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Luminescence of a Cooper Pair

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This Letter theoretically discusses the photon emission spectra of a superconducting $p-n$ junction. On the basis of the second order perturbation theory for electron-photon interaction, we show that the recombination of a Cooper pair with two $p$-type carriers causes enhancement of the luminescence intensity. The calculated results of photon emission spectra explain characteristic features of observed signal in an recent experiment. Our results indicate high functionalities of superconducting light-emitting devices.

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Light-emitting diodes (LEDs) usually fabricated on semiconductors have been an important element of modern technologies. Recent researches seem to focus on producing a well controlled photon and an entangled photon pair [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled quantum states because of its coherent nature [3–5]. Superconducting LEDs [6] have been originally proposed [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled states because of its coherent nature [3–5]. Superconducting LEDs [6] have been originally proposed [1,2] for realizing quantum information. Superconducting devices have a great advantage in producing entangled states because of its coherent nature [3–5].

The radiative recombination of Cooper pairs has been observed recently in a InGaAs/InP $p-n$ junction attaching onto a superconductor Nb [9]. The electroluminescence becomes drastically large at low temperatures below the superconducting transition temperature $T_c$ of Nb electrode. Surprisingly the degree of enhancement in the luminescence intensity is 1 order of magnitude. Although the experiment has shown clearly effects of superconductivity on the radiative recombination, the mechanism has been an open question. This Letter theoretically addresses this issue. We study the emission spectra of photon in a superconducting $p-n$ junction by using the second order perturbation expansion for electron-photon interaction. In the second order, we find that a peculiar recombination process to superconductivity enhances the luminescence intensity. In that recombination process, two electrons recombine with two $p$-type carriers as a Cooper pair. The theoretical results explain characteristic features of the experimental findings [9]. This Letter not only figures out a mechanism of the large luminescence intensity but also gives a guide for designing highly functional superconducting light-emitting devices.

Let us consider a $p$-type semiconductor-superconductor junction under the applied bias voltage $eV_{ph}$ as shown in Fig. 1(a). The energy is measured from the horizontal line indicated by “0”. The sign of energy in a $p$-type semiconductor is chosen to be opposite to that in a superconductor. We assume that a semiconductor and a superconductor are in their local equilibrium which are characterized by the local chemical potential $\mu_p$ and $\mu_n$, respectively. The edges of the conduction and valence bands are $E_c$ and $E_v$, respectively. In what follows, we use a unit of $\hbar = k_B = c = 1$, where $k_B$ is the Boltzmann constant and $c$ is

![FIG. 1 (color online). Schematic energy diagram of forward biased $p-n$ junctions. A theoretical model used for calculation is shown in (a). In (c), a realistic junction in experiments is illustrated. Predicted results of the photon spectra are shown in (b) and (d).](image-url)
the speed of light. The $p$-type semiconductor is described by

$$H_p = \sum_{k,\sigma} \varepsilon_p(k)b_{k,\sigma}^\dagger b_{k,\sigma},$$

(1)

where $\varepsilon_p(k) = k^2/(2m_p) + E_v + eV_{sd}/2$, $m_p$ is the effective mass, and $b_{k,\sigma}^\dagger$ ($b_{k,\sigma}$) is the creation (annihilation) operator of a $p$-type carrier with a wave number $k$ and spin $\sigma = \uparrow$ or $\downarrow$. The photon states are described by

$$H_{ph} = \sum_q \omega_q \left( a_q^\dagger a_q + \frac{1}{2} \right),$$

(2)

where $a_q^\dagger$ ($a_q$) is the creation (annihilation) operator of a photon with a wave number $q$ and an energy $\omega_q$. The normal state in a metal is described by

$$H_{nn} = \sum_{k,\sigma} \left( \frac{k^2}{2m_n} + E_c + \frac{eV_{sd}}{2} \right) c_{k,\sigma}^\dagger c_{k,\sigma},$$

(3)

where $c_{k,\sigma}^\dagger$ ($c_{k,\sigma}$) is the creation (annihilation) operator of an $n$-type carrier and $m_n$ is the effective mass. The electron-photon interaction Hamiltonian in the dipole approximation is given by

$$H_I = \sum_{k,q,\sigma} B_{k,q} b_{k-q,\sigma} c_{k,\sigma} a_q^\dagger + \text{H.c.},$$

(4)

where $B_{k,q}$ is the coupling energy. On the basis of the second order perturbation theory, the number of emitting photon $N_{ph} = \sum_q a_q^\dagger a_q$ is calculated as

$$\langle N_{ph} \rangle = \langle N_{ph}(1) \rangle + \langle N_{ph}(2) \rangle,$$

