EPHOU-12-007
OU-HET 759/2012

Stability of Leptonic Self-complementarity

Naoyuki Haba[1], Kunio Kaneta[1,2], and Ryo Takahashi[1]
[1] *Department of Physics, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan and*
[2] *Department of Physics, Graduate School of Science, Osaka University, Toyonaka, 560-0043, Japan*

We investigate a stability of leptonic self-complementarity such that sum of three mixing angles in lepton sector is 90 degrees. Current experimental data of neutrino oscillation indicates that the self-complementarity can be satisfied within 3σ ranges of each mixing angles. Thus the self-complementarity may be a key to study a flavor physics behind the standard model, and important to discuss its stability. We analyze renormalization group equations in a context of minimal supersymmetric standard model for the self-complementarity. It is seen that one of Majorana phases plays an important role for the stability of self-complementarity. We find some stable solutions against quantum corrections at a low energy. An effective neutrino mass for neutrino-less double beta decay is also evaluated by the use of neutrino parameters giving rise to the stable solutions.

PACS numbers: 14.60.Pq, 12.60.Jv

Neutrino oscillation experiments established that there are two large mixing angles (θ_{12} and θ_{23}) of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix in lepton sector. Then a non-vanishing θ_{13} in the PMNS has been reported by recent long baseline and reactor neutrino experiments [1]. These results can be interpreted by three flavor mixing of neutrinos. Regarding with neutrino masses m_i $(i = 1, 2, 3)$, the neutrino oscillation experiments determine only two mass squared differences, $\Delta m^2_{21} \equiv |m_2|^2 - |m_1|^2$ and $|\Delta m^2_{31}| \equiv ||m_3|^2 - |m_1|^2|$. Therefore, two types of neutrino mass hierarchy are allowed, i.e. normal hierarchy (NH) $m_1 < m_2 < m_3$ and inverted hierarchy (IH) $m_3 < m_1 < m_2$. Further, neutrino experiments have not determined whether the neutrinos are Dirac or Majorana particles. Clearly, the nature of neutrinos would be a key to find physics beyond the standard model (SM).

In theoretical side of neutrino physics, various approaches have been discussed in order to investigate hidden flavor structure behind the SM, e.g. introductions of flavor symmetry, mass (matrix) texture analyses, and searches for exotic relations among flavor mixing angles etc.. In this work, we focus on a leptonic self-complementarity [2] (see also [3] for related discussions) as

$$\theta_{12} + \theta_{23} + \theta_{13} = \frac{\pi}{2} = 90^\circ. \tag{1}$$

The current experimental data of neutrino oscillation indicates that the self-complementarity can be satisfied within 3σ ranges of each mixing angles. Therefore, the self-complementarity may be a key to investigate a flavor physics behind the SM, and important to discuss its stability.

We start with effective Yukawa interaction and Weinberg operator at a low energy scale such as electroweak (EW) scale $\Lambda_{\rm EW}$ in a context of minimal supersymmetric standard model (MSSM),

$$\mathcal{L}_Y = -y_e \overline{L_L} H_d e_R + \frac{\kappa}{2}(H_u L_L)(H_u L_L) + h.c., \tag{2}$$

where L_L are left-handed lepton doublets, e_R are right-handed charged leptons, $H_u(H_d)$ is up(down)-type Higgs, y_e is Yukawa matrix of charged leptons, and $\kappa(H_u L_L)(H_u L_L)$ is the Weinberg operator, which can be effectively induced by integrating out a heavy particle(s). One of examples to obtain this operator is seesaw mechanism. Typical scale of the seesaw mechanism is $\mathcal{O}(10^{14})$ GeV. Therefore, note that the effective coupling κ is having mass dimension -1 and $\kappa^{-1} \sim \mathcal{O}(10^{14})$ GeV. Such a heavy mass scale can realize tiny active neutrino mass scales through the seesaw mechanism. In this work, we utilize an useful parameterization for the PMNS matrix as [4]

$$V_{\rm PMNS} \equiv V_{eL}^\dagger V_\nu D_p = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c \equiv \cos\theta_{ij}$ $(i, j = 1, 2, 3;\ i < j)$, δ is a Dirac phase, and D_p is a diagonal phase matrix including two Majorana phases, ρ and σ. An neutrino mass matrix M_ν can be diagonalized as $V_\nu^\dagger M_\nu V_\nu^* = M_\nu^{\rm diag} \equiv {\rm Diag}\{m_1, m_2, m_3\}$ with $m_i \equiv \kappa_i v_u^2$ where v_u is vacuum expectation value of up-type Higgs.

