale \( m_\nu \sim 0.1 \text{ eV} \), a GUT scale \( M_{\text{GUT}} \sim 10^{16} \text{ GeV} \), and a TeV scale \( M_{\text{NP}} \sim 1 \text{ TeV} \) which is a new physics scale beyond the standard model (SM) and regarded as supersymmetry (SUSY) in this article. Among these three scales, we notice a fascinating relation,

\[
M_{\text{NP}}^2 \approx m_\nu \cdot M_{\text{GUT}}. \tag{1.1}
\]

Is this relation an accident, or providing a clue to the underlying new physics? We take a positive stance toward the latter possibility.

As for a neutrino mass \( m_\nu \), its smallness is still a mystery, and it is one of the most important clues to finding new physics. Among many possibilities, a neutrino mass doublet model suggests an interesting explanation of the smallness by a tiny vacuum expectation value (VEV) \([1–15]\). This VEV from a neutrino mass doublet is of \( O(0.1) \text{ eV} \) that is suitable for Dirac neutrino mass \([3,4,6,8]\), and we focus on this situation here.\(^1\) We can see that the relation of Eq. (1.1) is realized by two fundamental parameters, \( M_{\text{GUT}} \) and \( M_{\text{NP}} \) \([16]\).

The neutrino mass is much smaller than other fermions, since its origin is the tiny VEV from the different (neutrinophilic) Higgs doublet. The introduction of \( Z_2 \) symmetry distinguishes the neutrinophilic Higgs from the SM-like Higgs, where \( m_\nu \) is surely generated only through the VEV of the neutrinophilic Higgs. The SUSY extension of the neutrinophilic Higgs doublet model is considered in Refs. \([7,11,12,15]\). Since the neutrino Yukawa couplings are not necessarily tiny anymore, some related research has been done, such as, collider phenomenology \([8,10]\), low energy thermal leptogenesis \([11,12]\), cosmological constraints \([13]\),\(^2\) and so on.

\(^1\)The Majorana neutrino case is also possible \([1,2,5,9–12]\), but additional scales of Majorana masses make the realization of Eq. (1.1) complicated.

\(^2\)The setup in Ref. \([13]\) is different from the usual neutrinophilic Higgs doublet models, since it includes a light Higgs particle.

On the other hand, SUSY is the most promising candidate of new physics beyond the SM because of the excellent success of gauge coupling unification. Thus, the SUSY SM well fits the GUT scenario as well as the existence of a dark matter candidate.

There are some attempts that try to realize the relation in Eq. (1.1). One example is to derive \( m_\nu \) from a higher dimensional operator in the SUSY framework \([17,18]\). Another example is to take a setup of matter localization \([19]\) in a warped extra dimension \([20]\). These scenarios are interesting, but the model in this article is much simpler and contains no additional scales other than \( M_{\text{NP}}, m_\nu, \) and \( M_{\text{GUT}} \). (For other related papers, see, for example, \([21,22]\).)

In this article, we investigate the phenomenology of a SUSY \( SU(5) \) GUT with the neutrinophilic Higgs \( (SU(5)_{\nu}) \) model proposed in Ref. \([16]\).\(^3\) Usually, SUSY neutrinophilic Higgs doublet models have the tiny mass scale of soft \( Z_2 \)-symmetry breaking \((\rho, \rho' = O(10) \text{ eV}) \) shown in Refs. \([11,12,15]\). This additional tiny mass scale plays a crucial role in generating the tiny neutrino mass, however, its origin is completely unknown (assumption). In other words, the smallness of \( m_\nu \) is just replaced by that of \( Z_2 \)-symmetry breaking mass parameters, and this is not an essential explanation of tiny \( m_\nu \). This is a common and serious problem that exists in neutrinophilic Higgs doublet models in general. It can be noted that this problem can be solved by Ref. \([16]\), where two scales of \( M_{\text{GUT}} \) and \( M_{\text{NP}} \) naturally induce the suitable magnitude of \( m_\nu \) through the relation of Eq. (1.1), and does not require any additional scales. The model contains a pair of new neutrinophilic Higgs doublets with GUT-scale masses, and the \( Z_2 \) symmetry is broken by the TeV-scale dimensionful couplings of these new doublets to the ordinary SUSY Higgs doublets. Once the ordinary Higgs doublets obtain VEVs \((v_{u,d})\) by the usual electroweak symmetry breaking, they trigger