(5)

$$\langle N_{ph}(1) \rangle = \int_{-\infty}^{t_1} dt_1 \int_{-\infty}^{t_1} dt_2 \langle \chi_0 | H_I(t_1) N_{ph} H_I(t_2) | \chi_0 \rangle,$$

(6)

$$\langle N_{ph}(2) \rangle = \int_{-\infty}^{t_1} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_1} dt_3 \int_{-\infty}^{t_1} dt_4 I(2),$$

(7)

$$I(2) = \langle \chi_0 | H_I(t_1) H_I(t_2) N_{ph} H_I(t_3) H_I(t_4) | \chi_0 \rangle,$$

(8)

where $|\chi_0\rangle \rightarrow |0\rangle \otimes |N\rangle \otimes |P\rangle$.

The BCS theory describes superconducting states,

$$H_{ns} = \sum_{k,\sigma} E_k \gamma_{k,\sigma}^\dagger \gamma_{k,\sigma},$$

(9)

where $E_k = \sqrt{\varepsilon_n(k) + \Delta^2}$, $\xi_n(k) = k^2/2m_n - \mu_n$, $\Delta$ is the pairing potential, and $\gamma_{k,\sigma}^\dagger$ ($\gamma_{k,\sigma}$) is the creation (annihilation) operator of a Bogoliubov quasiparticle. We try to consider effects of superconductivity through the Bogoliubov transformation [10]. The description in Eq. (10), however, is valid within a small energy scale near the Fermi level which is at $\mu_n = E_c + eV_{sd}/2 + \mu_n$ measured from 0.

To apply the BCS theory to the present issue, a rule is necessary to describe the operator in the interaction picture. The transformation connects an electron operator and Bogoliubov operators by

$$c_{k,\sigma}^\dagger(t) = e^{i\mu\gamma(t)} (u_k e^{i\xi_n(k)/2} - s_{-\sigma} v_k e^{-i\xi_n(k) - k,\sigma}),$$

(10)

in $(\chi_0) \cdots (\chi_0)$, where $u_k(\nu_k) = [(1 + (-1)^n) \xi_n(k)/E_k]/[2^{1/2}]$, $s_{\sigma} = 1(-1)$ for $\sigma = \uparrow(\downarrow)$, and $\sigma$ means the opposite spin to $\sigma$. The thermal average of operators is carried out in the local equilibrium. In a $p$-type semiconductor, for instance, the average of operators is calculated in

$$H_p' = \sum_{k,\sigma} \xi_n(k) b_{k,\sigma}^\dagger b_{k,\sigma},$$

(11)

instead of Eq. (1) with $\xi_n(k) = k^2/2m_p - \mu_p$. In a superconductor, the average of the Bogoliubov operators is calculated in Eq. (10). In Eq. (9), $|P\rangle$ means the state vector of $p$-type carrier in the local equilibrium and $|N\rangle$ indicates the BCS state in the local equilibrium.

The time average of the photon number $\langle N_{ph} \rangle$ corresponds to the luminescence intensity and it in the first order perturbation expansion results in

$$\langle N_{ph}(1) \rangle = 2\pi \sum_{k,q,\sigma} |B_{k,q}|^2 f_k q q' \delta(\omega - E_k)$$

$$+ v_k^2 (1 - f_k q) \delta(\omega - E_k),$$

(12)

where $\omega = \omega_q - E_g - \mu_n - \mu_p - \xi_n(k - q)$, $E_g = E_v + E_c + eV_{sd}$, $f_k = [1 - \tanh(E_k/2T)]/2$, and $f_k = [1 - \tanh[\xi_n(k)/2T]]/2$. This result recovers the photon spectra in a normal $p-n$ junction by tuning $\Delta \rightarrow 0$, which means that $E_k \rightarrow -\xi_n(k)$, $u_k \rightarrow 0$, $v_k \rightarrow 1$ for $k < k_F$ and $E_k \rightarrow -\xi_n(k)$, $u_k \rightarrow 1$, $v_k \rightarrow 0$ for $k > k_F$ with $k_F$ being the Fermi wave number satisfying $k_F^2/2m_n = \mu_n$. The threshold of spectra is $E_g$ and the width of spectra is given by $\mu_n + \mu_p$. The spectra in Eq. (13) have a broad profile reflecting the quasiparticle density of states as schematically shown in Fig. 1(b). Equation (13) depends on temperature through the Fermi distribution function and the pair potential. We have, however, numerically confirmed that the dependence is very weak and that Eq. (13) is almost constant below and near above $T_c$. The general features of spectra are determined by the energy scales such as $E_g$, $\mu_n$, and $\mu_p$. They are much larger than $\Delta$.