Next, we consider renormalization group equations (RGEs) in the MSSM. The RGE of κ is given by

$16\pi^2(d\kappa/dt) = \alpha\kappa + [(y_e y_e^\dagger)\kappa + \kappa(y_e y_e^\dagger)^T]$ with $\alpha \equiv 6[-g_1^2/5 - g_2^2 + \mathrm{Tr}(y_u^\dagger y_u)]$ where g_i are gauge coupling constants, t is an arbitrary renormalization scale as $t \equiv \ln(\mu/\Lambda)$, and Λ is a high energy scale such as the seesaw scale [5, 6]. One can also obtain RGEs of θ_{ij} in a diagonal basis of y_e as $d\theta_{ij}/dt = F_{ij}(\theta_{12},\theta_{23},\theta_{13},\kappa_i,\delta,\rho,\sigma,y_\tau;t)$ where right-hand side (RHS) of this equation is given in [6] (see also [7] for other discussions of mixing angles under the RGEs). Now we turn to the self-complementarity relation (1) and investigate the following equation,

$$\frac{d}{dt}\sum_{ij}\theta_{ij} = \sum_{ij} F_{ij}(\theta_{12},\theta_{23},\theta_{13},\kappa_i,\delta,\rho,\sigma,y_\tau;t) \equiv F(\theta_{12},\theta_{23},\theta_{13},\kappa_i,\delta,\rho,\sigma,y_\tau;t), \quad (4)$$

where ij is summed over 12, 23, and 13. The function F is described by 3 mixing angles, 3 effective couplings for the light neutrino masses (or equivalently light neutrino masses m_i), 3 CP-phases, a Yukawa coupling of τ, and renormalization scale. Then once we impose (1) on (4) at an energy scale t_0, one of mixing angles in F is removed as e.g. $\tilde{F}(\theta_{12},\theta_{23},\kappa_i,\delta,\rho,\sigma,y_\tau;t_0)$. We now focus on an equation,

$$\tilde{F}(\theta_{12},\theta_{23},\kappa_i,\delta,\rho,\sigma,y_\tau;t_0) = 0. \quad (5)$$

This equation means that once the equation is satisfied at an energy scale t_0, the self-complementarity is also satisfied at all other energy scales t, i.e. the self-complementarity is stable against quantum corrections, if running effects of parameters except for mixing angles are tiny. In fact, we can find consistent solutions of (5) with experiments for both NH and IH cases. According to the latest experimental data of neutrino oscillation [8]

$$31.3^\circ \lesssim \theta_{12} \lesssim 37.5^\circ, \quad (6)$$
$$38.6^\circ \lesssim \theta_{23} \lesssim 53.1^\circ, \quad (7)$$
$$7.0^\circ(7.3^\circ) \lesssim \theta_{13} \lesssim 10.9^\circ(11.1^\circ), \quad (8)$$

at 3σ level for the NH(IH), the (1) can be satisfied.

