A Social Discounting Model based on Tsallis Statistics.

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Abstract:
Social decision making (e.g. social discounting and social preferences) has been attracting attention in economics, econophysics, social physics, behavioral psychology and neuroeconomics. This paper proposes a novel social discounting model based on the deformed algebra developed in the Tsallis' non-extensive thermostatistics. Furthermore, it is suggested that this model can be utilized to quantify the degree of consistency in social discounting in humans and analyze the relationships between behavioral tendencies in social discounting and other-regarding economic decision-making under game-theoretic conditions. Future directions in the application of the model to studies in econophysics, neuroeconomics, and social physics are discussed.

Keywords: Altruism, Neuroeconomics; Econophysics; Social discounting; Social preferences; Tsallis's statistics

1. **Introduction:**

Social decision-making (Fehr and Camer, 2007; Sanfey, 2007) has been a major topic in social physics, econophysics, and neuroeconomics, in addition to decision under uncerainty (Schinckus, 2009; Takahashi, 2009b) and temporal discounting (Takahashi, 2009a). Studies in behavioral economics have revealed that humans and non-human animals discount the value of rewards as the social distance of the recipient becomes larger (referred to as "social discounting")(Jones and Rachlin, 2006; Osinski, 2009). Recent studies have started to unify theoretical frameworks regarding social preferences, social discounting, decision under risk and uncertainty, and temporal discounting (Jones and Rachlin, 2009). Accumulating evidence indicates the importance of utilization of social discounting model incorporating social distance to social and economic decision-making processes in the field of neuroeconomics: *e.g.,* (i) in economics, experimental economists Hoffman et al., (1996) demonstrated that perception of social distance affected other-regarding behavior in the dictator game, and an economist Akerlof proposed a model which incorporates the role of social distance in social decision-making (Akerlof, 1997), (ii) temporal and probability discounting models have been extensively studied in the field (Weber and Huettel, 2008) and (iii) one recent neuroimaging study reported that social distance is actually represented in human parietal cortex in the brain (Yamakawa et al., 2009). However, in studies in neuroeconomics, the model of social discounting has not fully been utilized. I therefore introduce, in this paper, a novel framework for social discounting utilizing the q-exponential function based on Tsallis' statistics (Tsallis, 1994), of which usefulness has been examined in our previous behavioral studies regarding temporal discounting (Takahashi et al., 2007; Takahashi, 2007b; Takahashi et al., 2008a, 2008b; Takahashi et al., 2009; Takahashi, 2009a). Notably, the q-exponential function is a well-studied function in a deformed algebra inspired by and developed in Tsallis' non-extensive thermodynamics (Tsallis, 1994).

This paper is organized in the following manner. In Section 2, I briefly introduce the psychophysical equivalence of delay and social distance in decision-making processes, based on recent findings in behavioral psychology. In Section 3, I introduce a novel social discounting model based on Tsallis' statistics utilizing the q-exponential function, and explain behavioral neuroeconomic interpretations for the parameters in the model. In Section 4, some conclusions from this study and future study directions by utilizing the present social discounting model are discussed.

2. **A psychophysical equivalence of delay and social distance in decision making**

In econophysics, temporal discounting (delay discounting) has been investigated (Cajueiro, 2006; Takahashi et al., 2007). Also, in the rapidly evolving field of neuroeconomics, neurobiological correlates of temporal discounting has been attracting attention (McClure et al., 2004; Kable & Glimcher, 2007; Hwang et al., 2009; Kim et al., 2009; Takahashi, 2009a). A choice alternative available only after a given delay is worth less (devalued) than one of the same nominal amount available immediately. Such alternatives are said to be discounted by time (delay discounting). The more delayed a reward is, the lower is its delay-discounted value. The function relating discounted value to delay is called a temporal discount function. Delay discounting is known to be well described by the following hyperbolic equation (proposed originally by Mazur, 1987):

