Confined light composed of a single localized mode inside photonic crystals for a qubit

H. Nihei · A. Okamoto

Abstract We have demonstrated that light is confined in a single mode near a very small defect (such as an impurity atom) embedded in photonic crystals by coherent control using a dark line that is a spectrum singularity leading to the complete quenching of emission. In this demonstration, we have clarified the strong confinement and a very short response time, as compared with the widely used tunable PBG method. These features are useful for keeping the memory in a two-state quantum system such as a quantum bit (qubit) and for processing quantum information.

Keywords photonic crystals, photonic band gap, quantum information processing, quantum bit

1. Introduction

The trapping and confinement of light near a very small defect embedded in three-dimensional (3D) photonic crystals are expected to be the basis of achieving nano-optical memory devices including a quantum bit (qubit) (S. John 2009; X. Ma et al. 2009). In a photonic crystal, due to the presence of a 3D photonic band gap (PBG), there is no propagating electromagnetic wave in any direction in space, which provides the light confinement effect. Such a light confinement has been experimentally achieved by combing 3D photonic crystals and surface 2D PBG structures (K. Ishizaki et al. 2009). More recently, light confinement has been observed using a lithium niobate photonic crystal cavity (J. Dahdah et al. 2011) and strongly confined Anderson-localized cavity modes have been generated by deliberately adding disorder to photonic crystal waveguides (L. Sapienza et al. 2010). In general, the light confinement is accompanied by an anomalously large vacuum Rabi splitting. Furthermore, Rabi splitting causes the formation of several localized modes in confined light (M. Woldeyohannes et al. 1999). It is increasingly desired to form confined light composed of a single localized mode. This provides a robust mechanism for all-optical PBG microchips, such as optical logic gates including quantum logic gates.

One approach is band gap control using a tunable PBG (S. Kubo et al. 2004), where the generation and elimination of specific localized modes in confined light are controlled by opening and closing the PBG to generate a single-mode confined light. The tunable PBG has been achieved by infiltrating nematic liquid crystals into PBG structures (such as inverse opals and PBG fibers) by applying an electric field (K. Busch et al. 1999; M. Haurylaua et al. 2006). However, in the process of closing the PBG, a part of the energy of the localized mode is transferred to that of the propagating light mode traveling away from photonic crystals, leading to the energy loss of confined light. Another approach to controlling confined light is the coherent control method using two lasers: one is the steady-state coherent control laser coupling the upper levels of an embedded defect (such as an impurity atom) and the other is the pump laser pulse used to generate an excited state of the atom (T. Quang et al. 1997; M. Woldeyohannes et al. 1999). Moreover, we have proposed the coherent control using a dark line (a spectrum singularity leading to the complete quenching of emission at certain values of emitted photon frequencies), where confined light can be controlled without

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closing the PBG (H. Nihei et al. 2008). Here, we have demonstrated that the coherent control provides a very high spectral selectivity by calculating the emission spectrum and an optical switching in two or more localized modes in confined light.

In this paper, we demonstrate that light is confined in a single mode near a very small defect (such as an impurity atom) embedded in photonic crystals by coherent control using a dark line. Here, we clarify the strong confinement and a very short response time, as compared with the tunable PBG method. These features come from the fact that confined light is controlled without closing the PBG, where confined light remains localized near the atom, even during the control of confined light. This is useful for keeping the memory in a two-state quantum system such as a qubit and for processing quantum information.

2. Defect embedded in photonic crystals

A defect embedded in 3D photonic crystals is assumed to be a single impurity atom with a three-level energy configuration with two upper levels ($|2\rangle$ and $|3\rangle$) and a ground level $|1\rangle$. At the initial time t=0, a pump laser pulse is used to prepare the atom in the form of a coherent superposition of the two upper levels $|2\rangle$ and $|3\rangle$;

$$|\psi(0)\rangle = \cos(\theta/2)|2\rangle + e^{i\phi}\sin(\theta/2)|3\rangle, \qquad (1)$$

where θ and ϕ are determined by the phase and the area of the pump pulse laser. Such a state vector composed of a two-state system can be graphically represented by a point on the Bloch sphere, as shown in Fig. 1.