The results of the second order perturbation are given by

$$I(2) = \sum_{k_1 \cdots k_4, q_1 \cdots q_4, \sigma_1 \cdots \sigma_4} e^{-i\Omega_{k_1} - i\Omega_{k_2} - i\Omega_{k_3} + i\Omega_{k_4}}$$

$$\times B_{k_1, q_1}^* B_{k_2, q_2}^* B_{k_3, q_3} B_{k_4, q_4} Q_{ph} Q_{P} Q_{N},$$

(13)

with $\Omega_{k, q} = \omega_{q_j} - e\epsilon(k_j - q_j) - \mu_n$. The average of the operators $Q_{ph}$, $Q_P$, and $Q_N$ are calculated as follows,

$$Q_{ph} = \sum_{q_5} \langle 0 | a_{q_1} a_{q_2} a_{q_3} a_{q_4} a_{q_5} | 0 \rangle$$

$$= 2(\delta_{q_1 q_2} \delta_{q_3 q_4} + \delta_{q_1 q_4} \delta_{q_2 q_3}),$$

(14)
\[ Q_P = \langle P | \frac{P}{p_{\sigma_1}} | \frac{P}{p_{\sigma_2}} | \frac{P}{p_{\sigma_3}} | b_{p_{\sigma_3}} | b_{p_{\sigma_2}} | b_{p_{\sigma_1}} | P \rangle, \]
\[ = \int_{p_1}^{P} \int_{p_1}^{P} [\delta_{14}^{q} \delta_{15}^{q} \delta_{16}^{q} \delta_{17}^{q} \delta_{18}^{q}], \]  
(16)
where \[ \delta_{ij}^q = \delta_{q_i-q_j}, \]  
\[ \delta_{ij} = \delta_{\sigma_i-\sigma_j}, \]  
\[ \delta_{ij}^p = \delta_{p_i-p_j}, \]  
and \[ p_j = k_j - q_j. \]  
By applying the Bogoliubov transformation, we find,
\[ Q_N = \langle N | (u_k e^{iE_k^{\dagger} \gamma_k^+} \gamma_k, \sigma_k - \sigma_1 u_k e^{-iE_k^{\dagger} \gamma_k, \sigma_k} \rangle \]
\[ \times (u_k e^{iE_k^{\dagger} \gamma_k^+} \gamma_k, \sigma_k - \sigma_2 u_k e^{-iE_k^{\dagger} \gamma_k, \sigma_k} \rangle \]
\[ \times (u_k e^{iE_k^{\dagger} \gamma_k^+} \gamma_k, \sigma_k - \sigma_3 u_k e^{iE_k^{\dagger} \gamma_k^+} \gamma_k, \sigma_k \rangle \]
\[ \times (u_k e^{iE_k^{\dagger} \gamma_k^+} \gamma_k, \sigma_k - \sigma_4 u_k e^{iE_k^{\dagger} \gamma_k^+} \gamma_k, \sigma_k \rangle |N\rangle, \]  
(17)
which gives 12 terms. In what follows, we extract the most dominant contribution in Eq. (17). The average of \[ Q_N \]  
includes the following four terms
\[ Q_N(S) = u_{k_1} u_{k_2} u_{k_3} u_{k_4} \delta_{\sigma_1, \sigma_2} \delta_{\sigma_3, \sigma_4} \delta_{k_1, k_2} \]
\[ \times \delta_{k_1, -k_2} \sigma_j^{k_1, -k_1} e^{iE_k^{l_1} (1 - f_k^{l_1})} e^{-iE_k^{l_1} (1 - f_k^{l_1})} \]
\[ + e^{-iE_k^{l_1} (1 - f_k^{l_1})} e^{iE_k^{l_1} (1 - f_k^{l_1})} \]
\[ - e^{-E_k^{l_1} (1 - f_k^{l_1})} e^{-iE_k^{l_1} (1 - f_k^{l_1})} \]
\[ - e^{iE_k^{l_1} (1 - f_k^{l_1})} e^{-iE_k^{l_1} (1 - f_k^{l_1})} \], \]  
(18)
Equation (18) describes effects of superconductivity on the emission spectra because \[ \delta_{\sigma_i, \sigma_j} \delta_{k_i, -k_2} \]  
means the destruction of two electrons as a Cooper pair. A recombination process in \[ Q_N(S) \]  
is schematically illustrated in Fig. 2(a). The remaining eight terms in \[ Q_N \]  
describe the emitting processes shown in Fig. 2(b) and give the luminescence intensity proportional to \[ \langle N_{ph} \rangle \frac{(1)}{2}. \]  
We will show that \[ Q_N(S) \]  
gives large contribution to the emission spectra at \[ \delta q = q_2 = q_4 = 0 \]  
(\[ \delta q = q_3 = q_4 = 0 \]  
in other words)