$$V(D)=\frac{V(0)}{1+kD}, \qquad \text{(Equation 1)}$$

where V(D) is the discounted value of the delayed reward at delay D, V (0) is the (undiscounted) value of the immediate reward at delay D=0, D is its delay until receipt, and k is a free parameter measuring the degree to which the delayed reward is discounted. Equation 1 predicts that if a person chooses a larger, later reward V_2 (the subjective value of option 2) at delay D_2 over a smaller, sooner reward V_1 (the subjective value of option 1) at delay D_1 (i.e., $D_2 > D_1 > 0$, $V_1(D_1) < V_2(D_2)$, V_1:smaller sooner reward; V_2: larger later reward), preference (i.e., the inequality $V_1 < V_2$) may reverse at a certain time-point, as time passes. In other words, for some $\Delta t > 0$, although $D_2 - \Delta t (:= D_2') > D_1 - \Delta t (:= D_1')$ (>0) still holds, the reversed inequality $V_1(D_1') > V_2(D_2')$ holds. The combination of the two inequalities: "$V_1(D_1) < V_2(D_2)$ and $V_1(D_1') > V_2(D_2')$" indicates preference reversal over the time-interval of Δt, in that although people prefer larger later reward V_2 to smaller sooner reward V_1 before the time passage Δt, they prefer smaller sooner reward V_1 to larger later reward V_2 after the time-passage of Δt. This tendency is referred to as time-inconsistency in behavioral economics (Frederick et al., 2002). This paradoxical phenomenon occurs due to the time-dependency of the time-discount rate $-V(D)'/V = k/(1+kD)$ in equation 1 (the time-discount rate is a decreasing function of delay D). The degree to which the agent demonstrates the "preference reversal" over time can be parametrized by utilizing a q-exponential temporal discount model based on Tsallis' statistics (see equation 4 in Section 3) as a deviation of the exponential discounting which is measured with q-parameter (Cajueiro, 2006: Takahashi et al., 2007; Takahashi, 2007b; Takahashi et al., 2008a, 2008b; Takahashi et al., 2009).

In econophysics (Tsallis et al., 2003) and neuroeconomics (Fehr & Camrer, 2007; Sanfey, 2007), decision making with social interactions under game-theoretic conditions (von Neumann and Morgenstern, 1944) and economic transactions (Anteneodo et al., 2002), such as social discounting (Jones & Rachlin, 2005), has been attracting attention. There is reason to expect that social discounting may also be related to time discounting. Rachlin and Raineri (1992) speculated that social discounting might be described by an equation (a hyperbolic social discounting model) with the form of Equation 1, as follows:

$$\mathrm{V(N)} = \frac{V(0)}{1+sN}, \qquad \text{(Equation 2)}$$

where V(0) and V(N) are as in Equations 1 and 2, N is a measure of social distance, and s is a constant measuring degree of social discounting. Just as (in Equation 1) a larger k describes more impulsive (or less self-controlled) choices, so (in Equation 2) a larger s would describe more selfish (or less altruistic) choices. Jones & Rachlin (2006) further experimentally tested the applicability of Equation 2 to social decision-making, and observed that the hyperbolic social discounting model better fit participants' social decision behavior than an exponential social discounting model:

$$\mathrm{V(N)=V(0)exp(-sN)}. \qquad \text{(Equation 3)}$$

Therefore, it can be said that human social decision-making has "social inconsistency". I have previously proposed that non-reciprocal altruism observed in humans might result from this inconsistency in social decision-making (Takahashi, 2007a). Rachlin's group further examined the psychophysical equivalence of delay and social distance in human decision-making (Rachlin & Jones, 2008). In the study, they reported that social distance and delay are psychophysically equivalent in human psychological space and both delay and social discount functions have hyperbolic forms. Moreover, Jones and Rachlin demonstrated altruism in the public good game was associated with small degrees of (i.e., shallow) social discounting (Jones & Rachlin, 2009), consistent with my previous hypothesis (Takahashi, 2007a).

I have previously proposed a probabilistic choice model (Takahashi, 2007c) utilizing the q-exponential function (Martinez et al., 2008) developed in Tsallis' statistics, based on the mathematical equivalence, originally proposed by Rachlin and Raineri (1991), of delay and probability under the assumption of ergodicity. Also, recent behavioral economic studies including ours revealed that psychophysics of time-perception determines the functional form (e.g., degree of time-inconsistency) of delay-discount function (Takahashi et al., 2008a; Zauberman et al., 2009), supporting my previous hypotheses (Takahashi, 2005; Takahashi, 2006). Taken together, it appears to be a promising direction to establish social discounting models based on the psychophysical equivalence of delay and social distance, by utilizing the q-exponential function based on Tsallis' statistics.