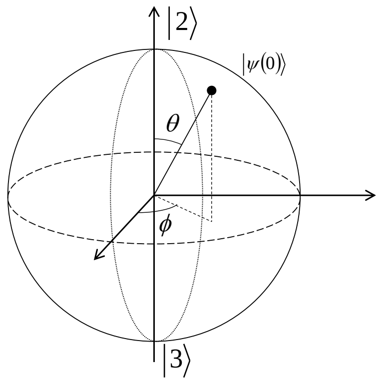

Fig. 1: Bloch sphere representation of a two-state system.

For t>0, the coherent superposition of the two upper levels

$$|\phi(t)\rangle = a_3(t)|3\rangle + a_2(t)|2\rangle, \qquad (2)$$

can act as an optical memory to encode quantum information, which is a qubit, where $a_k(t)$ (k=2, 3) is a complex amplitude and $|a_k(t)|^2$ gives the probability of finding the atom in level $|k\rangle$. Using Eq. (2), the state vector of the atom for t>0 can be written as

$$|\psi(t)\rangle = |\phi(t)\rangle|\{0\}\rangle + \sum_{\vec{k}\lambda} a_{1\vec{k}\lambda}(t)|1\rangle|\{\vec{k}\lambda\}\rangle, \qquad (3)$$

where the state vector $|k\rangle|\{0\}\rangle$ represents the atom in the upper states $|k\rangle$ and the vacuum electromagnetic field. The state vector $|1\rangle|\{\vec{k}\lambda\}\rangle$ represents the atom in the ground state $|1\rangle$ and a single photon in the mode $\{\vec{k}\lambda\}$, where $\vec{k}\lambda$ is the product of the wave vector $\vec{k}$ and he polarization index λ (=1, 2). Furthermore, the complex amplitudes $a_k(t)$ and $a_{1\vec{k}\lambda}(t)$ can be written as

$$\begin{cases} a_k(t) = b_k(t)e^{-i\omega_k t} & ,(k=2,3) \\ a_{1\vec{k}\lambda}(t) = b_{1\vec{k}\lambda}(t)e^{-i(\omega_{\vec{k}}+\omega_1)t} & \end{cases}, \qquad (4)$$

where $b_k(t)$ and $b_{1\vec{k}\lambda}(t)$ are the slowly varying envelope approximation (SVEA) amplitudes of $a_k(t)$ and $a_{1\vec{k}\lambda}(t)$, respectively.

To keep the memory $|\phi(t)\rangle$ in the state vector $|\psi(t)\rangle$ for $t>0$, it is necessary to maintain highly excited states (namely, $|a_2(t)|^2 + |a_3(t)|^2 \cong 1$), which may be guaranteed by the formation of a photon-atom bound state, where the emitted photon remains partially localized in the vicinity of the emitting atom. This can be achieved by using PBG, so that we assume photonic crystals with a high-pass single PBG (M. Woldeyohannes et al. 1999) and that the transition frequency of the atom ($\omega_{21} = \omega_2 - \omega_1$) is deep inside the PBG ($\omega_{21} << \omega_c$). The other frequency ($\omega_{31} = \omega_3 - \omega_1$) is assumed to be near (but inside) the PBG, where we denote the detuning as $\delta = \omega_{31} - \omega_c$. Furthermore, to control the quantum information $|\phi(t)\rangle$, we assume the coherent control laser coupling the two upper levels ($\omega_{32} = \omega_3 - \omega_2$). These photonic crystal and defect systems have been widely used (T. Quang et al. 1997; M. Woldeyohannes et al. 1999; H. Nihei et al. 2005; H. Nihei et al. 2008). Such a model can describe the essential physics of the optical properties of photonic crystals and may enable the realization of experimental systems such as an ensemble of CdSe-ZnSe quantum dots embedded in an inverse opal consisting of air spheres in TiO_2 (S. C. Cheng et al. 2009).

