Empirical estimation of consistency parameter in intertemporal choice based on Tsallis' statistics.

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Summary

Impulsivity and inconsistency in intertemporal choice have been attracting attention in econophysics and neuroeconomics. Although loss of self-control by substance abusers is strongly related to their inconsistency in intertemporal choice, researchers in neuroeconomics and psychopharmacology have usually studied impulsivity in intertemporal choice using a discount rate (e.g., hyperbolic k); with little effort being expended on parameterizing subject's inconsistency in intertemporal choice. Recent studies using Tsallis' statistics-based econophysics have found a discount function (i.e. q-exponential discount function), which may continuously parameterize a subject's consistency in intertemporal choice. In order to examine the usefulness of the consistency parameter ($0 \leq q \leq 1$) in the q-exponential discounting function in behavioral studies, we experimentally estimated the consistency parameter q in the Tsallis' statistics-based discounting function by assessing the points of subjective equality (indifference points) at seven delays (one week-25 years) in humans (N=24). We observed that most (N=19) subjects' intertemporal choice was completely inconsistent (q=0, i.e. hyperbolic discounting), the mean consistency ($0 \leq q \leq 1$) was smaller than 0.5, and only one subject had a completely consistent intertemporal choice (q=1, i.e. exponential discounting). There was no significant correlation between impulsivity and inconsistency parameters. Our results indicate that individual differences in consistency in intertemporal choice can be parameterized by introducing a q-exponential discount function and most people discount delayed rewards hyperbolically, rather than exponentially (i.e. mean q is smaller than 0.5). Further, impulsivity and inconsistency in intertemporal choice can be considered as separate behavioral tendencies. The usefulness of the consistency parameter q in psychopharmacological studies of addictive behavior was demonstrated in the present study.

1. **Introduction:**

Delay discounting in intertemporal choice refers to the devaluation of a delayed reward compared to the value of an immediate reward [1,2]. Studies in psychopharmacology, psychiatry, behavioral neuroscience, and neuroeconomics have demonstrated that addiction is associated with rapid discounting of delayed rewards [1]. Moreover, inconsistency in intertemporal choice (observed as preference reversal over time), which should never exist in rational decision-makers, according to Becker and Murphy's economic theory of rational addiction [3], is associated with loss of self-control in substance misusers and addicts [1, 2].

Dynamically consistent temporal discounting (exponential discounting), which is typicaly assumed in neoclassical economic theory, follows the exponential equation [1,2,3,4]:

$$V(D) = Aexp(-kD) \qquad \text{(Equation 1)}$$

where V is the subjective value of a reward, A is the (objective) amount of the reward (=a subjective value of an immediate reward), and D is the length of delay until the delivery of reward. The free parameter k is an index of the degree of discounting, i.e., larger k values correspond to steeper delay discounting. The exponential discount function is the same as an equation describing a decay of radioactive matters; in other words, a devaluation rate (=discount rate, defined below in equation 3) is constant and independent of D, which confirms the absence of inconsistency in intertemporal choice. However, psychophramacological, neuroeconomic, and behavioral ecological studies [1,2] have shown that subjects often display preference reversals as time passes, because the following hyperbolic discount equation significantly fits their behavioral data better than Equation 1:

$$V(D)= A/(1+jD). \qquad \text{(Equation 2)}$$

where j is a free parameter. Large j values again correspond to rapid discounting. [2]. It should be noted that a discount rate, an indicator of impulsivity in intertemporal choice, is defined independently of the shape of discount functions, as:

$$\text{(a discount rate)} := |(dV(D)/dD)/V| \qquad \text{(Equation 3).}$$

In exponential discounting, a discount rate = k (an exponential discount rate) is constant over time. In contrast, in hyperbolic discounting, the discounting rate =j *(1+jD)* (a hyperbolic discount rate) is a decreasing function of delay ("increasing patience"), resulting in preference reversal as time passes; psychologically speaking, subjects who discount hyperbolically overestimate patience in intertemporal choice behavior in the future [1,2,3].

2. **The q-exponential discount function**

Recent efforts in psychopharmacology and neuroeconomics to understand neuropsychological mechanisms underlying intertemporal choice have been expended mainly on neuropsychological factors which modulate impulsivity in intertemporal choice parametrized by a (hyperbolic) discount rate alone [1,5,6,7,8]. The lack of significant attention to inconsistency in intertemporal choice is also reflected in the utilization of a discount parameter which only reflects impulsivity but not inconsistency in intertemporal choice (i.e. AUC, Area Under the discount Curve) [9], in psychopharmacological studies. This situation has its limitations, for problematic self-destructive behaviors by substance abusers are strongly associated with their inconsistency (a changeable discount rate), rather than impulsivity (a large discount rate) in intertemporal choice [1,2,10,11] (note that Becker and Murphy's hypothetical rational addicts with a large discount rate do not act against their self-interest because they are consistent in intertemporal choice [3]). This problem with behavioral studies of intertemporal choice is partly due to the absence of good behavioral measures/parameters of (in)consistency in intertemporal choice. To date, qualitative assessment of inconsistencies in intertemporal choice has been performed by comparing the paramers (R-square values) of goodness of fit between exponential and hyperbolic functions, although the R-square value is problematic in nonlinear curve regression with multiple parameters. A major reason for this situation has been that we have not had a discounting function with partial consistency (an intermediate of exponential and hyperbolic functions). An obvious remedy would be to have a continuous parameter of consistency in intertemporal choice.