\(^3\)A similar model was suggested in Ref. \([23]\), where lepton flavor violation was also roughly estimated.
VEVs for the neutrinophilic Higgs doublets of \( \nu_{a,d} M_{NP}/M_{GUT} \sim m_\nu \). Then, \( O(1) \) Yukawa couplings of the neutrinophilic doublets to \( LN \) (L: lepton doublet, N: right-handed neutrino) give neutrino masses of the proper size. We can also obtain a GUT embedding of the SUSY neutrinophilic Higgs doublet model, which realizes the relation, \( m_\nu \sim \nu_{a,d} M_{NP}/M_{GUT} \), dynamically. As a remarkable feature of this model, accurate gauge couplings can be unified in keeping with a proton stability. Flavor changing processes are also a sensible aspect of this model. In general, flavor violation in the charged lepton sector is related to that in the quark sector because the lepton doublet and the right-handed down-type quark are contained in the same multiplet in \( SU(5) \) GUT. Particularly, neutrino oscillation directly contributes flavor violations in both sectors. One of our purposes is to evaluate such flavor violating processes.

This paper is organized as follows. In Sec. II, we review the SUSY \( SU(5) \)\(_H\) model. In Secs. III and IV, we discuss gauge coupling unification, and investigate flavor violations in the SUSY \( SU(5) \)\(_H\) model. These sections are the main parts of this article. In Sec. V, we present a summary.

II. SUSY \( SU(5) \) GUT WITH NEUTRINOPHILIC HIGGS

Before showing the SUSY \( SU(5) \)\(_H\) model [16], we show the SUSY neutrinophilic Higgs doublet model. This has a specific parameter region which is different from Refs. [11,12,15]. We introduce \( Z_2 \) parity, where only vector-like neutrinophilic Higgs doublets and right-handed neutrino have odd charge. The superpotential of the Higgs sector is given by

\[
W_h = \mu H_u H_d + M H_v H_{\nu'} - \rho H_u H_{\nu'} - \rho' H_v H_d. \tag{2.1}
\]

\( H_v \) (\( H_{\nu'} \)) is a neutrinophilic Higgs doublet, and \( H_v \) has a Yukawa interaction of \( LH, N \), which induces a tiny Dirac neutrino mass through the tiny VEV of \( \langle H_v \rangle \). This is the origin of the smallness of the neutrinophilic mass, and we concentrate on a Dirac neutrino scenario, i.e., \( m_\nu \approx \langle H_v \rangle = O(0.1) \) eV. On the other hand, \( H_{\nu'} \) does not couple with any matter. \( H_u \) and \( H_d \) are Higgs doublets in the minimal supersymmetric standard model, and quarks and charged lepton obtain their masses through \( \langle H_u \rangle \) and \( \langle H_d \rangle \). Note that this structure is guaranteed by the \( Z_2 \) symmetry. Differently from conventional neutrinophilic Higgs doublet models, here we take \( M \) as the GUT scale and \( \mu, \rho, \rho' \) \( O(1) \) TeV. The soft \( Z_2 \)-parity breaking parameters, \( \rho \) and \( \rho' \), might be induced from the SUSY breaking effects (see below), and we regard \( \rho \) and \( \rho' \) as the mass parameters of new physics scale, \( M_{NP} = O(1) \) TeV. We remark that the usual SUSY neutrinophilic Higgs doublet models take \( \rho, \rho' = O(10) \) eV (for \( O(1) \) TeV \( B \) terms) [11,12,15]. This additional tiny mass scale plays a crucial role in generating the tiny neutrino mass; however, its origin is just an assumption. Thus, the smallness of \( m_\nu \) is just replaced by that of \( \rho \) and \( \rho' \). This is a common and serious problem that exists in neutrinophilic Higgs doublet models in general. The present model solves this problem, in which two scales of \( M_{GUT} \) and \( M_{NP} \) induce the suitable magnitude of \( m_\nu \) dynamically, and does not require any additional scales, such as \( O(10) \) eV. This is one of the most important points in this model.