3. Time evolution of a qubit

The amplitude $a_k(t) = b_k(t)e^{-i\omega_k t}$ that determines the time evolution of a qubit $|\phi(t)\rangle$ develops, according to the Schrödinger equation, which has been given by the sum of the three components as

$$b_k(t)e^{-i\omega_k t} = \left\{ \sum_{j=1}^{2} b_{kj}(t) + b_{qk}(t) \right\} e^{-i\omega_k t}, \qquad (5)$$

(H. Nihei et al. 2002). The component $b_{kj}(t)e^{-i\omega_k t}$ takes the form $b_{kj}(t) = Q_{kj}P_j(t)$ with

$$\begin{cases} Q_{3j} = \left(u_j^2 + \delta\right)\cos\theta + i\Omega e^{i\phi}\sin\theta \\ Q_{2j} = \left(u_j^2 + \alpha u_j + \delta\right)e^{i\phi_p}\sin\theta - i\Omega e^{i\phi_c}\cos\theta \\ P_j(t) = \dfrac{\prod_{p=1}^{4}\left(\sqrt{u_j^2} + u_p\right)}{\left(u_j^2 - u_l^2\right)\left(u_j^2 - u_m^2\right)\left(u_j^2 - u_n^2\right)} e^{i\left(u_j^2+\delta\right)t}, \quad (k \neq l \neq m \neq n), \end{cases} \qquad (6)$$

where Ω is the Rabi frequency of the coherent control laser and u_j is the root of the cubic equation $x^4 + \alpha x^3 + 2\delta x^2 + \alpha\delta x - \left(\Omega^2 - \delta^2\right) = 0$ with the scaled parameter α, which takes the form $\alpha \approx \omega_{31}^{5/2} d_{31}^2 / \left(12\pi\varepsilon_0\hbar c^3\right)$. The phases of the pump laser pulse and coherent control laser are denoted by ϕ_p and ϕ_c, respectively. The amplitudes $b_{qk}(t)$ are the branch cut contribution in the calculation of complex integrodifferential equations that come from the Schrödinger equation, which decay to zero as $t \to \infty$, faster than the other components (M. Woldeyohannes et al. 1999).

The formation of the three components in the amplitude $b_k(t)$ comes from a Rabi splitting, where the components $b_{kj}(t)e^{-i\omega_k t}$ (j=1, 2) represent the localized modes in the emitted field. Their frequencies ω_{Lk} that are given by $b_{kj}(t)e^{-i\omega_k t} = |b_{kj}(t)|e^{-i\omega_{Lk} t}$ satisfy the relation $\omega_{L2} < \omega_{L1} < \omega_c$, which means that the frequencies ω_{Lk} are inside the PBG. Therefore, the components $b_{kj}(t)e^{-i\omega_k t}$ remain localized near the atom and their amplitudes $|b_{kj}(t)|^2$ do not decay to zero, which remain constant at its initial value $|b_{kj}(0)|^2$. The other component $b_{qj}(t)e^{-i\omega_k t}$ is a quasi-dressed state near the edge frequency of the PBG, which decays to zero in a very short time (picosecond order).

Figure 2 shows the time evolution of the qubit $|\phi(t)\rangle$ using the Bloch sphere representation. The norm (the length from the origin) of the qubit $|\phi(t)\rangle$ is given by $R(t) = \sqrt{|a_3(t)|^2 + |a_2(t)|^2}$. The two phases that determine the position of the qubit in the Bloch sphere are $\theta_B(t) = 2\arccos(|a_3(t)|/R(t))$ and $\phi_B(t) = \arg(e^{i\omega_{32}} a_3(t)/a_2(t))$.

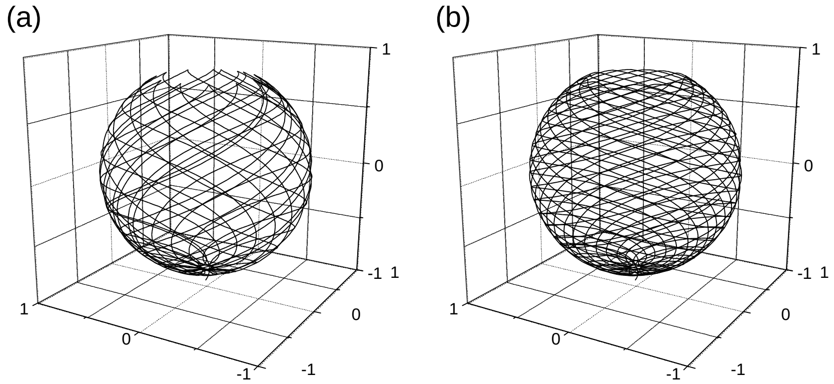

Fig. 2: Time evolution of the qubit.