Recent advances in econophysics and Tsallis' statistical physics (a type of nonextensive thermodynamics), have discovered a q-exponential function, which is a generalization of a usual exponential function [12]. The q-exponental function is defined as:

$$\exp_q(x):=[1+(1-q)x]^{1/(1-q)} \qquad \text{(Equation 4)}$$

where q is a parameter ($0 \leq q \leq 1$), and $\exp_q(x) \rightarrow \exp(x)$ (a usual exponential function) when $q \rightarrow 1$ (proven by utilizing l'Hopital's theorem) [12].

A recent econophysical study on intertemporal choice [13] proposed the q-exponential discount function:

$$V(D)=A/ \exp_q(kD)=A/[1+(1-q)kD]^{1/(1-q)} \qquad \text{(Equation 5)}$$

where A is the objective amount of a reward and k is a parameter of impulsivity at delay D=0 (q-exponential discount rate). We can easily see that when q=0, equation 5 is exactly the same as a hyperbolic discount function (eqation 2), while $q \rightarrow 1$, is the same as an exponential discount function (equation 1). Therefore, it can be hypothesized that this q-parameter can be used to assess a subject's consistency in intertemporal choice in behavioral studies (q=0, with complete inconsistency corresponding to hyperbolic discounting; while q=1, complete consistency, corresponds to exponential discounting). Regarding impulsivity parameters in the discount functions above, only for small kD, impulsivity may be parametrized with k in the exponential discount function (equation 1), because exp(-kD) is approximately equal to 1/(1+kD) for small kD, shown from first-order Taylor expansions of exp(-kD) and 1/(1+kD) in terms of kD (=A(1-kD)). In contrast, kD is arbitrarily taken in the q-exponential discount function, showing the advantage of the q-exponential discount function. To date, no study to our knowledge has actually estimated a human subject's q-parameter in intertemporal chocie.

3. Behavioral data

We conducted a behavioral experiment assessing subjects' discount behavior, by adopting exactly the same procedure of previous studies [1,5] investigating the relationship between addiction and discounting in humans (briefly, A=100,000 yen, with 7 delays ranging from 1 week to 25 years). Subjects were Hokkaido university students (N=24). After conducting the experiment, we estimated, by utilizing nonlinear curve fitting procedure of R statistical language with the constraint of q>0, subjects' impulsivity and inconsistency in intertemporal choice (i.e., *k* and *q* values in equation 5, respectively). The data are presented in Table 1. The data shows that most (79%) subjects' q-parameter was 0, indicating that most people have completely inconsistent intertemporal choice. Also, the group average of q was smaller than 0.5, indicating that average discounting behavior is hyperbolic, rather than exponential, as demonstrated by previous studies [1,2,4]. There was no significant correlation between estimated q and k values ($p>0.05$, Pearson's product-moment correlation analysis), implying that inconsistency and impulsivity are distinct behavioral tendencies; in other words, more impulsive subjects may not always be more inconsistent in intertemporal choice. It should be noted here that when other parameters of impulsivity, e.g., k in the exponental discount function (equation 1) and j in the (simple) hyperbolic discount function (equation 2), no significant correlation between inconsistency and impulsivity was observed ($p>0.05$, Pearson's product-moment correlation analysis), again strongly indicating that human agents' inconsistency and impulsivity in intertemporal choice are distinct. Further,it is interestng to note that only one subject's q-value was 1, suggestng that maybe less than 5% of people are rational decision makers in the economic sense.

4. **Conclusions and implications for behavioral and neuroeconomics**

Our empirical estimation of q-parameter in q-exponential discounting shows that q can express each subject's inconsistency in a continuous manner (with smaller q values indicating more inconsistent intertemporal choices). Future studies should examine whether substance abusers are more inconsistent in intertemporal coice, in addition to impulsivity in intertemporal choice, by utilizing the q-exponential discount function (equation 5). Further, to examine what types of psychopharmacological/behavioral treatments can increase consistency (q-parameter) in such behavior, we may be able to help identify more effective medical treatments for addiction and substance misuse. Moreover, because temporal discounting may be related to risk attitude in decision-making under risk/uncertainty [14], investigating the relationship between the inconsistency parameter in the q-exponential discount function and the risk attitude parameter also based on Tsallis' statistics [15] is important in future studies in econophysics and neuroeconomics.

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Table 1 Characteristics of consistency and impulsivity parameters in q-exponential discount function (equation 5).

ID	q: consistency	k: impulsivity
1	0.123669	0.00757
2	0	0.0001586
3	0	0.001074
4	0	0.00007456
5	0.7383	0.0007525
6	0	0.0006038
7	0	0.0017104
8	0	0.001688
9	0	0.00008109
10	0	0.003362
11	0	0.0021854
12	0	0.0001221
13	0	0.0004698
14	0	0.0001388
15	0.48532	0.01004
16	0	0.000000213
17	0.5492765	0.0017443
18	0	0.001643
19	0	0.00004921
20	0	0.00003236
21	1	0.00008159
22	0	0.000235
23	0	0.0001244
24	0	0.0000513
average	0.1207	0.001416351
S.E.M	0.05748	0.000514473

19 of 24 subjects were completely inconsistent in intertemporal choice. Averaged consistency parameter (0.1207) was smaller than 0.5, indicating that representative human interteporal choice is hyperbolic, rather than exponential. There was no significant correlation between consistency and impulsivity parameters. Note that q=0 and 1 correspond to hyperbolic and exponential discounting, and smaller q values indicate more changeable discount rates (inconsistency in intertemporal choice), and larger k values indicate stronger impulsivity at D=0. S.E.M.=standard error of mean.