Amazingly, stationary conditions make the VEVs of neutrinophilic Higgs fields to be

\[
\nu_\nu = \frac{\rho v_u}{M}, \quad \nu_{\nu'} = \frac{\rho' v_d}{M}. \tag{2.2}
\]

It is worth noting that they are induced dynamically through the stationary conditions, and their magnitudes are surely of \( O(0.1) \) eV. Since the masses of neutrinophilic Higgs \( H_v \) and \( H_{\nu'} \) are superheavy as the GUT scale, there are no other vacua (such as, \( \nu_{a,d} \sim v_{\nu,\nu'} \) except for \( \nu_{a,d} \gg v_{\nu,\nu'} \) [15]. Also, their heaviness guarantees the stability of the VEV hierarchy, \( v_{a,d} \gg v_{\nu,\nu'} \), against radiative corrections [14,15]. This is because in the effective potential, \( H_v \) and \( H_{\nu'} \) inside loop diagrams are suppressed by their GUT-scale masses.

The model suggested in Ref. [16] has the GUT-scale mass of the neutrinophilic Higgs doublets in Eq. (2.1), so that it is naturally embedded into a GUT framework, and it is the SUSY \( SU(5) \)\(_H\) model. A superpotential of a Higgs sector at the GUT scale is given by

\[
W_GUT^H = M_0 \mu \Sigma^2 + \lambda \mu \Sigma^3 + H \Sigma \tilde{H} + \Phi_H \Sigma \tilde{\Phi}_\nu - M_1 H \Sigma \tilde{H} - M_2 \Phi_H \tilde{\Phi}_\nu, \tag{2.3}
\]

where \( \Sigma \) is an adjoint Higgs whose VEV reduces the GUT gauge symmetry into the SM. \( \Phi_H \) (\( \Phi_{\nu} \)) is a neutrinophilic Higgs of (anti-)fundamental representation, which contains \( H_v \) (\( H_{\nu'} \)) in the doublet component [while the triplet component is denoted as \( T_{\nu} (\tilde{T}_{\nu'}) \)]. \( \Phi_H \) and \( \Phi_{\nu} \) are odd under the \( Z_2 \) parity. \( H \) (\( \tilde{H} \)) is a Higgs of (anti-)fundamental representation, which contains \( H_u \) (\( H_d \)) in the doublet component [while the triplet component is denoted as \( T (\tilde{T}) \)]. The VEV of \( \Sigma \) and \( M_{0,1,2} \) are all of \( O(10^{10}) \) GeV; thus, we encounter the so-called triplet-doublet (TD) splitting problem. Some mechanisms have been suggested for a solution of TD splitting, but here we show a case where the TD splitting is realized just by a fine-tuning between \( \langle \Sigma \rangle \) and \( M_1 \). That is, \( \langle \Sigma \rangle - M_1 \) induces the GUT scale masses of \( T, \tilde{T} \) in keeping with weak scale masses of \( H_u, H_d \). This is a serious fine-tuning, so that we cannot expect a simultaneous fine-tuned cancellation between \( \langle \Sigma \rangle \) and \( M_2 \). Thus, we consider the case where the TD splitting only works in \( H \) and \( \tilde{H} \), while it does not work in \( \Phi_{\nu} \) and \( \tilde{\Phi}_{\nu} \). This situation makes Eq. (2.3) become

\[
W_{\text{eff}}^H = \mu H_u H_d + M_H H_{\nu'} + M^T \tilde{T} + M'' T_{\nu} \tilde{T}_{\nu}. \tag{2.4}
\]

This is the effective superpotential of the Higgs sector below the GUT scale, and \( M, M', M'' \) are of \( O(10^{16}) \) GeV, while \( \mu = O(1) \) TeV.
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Now let us consider an origin of the soft $Z_2$-parity breaking terms, $\rho H_u H_\nu^c$ and $\rho' H_u H_\nu^c$, in Eq. (2.1). They play a crucial role in generating the marvelous relation in Eq. (1.1) as well as the tiny Dirac neutrino mass. Since the values of $\rho$, $\rho'$ are of order $O(1)$ TeV, they might be induced from the SUSY breaking effects. We consider some possibilities for this mechanism. One example is to take a noncanonical Kähler of $[S^4(H_u H_\nu^c + H_u H_\nu^c) + \text{H.c.}]_D$, where the $F$ term of $S$ could induce the $\rho$ and $\rho'$ terms effectively through the SUSY breaking scale as in Giudice-Masiero mechanism [24]. There might be other models that induce the $\rho$ and $\rho'$ terms in Eq. (2.1) without a singlet $S$.