Figure 2(a) shows the case for the Rabi frequency $\Omega = 5\alpha^2$, the detuning $\delta = -10\alpha^2$, the phases $\phi = \theta = 0$ and the frequency $\omega_{32} = 2|\delta|$. At t=0, the atom is fully excited, so that the norm of the qubit for t=0 is R(0)=1. In the course of a short time on the picosecond order, the excited state partially decays to the ground state $|1\rangle$, so that the norm $R(t)$ is reduced to 0.85, where the qubit makes one or two rotations on the Bloch sphere. This behavior is expressed by the quasi-dressed state $b_{qk}(t)e^{-i\omega_k t}$, where the amplitude $|b_{qk}(t)e^{-i\omega_k t}|$ decays to zero in this short time.

After that, the decay is suppressed due to the presence of the PBG. The norm of the qubit $|\phi(t)\rangle$ remains $R(t)$=0.85, showing the rotation in the Bloch sphere. The rotation is characterized by the two phases $\theta_B(t)$ and $\phi_B(t)$. The phase $\theta_B(t)$ shows the variation due to the energy transfer between the two upper levels, which comes from the quantum interference between the two localized modes. This leads to the wide rotation of the qubit $|\phi(t)\rangle$ in the Bloch sphere, as shown in Fig. 2. The mesh of the rotation is determined by the phase $\phi_B(t)$ depending on the frequency ω_{32}. Figure 2(b) shows the case of $\omega_{32} = 3|\delta|$, where the norm remains 0.85, which is still the same as that shown in Fig. 2(a).

4. Single-mode confined light

Figure 3(a) shows the time evolution of the qubit $|\phi(t)\rangle$ during the formation of single-mode confined light by the tunable PBG method. Here, we assume that one of the two localized modes ($b_{k1}(t)e^{-i\omega_k t}$) is moved out of the PBG by closing the PBG. As a result, only the single localized mode ($b_{k2}(t)e^{-i\omega_k t}$) remains inside the PBG. This closing of the PBG corresponds to the change in the detuning from $-10\alpha^2$ to 0.

Figure 3(a) shows that from t=0, the qubit $|\phi(t)\rangle$ exhibits random rotations inside the Bloch sphere. In this process, the mode $b_{k1}(t)e^{-i\omega_k t}$ that moved out of the PBG travels away from the photonic crystals in the form of a traveling pulse. As a result, in addition to the quasi-dressed state $b_{qk}(t)e^{-i\omega_k t}$, the amplitude of $b_{k1}(t)e^{-i\omega_k t}$ also decays to zero, where the qubit makes about ten or more rotations on the Bloch sphere.

After the random rotation, confined light composed of a single localized mode remains near the atom. As a result, the qubit $|\phi(t)\rangle$ exhibits circular rotations with the fixed phase $\theta_B(t)$. Such circular rotations are necessary for the Hadamard transformation operation for a phase rotation gate that is one of the universal logic gates for quantum computing. However, in addition to the quasi-dressed state $b_{qk}(t)e^{-i\omega_k t}$, the localized mode $b_{k1}(t)e^{-i\omega_k t}$ that moved out of the PBG also travels away from the photonic crystals, so that the energy loss of confined light is enhanced, as compared with that shown in Fig. 2, and the norm $R(t)$ is reduced to 0.55.

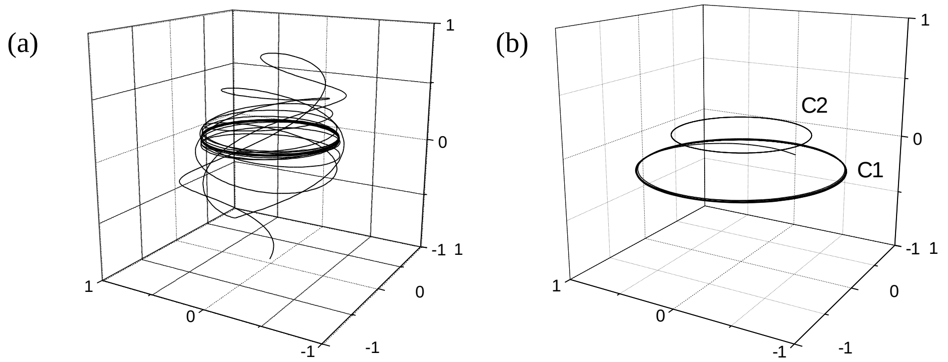

Fig. 3: Time evolution of the qubit during the formation of single-mode confined light.