**FIG. 2** (color online). Recombination processes in the second order perturbation expansion, where solid, broken, and wavy lines represent the propagation of an electron, a \( p \)-type carrier, and a photon, respectively. In (a), a recombination of a Cooper pair in \( Q_N(S) \) is shown. In (b), a recombination process other than \( Q_N(S) \) is illustrated.
The superconductor is attached to an

due to elastic impurity scatterings \( \tau_0 \). At \( T = 0 \), we obtain
\[
I_0 = I_{0(0)}(0) 2 \alpha^2 / (\sqrt{1 + \alpha^2}(\alpha + \sqrt{1 + \alpha^2})),
\]
where \( \alpha = \alpha_{\Delta, \Delta_0} \) is the pair amplitude at the zero temperature and
\( I_{0(0)}(0) = \pi N_0 / 2 \Delta_0 \) is Eq. (20) at \( T = 0 \) and \( 1 / \tau = 0 \). At
\( T \leq T_c \), we find
\[
\frac{I_0}{I_{0(0)}} = \left\{ \begin{array}{ll}
\frac{c_0 \alpha^2 (\Delta / \Delta_0)^2 \Delta_0 / T}{\alpha \alpha \Delta_0} & \alpha \ll 1, \\
\frac{\alpha^4 \Delta_0 / \Delta}{\alpha^3 \Delta_0} & \alpha \gg 1,
\end{array} \right.
\]
where \( c_0 \) is a constant of the order of unity. In Fig. 3(a), we show \( I_0 \) as a function of temperature for several choices of \( \alpha \), where we describe the dependence of \( \alpha \) on temperature by the BCS theory. The amplitude of \( I_0 \) at \( T = 0 \) is suppressed in the dirty limit as shown in a result with \( \alpha = 0.2 \). The amplitude at \( T = 0 \) increases with increasing \( \alpha \). At
\( \alpha = 1 \), \( I_{0(0)}(0) \) has almost the same amplitude as \( I_{0(0)}(0) \). When we increase \( \alpha \) up to 2.0, the results show a bump just below \( T_c \). Next we consider inelastic scatterings described by \( 1 / \tau_{ie} = C_{ie} (T / T_c) \), where \( C_{ie} \) is a coupling constant and \( p \) depends on scattering sources such as \( p = 1 \) for electron-phonon scatterings and \( p = 2 \) for repulsive electron-electron interaction. In Fig. 3(b), we calculate \( I_0 \) for several choices of \( C_{ie} \) and \( p \). Since \( 1 / \tau_{ie} \to 0 \) at \( T = 0 \), the amplitude is close to \( I_{0(0)}(0) \) at \( T = 0 \). When we decreases \( C_{ie} \), the bump appears below \( T_c \). For \( 1 / \tau \leq \Delta_0 \), the luminescence intensity at \( T \leq T_c \) is then given by
\[
\overline{I_{ph}(2)} = 4 \pi c_0 |B|^4 N_0 \left( \frac{\Delta}{T} \right)^2 \sum_q \delta(\Omega_{k, q}).
\]

Finally we modify Eq. (22) to describe the photon spectra in realistic junctions as shown in Fig. 1(c). A superconductor is attached to an \( n \)-type semiconductor whose thickness \( L_w \) is about 30–50 nm [9]. The proximity effect enables the penetration of Cooper pairs into the \( n \)-type semiconductor. In experiments, photons are emitted mainly from a quantum well which is sandwiched by the \( p \)- and \( n \)-type semiconductor. The pair amplitude in the quantum well can be proportional to \( \Delta \epsilon^{-L_w/\xi_T} \) with \( \xi_T \) is the diffusion constant in the \( n \)-type semiconductor. The quantum well would be replaced by a quantum dot near future. The level in the quantum well (dot) \( E_w \) should coincide with the Fermi level in the \( n \)-type semiconductor \( \mu_n \). Namely, \( |E_w - \mu_n| \) must be less than both the Thouless energy \( E_{Th} = D / L_w^2 \) and \( \Delta \). This resonant condition is particularly important for a Cooper pair to penetrate into the quantum well (dot). The emission spectra have a peak at \( \alpha_0 \) and the peak width is given by \( \Gamma = \sqrt{2 \nu_0} T_n \), where \( T_n \) is the transfer integral between the quantum well (dot) and the semiconductor. The argument above is summarized by an equation for \( T \leq T_c \)
\[
\overline{I_{ph}(2)} = |B|^4 N_0 \sum_q \frac{\Delta^2 \epsilon^{-2L_w/\xi_T}}{(\Omega_q - \Omega_0)^2 + (\Gamma)^2}.
\]