### III. GAUGE COUPLING UNIFICATION AND PROTON DECAY

In this section, we discuss a characteristic feature of the gauge coupling unification and the proton decay in the SUSY $SU(5)_{H_u}$ model by focusing on the role of $T_\nu$ and $\tilde{T}_\nu$. For the minimal SUSY $SU(5)$ GUT model, in order to unify the gauge couplings, the mass of $T$ and $\tilde{T}$ should be lighter than the GUT scale as $3.5 \times 10^{14}$ GeV $\leq M' \leq 3.6 \times 10^{15}$ GeV due to threshold corrections [25]. However, to avoid rapid proton decay, $M'$ must be heavier than the GUT scale ($M' > M_{\text{GUT}}$). Hence, it is difficult to achieve both accurate gauge coupling unification and enough proton stability in the minimal SUSY $SU(5)$ GUT.

The situation changes in the SUSY $SU(5)_{H_u}$ model. In this model, a superpotential of the Yukawa sector is given by

$$ W_Y = \frac{1}{4} f_{ui} \psi_i \psi_j H + \sqrt{2} f_{di} \psi_i \phi_i \bar{H} + f_{\nu_i} \eta_i \phi_j \Phi_\nu, \quad (3.1) $$

at the GUT scale, where $i$ and $j$ are family indices, $\psi_i$, $\phi_i$, and $\eta_i$ are 10-plet, 5-plet, and singlet in the $SU(5)$ gauge group, respectively, which are written in terms of minimal supersymmetric standard model fields as

$$ \psi_i = \{ Q_i, e^{-i\phi_i} U_i, (V_{\text{KM}})_{ij} \hat{E}_j \}, $$

$$ \phi_i = \{ (V_D)_{ij} \hat{D}_j, (V_D)_{ij} \hat{L}_j \}, $$

$$ \eta_i = \{ e^{-i\phi_i} N_i \}. $$

Since Yukawa couplings are written as

$$ f_{ui} = f_u e^{i\delta_{ui}}, $$

$$ f_{di} = (V_{\text{KM}})_{ik} f_{dj} (V_D^\dagger)_{kj}, $$

$$ f_{\nu_i} = f_\nu e^{i\delta_{\nu_i}}, $$

the superpotential in this basis is given by

$$ W_Y = f_u Q_i \hat{U}_i H_u + (V_{\text{KM}})_{ij} f_{di} Q_i \hat{D}_j H_u + f_d \hat{E}_i L_i H_d $$

$$ + f_u (V_{\text{KM}})_{ij} \hat{E}_j T - \frac{1}{2} f_u Q_i Q_i T $$

$$ + (V_{\text{KM}})_{ij} f_{dj} \hat{U}_j T - (V_{\text{KM}})_{ij} f_{di} Q_i L_j T $$

$$ - f_\nu (V_D)_{ij} \hat{N}_i L_i H_\nu + f_\nu (V_D)_{ij} \hat{N}_i \bar{D}_j T_\nu, \quad (3.4) $$

where CP phases, $\phi_u$ and $\phi_\nu$, are omitted, for simplicity. The terms from the fourth to seventh in Eq. (3.4) cause proton decay, which also exists in the minimal SUSY $SU(5)$ GUT. Thus, we should take $M' > M_{\text{GUT}}$ to avoid rapid proton decay. Meanwhile, the last term in Eq. (3.4) has nothing to do with the proton decay. Since $T_\nu$ and $\tilde{T}_\nu$ contribute beta functions of $SU(3)_c \times U(1)_Y$, accurate gauge coupling unification is achieved with the $T_\nu$ and $\tilde{T}_\nu$ threshold corrections with $3.5 \times 10^{14}$ GeV $\leq M'' \leq 3.6 \times 10^{15}$ GeV. Therefore, the SUSY $SU(5)_{H_u}$ model can realize not only accurate gauge coupling unification but also proton stability. Remembering that $M$ is the GUT scale, $O(1)$% tuning between $M$ and $M''$ is needed, but it can be happen. Or, no tuning is required when one of couplings is of $O(0.01)$, for example, a coupling of $S^4 H_u H_\nu^c$.