3. **A social discounting model based on Tsallis' statistics**

In order to continuously quantify both impulsivity and time-inconsistency in temporal discounting, in a distinct manner, the following time-discount function (a q-exponential time-discount function) has been proposed by Cajueiro (2006) and empirically examined in our previous studies (Takahashi et al., 2007; Takahashi, 2007b; Takahashi et al., 2008a, 2008b; Takahashi et al., 2009):

$$V(D)=V(0)/\ exp_q\ (kD)=V(0)/[1+(1-q)k_qD]^{1/(1-q)} \qquad \text{(Equation 4)}$$

where $exp_q()$ is the q-exponential function in the deformed algebra inspired by Tsallis' non-extensive thermodynamics, q is a real consistency parameter smaller than 1 for almost all people (Takahashi et al., 2007), and k_q is an impulsivity parameter (V and D have the same definitions as Equation 1). It is to be noted that larger q (<1) values (q closer to 1) correspond to more consistent intertemporal choice; namely, $q\rightarrow1$ corresponds to exponential discounting (complete time-consistency), while q=0, hyperbolic discounting (equation 1). By utilizing the q-exponential time-discount function, we have empirically shown that depressed patients are more impulsive and time-inconsistent in comparison to healthy controls (Takahashi et al., 2008b) and Westerners are more impulsive and time-inconsistent than Easterners (Takahashi et al., 2009). Considering that it is well established, as introduced above, that delay and social distance may be psychophysically equivalent in decision-making processes, it is a logical next step to combine the q-exponential discount model with the experimentally-confirmed psychophysical equivalence of delay and social distance.

Let us now put the social distance N introduced in Equation 2 (Section 2), instead of delay D, in Equation 4. We obtain:

$$V(N)=V(0)/\ exp_q\ (sN)=V(0)/[1+(1\text{-}q)k_qN]^{1/(1\text{-}q)} \qquad \text{(Equation 5)}$$

where N is the social distance between the agent and another subject, and k_q is a parameter of selfishness, i.e., larger k_q values indicate stronger selfishness, and q indicates the degree of “social inconsistency”. This q-exponential social discount model, which is a natural extension of the q-exponential discount function combined with the reported psychophysical equivalence of social distance and delay in human decision processes (Rachlin & Jones, 2008), may allow us to continuously parameterize agents' social consistency in game theoretic interactions and economic transactions. Furthermore, the q-exponential social discount rate= -V’(N)/V(N) is:

$$\text{q-exponential social discount rate}=\frac{k_q}{1+k_q(1-q)N}. \qquad \text{(Equation 6)}$$

Therefore, altruism indicated by V(N>0) is less steeply socially-discounted when N is larger. This behavior corresponds to social inconsistency in social discounting.

4. **Conclusions and implications for social physics, econophysics, and neuroeconomics**

As introduced in Section 3, q in the q-exponential social discounting model may be capable of expressing each subject's social inconsistency in interpersonal interactions in a continuous manner (with smaller q (<1) and k_q values indicating more social-inconsistent and altruistic social decision making, respectively). A promising research direction may be to examine the relationship between pure altruism and the “social inconsistency” parametrized with q in equation 5. As previously hypothesized (Takahashi 2007a), it may be plausible that people behave altruistically even towards mere chance acquaintances and strangers (i.e., people with large social distance N), because their altruism V(N) is not steeply discounted when $N\rightarrow\infty$ (hence the proverb: “a chance acquaintance is a divine ordinance”). Future neuroeconomic studies should examine this hypothesis by utilizing neuroimaging methods to measure the subject’s social distance N represented in the parietal cortex (Yamanaka et al., 2009) and conducting economic games which measure other-regarding behavior (e.g., the dictator game, the trust game, the prisoner’s dilemma game, the public good game, the ultimatum game, etc). Moreover, a recent study by a personality psychologist (Osinski, 2009) reported that the degree of social discounting is related to personality traits such as Agreeableness and Neuroticism. Therefore, future studies should examine whether parameters in the q-exponential social discount model (Equation 5) is also related to personality traits such as empathy (Wheelwright et al., 2006). Since the mathematical characteristics of q-algebra have extensively been examined and generalized (Nivanen et a., 2003) and Tsallis' statistics has been shown to be useful even outside traditional physics (Tsallis, 2002; Takahashi, 2009b), the present formulation of social discounting may readily be utilized in future studies related to social physics, econophysics, and neuroeconomics.

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