The curve C1 in Fig. 3(b) shows the time evolution of the qubit $|\phi(t)\rangle$ during the formation of single-mode confined light, where the coherent control method using the dark line effect (H. Nihei et al. 2008) is assumed to form a single-mode confined light. Here, a dark line is a singularity in the spectrum $S(\omega)$ leading to the complete quenching of emission at the certain value (ω_D) of emitted photon frequencies, $S(\omega_D)=0$. The dark line frequency is given by $\omega_D=\delta-\Omega\tan\theta$ for $\phi=\pi/2$. This means that the dark line ω_D is changed by applying the pump laser pulse (characterized by the phase θ in Eq. (1)) onto the atom. Furthermore, when the dark line ω_D is tuned to a localized mode, it is eliminated by the dark line effect, due to the complete quenching of emission.

In the case of the curve C1, we assume that the localized mode $b_{k1}(t)e^{-i\omega_k t}$ (the band edge side) is eliminated by the dark line effect, where the phase θ is determined by $\theta = \tan^{-1}\{(u_1^2+\delta)/\Omega\}$. The curve C1 immediately exhibits circular rotations in a very short time from t=0, as compared with the result of Fig. 3(a), where the amplitude $b_k(t)$ is given by the form $b_k(t)e^{-i\omega_k t}=\{b_{k2}(t)+b_{qk}(t)\}e^{-i\omega_k t}$. Here, the component $b_{k1}(t)e^{-i\omega_k t}$ tuned to the dark line is eliminated; however, the energy of the eliminated component $b_{k1}(t)e^{-i\omega_k t}$ remains confined near the atom, because its frequency is inside the PBG, so that the energy of the eliminated component $b_{k1}(t)e^{-i\omega_k t}$ is transferred not to the propagating mode but to the other localized mode $b_{k2}(t)e^{-i\omega_k t}$ of confined light. This energy transfer is caused simultaneously with the tuning operation of the dark line, so that the time required for performing the circular rotation is determined by the decay time of the quasi-dressed state $b_{qj}(t)e^{-i\omega_k t}$ in Fig. 2, which is on the picosecond order.

The curve C2 in Fig. 3(b) shows the time evolution of the qubit $|\phi(t)\rangle$ during the formation of single-mode confined light by the tunable PBG method for $t\to\infty$. Comparing C1 and C2, we find that the radius of C1 is sufficiently large, where the norms $R(\infty)$ of C1 and C2 are 0.85 and 0.55, respectively. Therefore, the loss coming from the decay to the ground state (1-$R(t)$) is reduced from 0.45 to 0.15 (34%). This is an important feature of the coherent control using the dark line, which is due to the fact that confined light is controlled without closing the PBG, where confined light remains localized near the atom even during the control of confined light. Such a highly excited state with a single frequency oscillation and a very short response time on the picosecond order are useful for keeping the memory in the two-state quantum system and for processing quantum information in a qubit, leading to a quantum logic gate.

5. Conclusion

We have theoretically demonstrated the formation of single-mode confined light near the impurity atom embedded inside photonic crystals. Furthermore, we paid our attention to the application of an atom such as a qubit, using the Bloch sphere representation. In this demonstration, first, we have shown that the state vector of the atom widely rotates in the Bloch sphere. In this case, confined light near the atom is composed of two or more localized modes. For the tunable PBG method and coherent control using a dark line, we have calculated the time evolution of the state vector during the formation of single-mode confined light. By comparing these two methods, we have found that the energy loss of confined light is strongly suppressed and its response time is very short in the case of the coherent control using the dark line. These features are due to the fact that confined light is controlled without closing the PBG, where confined light remains localized near the atom even during the control of confined light. This control of confined light is based on the tuning of the dark line to the localized mode by applying the pump laser pulse to the atom. It may be difficult to tune the dark line to the localized mode; however, the control of confined light without closing the PBG has the significant advantages of achieving low-loss and high-speed control. Such control of a two-state quantum mechanical system is useful for various applications such as qubits and quantum logic gates.

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