### IV. FLAVOR CHANGING PROCESSES

Flavor changing in the lepton sector is related to that in the quark sector, since $L$ and $D$ are contained in a same multiplet in $SU(5)_{H_u}$, where, mixing angles in $V_{\text{PMNS}}$ are expected to be large, and masses of left-handed slepton and right-handed down-type squark get sizable radiative corrections in off-diagonal elements of flavor space. Leading log approximation makes the off-diagonal elements

$$ (\delta m_L^2)_{ij} \approx - \frac{f_{vi}^2}{8\pi^2} (V_D^*_{ik} V_{D^*} L_{kj}) (3m_0^2 + A_0) \log \frac{M_P}{M}, \quad (4.1) $$

$$ (\delta m_D^2)_{ij} \approx - \frac{f_{vi}^2}{8\pi^2} (V_D^*_{ik} V_{D^*} L_{kj}) (3m_0^2 + A_0) \log \frac{M_P}{M}, \quad (4.2) $$

where $M_P$ is the Planck scale, $m_0$ and $A_0$ are universal scalar mass and universal scalar trilinear coupling in the mSUGRA scenario. Equation (4.1) originates from the loop diagram of $N$ and $H_\nu$, where an energy scale in the renormalization group equations runs from $M_P$ to $M$ ($H_\nu$, $H_\nu^c$ mass). On the other hand, Eq. (4.2) is induced from the loop diagram of $N$ and $T_\nu$, which runs from $M_P$ to $M''$ ($T_\nu$, $\tilde{T}_\nu$ mass). Notice that the loop effects in Eqs. (4.1) and (4.2) are different from those in $SU(5)$ with right-handed neutrinos ($SU(5)_{\text{RN}}$). In the $SU(5)_{\text{RN}}$ model, neutrinos are Majorana, and the counterparts of Eqs. (4.1) and (4.2) are given by

$$ (\delta m_L^2)_{ij} \approx - \frac{f_{vi} f_{vj}}{8\pi^2} (V_D^*_{ik} V_{D^*} L_{kj}) (3m_0^2 + A_0) \log \frac{M_P}{M_N}, \quad (4.3) $$

$$ (\delta m_D^2)_{ij} \approx - \frac{f_{vi}^2}{8\pi^2} (V_D^*_{ik} V_{D^*} L_{kj}) (3m_0^2 + A_0) \log \frac{M_P}{M'}, \quad (4.4) $$

where $M_N$ is the diagonal Majorana mass of $N_i (l = 1, 2, 3)$. The mass matrix of $N$ is diagonalized by the unitary matrix

\[ \text{See, for example, [26].} \]
VM [27], which does not appear in Eq. (4.4) because $M_{\nu}$ is usually assumed to be smaller than $M_{0}$ (> $M_{\text{GUT}}$). By comparing the SU(5)$_{\text{RN}}$ model with the SU(5)$_{H_{23}}$ model, we can find an advantage in the latter model. This is a predictability, that is, flavor changing processes are strongly predicted since there are no degrees of freedom of VM. This means that flavor violations in the charged lepton sector are directly related to those in the quark sector through the large flavor mixings in the neutrino sector. And, even if the mass matrix of the right-handed neutrinos is diagonal, magnitudes of $C_2^\sim L$ and $C_2^\sim D$ in the SU(5)$_{H_{23}}$ model are different from those in the SU(5)$_{\text{RN}}$ model. For example, Eq. (4.4) can be a few percent smaller than that in the SU(5)$_{H_{23}}$ model due to their log factors. The magnitude of the log factor is $\log \frac{M_{\mu}}{M_{\text{GUT}}}$, since $M_{\mu}$ must be larger than the GUT scale for the proton stability, and $M_{\text{GUT}}$ must be of $O(10^{14})$ GeV for accurate gauge coupling unification.

Let us show results of the numerical analyses in the $b \rightarrow s \gamma$ and $b \rightarrow \mu \gamma$ processes. Figures 1 and 2 show the correlations between the branching ratios of $B(b \rightarrow s \gamma)$ and $B(b \rightarrow \mu \gamma)$ with $\tan \beta = 10$ and $A_0 = 0$. In Fig. 1, $m_{1/2}$ is varied from 500 GeV to 800 GeV by 100 GeV. As for Fig. 2, $m_{1/2}$ is varied from 500 GeV to 1000 GeV by 100 GeV. $m_0$ is varied from 200 GeV to 1200 GeV for each line. Here, the Higgs mass, calculated by FeynHiggs [28–31], is varied around 118 GeV which is not excluded by ATLAS [32] and CMS [33]. In Figs. 1 and 2, $\sin^2 2 \theta_{13}$ is taken by 0 and 0.01.
respectively. We consider that the spectrum of neutrinos is hierarchical, and the $\nu_{e}$-Yukawa coupling is of $O(1)$. The current upper bound on $B(\mu \rightarrow e\gamma)$ is $2.4 \times 10^{-12}$ by the MEG experiment [34]. Figure 2 shows that large $\theta_{13}$ is restricted in $\mu \rightarrow e\gamma$. In this parameter region, $B(b \rightarrow s\gamma)$ does not change drastically because the $m_{1/2}$ dependence is larger than the $m_{0}$ dependence.

Figures 3 and 4 show the correlations between $B(b \rightarrow s\gamma)$ and $B(\tau \rightarrow \mu\gamma)$, whose parameters are the same as Figs. 1 and 2, respectively. $B(\tau \rightarrow \mu\gamma)$ does not reach the experimental upper bound in this parameter region. [The experimental upper bound for $B(\tau \rightarrow \mu\gamma)$ is $4.4 \times 10^{-8}$ by BABAR experiment [35].] Note that a ratio of $B(\tau \rightarrow \mu\gamma)/B(\mu \rightarrow e\gamma)$ depends largely on $\theta_{13}$, where other neutrino oscillation parameters are fixed. When $\theta_{13}$ becomes large, $B(\tau \rightarrow \mu\gamma)/B(\mu \rightarrow e\gamma)$ is closer to 10. This behavior is consistent with Ref. [23].

We do not consider the $\tau \rightarrow e\gamma$ process because the experimental upper bound for $B(\tau \rightarrow e\gamma)$ is $3.3 \times 10^{-8}$ [35] which is the same order as $B(\tau \rightarrow \mu\gamma)$. The ratio of $B(\tau \rightarrow e\gamma)/B(\tau \rightarrow \mu\gamma)$ is roughly proportional to $(V_{D})_{11}^{2}/(V_{D})_{22}^{2} < 1$. Hence, $B(\tau \rightarrow \mu\gamma)$ is more stringent constraint than $B(\tau \rightarrow e\gamma)$.

Finally, we comment on the Daya Bay experiment, which has measured a nonzero $\theta_{13}$ [36]. The best-fit value is given by $\sin^{2}2\theta_{13} = 0.092$, and such a large mixing angle gives a more stringent constraint in $\mu \rightarrow e\gamma$. Figure 5 shows the $m_{0}$ and $m_{1/2}$ dependence of $B(\mu \rightarrow e\gamma)$ with $\sin^{2}2\theta_{13} = 0.092$. We can see that $m_{1/2}$ should be larger than 2 TeV in order not to exceed the experimental bound in Fig. 5. As for neutrinoless double beta decay, it is forbidden in our setup because the neutrinos are Dirac fermion with lepton number conservation.

V. SUMMARY

Among three typical energy scales, a neutrino mass scale, a GUT scale, and a TeV (SUSY) scale, there is the marvelous relation of Eq. (1.1). In this paper, we have investigated the phenomenology of the SUSY $SU(5)_{H_{c}}$ model proposed in Ref. [16]. This model realizes the relation of Eq. (1.1) dynamically as well as the suitable Dirac neutrino mass through the tiny VEV of neutrino philic Higgs. First, we discussed the gauge coupling unification and the proton stability. Fascinatingly, the $SU(5)_{H_{c}}$ can realize not only accurate gauge coupling unification but also enough proton stability simultaneously, and this situation is hardly realized in the usual four-dimensional $SU(5)$ GUTs. Next, we investigated the correlations between $b \rightarrow s\gamma$ and $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$. We noted that $B(b \rightarrow s\gamma)$, $B(\mu \rightarrow e\gamma)$ and $B(\tau \rightarrow \mu\gamma)$ are correlated directly through the neutrino mixing in the $SU(5)_{H_{c}}$ model, which is an advantage of this model over the $SU(5)_{RN}$ model. As shown in Eq. (4.3), additional unknown degrees of freedom, parameters in $V_{M}$, are needed in the latter model. Therefore, flavor changing processes are strongly predicted in the $SU(5)_{H_{c}}$ model. As for the dependence of $\theta_{13}$, $B(\mu \rightarrow e\gamma)$ depends largely on it, so that $B(\mu \rightarrow e\gamma)$ is strongly limited in large $\theta_{13}$. On the other hand, we have shown that $B(b \rightarrow s\gamma)$ does not depend largely on $\theta_{13}$.

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