Mass spectra of neutrinos at a low energy are defined by $m_1 \equiv \sqrt{m_3^2 - |\Delta m_{31}^2|}$ and $m_2 \equiv \sqrt{m_3^2 - |\Delta m_{31}^2| + \Delta m_{21}^2}$ with best fit values $\Delta m_{21}^2 = 7.62 \times 10^{-5}$ eV^2 and $\Delta m_{31}^2 = 2.53 \times 10^{-3}$ eV^2 for the NH, and $m_1 \equiv \sqrt{m_2^2 - \Delta m_{21}^2}$ and $m_3 \equiv \sqrt{m_2^2 - |\Delta m_{31}^2| - \Delta m_{21}^2}$ with $\Delta m_{31}^2 = 2.40 \times 10^{-3}$ eV^2 for the IH. Therefore, the largest neutrino mass $m_3(m_2)$, of NH(IH) case is a free parameter in our analyses. We analyze in range of $\sqrt{|\Delta m_{31}^2|} \leq m_3 \leq 0.2$ eV ($\sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2} \leq m_2 \leq 0.2$ eV). The case of $m_3(m_2) = 0.2$ eV corresponds to a degenerate mass spectrum. In such case, $m_3(m_2)$ is bounded by a cosmological constraint on the sum of neutrino mass as $\sum m_i \lesssim 0.6$ eV [9], and thus as $m_3(m_2) \lesssim \sum m_i/3 \simeq 0.2$ eV. Therefore, this must be implied as upper bound on the largest neutrino mass. Here note that since input values of neutrino parameters at a low energy are used, the solutions of (5) are corresponding to ones at $t_0 \simeq \ln(\Lambda_{\rm EW}/\Lambda)$. The y_τ has been also approximated at a low energy as $y_\tau(\Lambda_{\rm EW}) = 10^{-2}$ in our analyses.

There are 7 parameters (3 mixing angles, 3 CP-phases, and 1 neutrino mass, m_3 or m_2) and 2 imposed equations ((1) and (5)). Therefore, number of free parameters is 5. Since it is however intricate to deal with all 5 parameters as completely free ones, we numerically analyze at some fixed neutrino masses as examples. According to our analyses, a CP-phase is important to give solutions of (5) with (1); we numerically found that there is no solution to satisfy (5) in cases of ($\delta = \rho = \sigma = 0$) and ($\delta \neq 0,\ \rho = \sigma = 0$) for both NH and IH, but ($\rho \neq 0,\ \delta = \sigma = 0$) and ($\sigma \neq 0,\ \delta = \rho = 0$) can give solutions of (5) in some cases of NH. In a case of NH with a minimal m_3 (i.e. $m_3 = \sqrt{|\Delta m_{31}^2|}$) and all cases of IH, neither ($\sigma \neq 0,\ \delta = \rho = 0$) nor ($\rho \neq 0,\ \delta = \sigma = 0$) can give the solution. Therefore, in the following, we focus on the cases of ($\rho \neq 0,\ \delta = \sigma = 0$) and ($\sigma \neq 0,\ \delta = \rho = 0$) for other cases of NH in detail. Now we have 2 free parameters (one of Majorana phases and one of mixing angles) in order to look for solution, i.e. once we fix one of Majorana phases and one of mixing angles, all values of our parameters are uniquely determined as we will explain below.

We have scanned over $0 \leq (|\rho| \text{ or } |\sigma|) \leq \pi$. Some results of numerical analyses are shown in FIG. 1 as examples. In the figures, vertical and horizontal axes are θ_{23} and θ_{12}, respectively, and shaded region mean that the self-complementarity (1) is correlatively satisfied within 3σ ranges of mixing angles (6)-(8). Lower(upper) slanting and right(left) sides are bounded by maximal(minimal) θ_{13} and θ_{12} at 3σ level, respectively. Note that an allowed range of θ_{23} becomes narrow as $41.6^\circ \lesssim \theta_{23} \lesssim 51.7^\circ$ compared to (7) due to (1). Both two lines in the figures are solutions of (5), which are contours of ρ or σ. FIG. 1 (a) is case of $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$ with ($\rho \neq 0,\ \delta = \sigma = 0$) for the NH, and there are 2 remaining parameters, ρ and one of mixing angles. In the case, 2 lines corresponding to $|\rho| \simeq 124.0^\circ$ and 143.0° are grazing shaded region at upper left described by A and lower right B points, respectively (there are 2 (different sign) solutions of ρ or σ in all cases, i.e. solutions are symmetric for reflection respect with π). Therefore, values of all 7 parameters in our analyses are uniquely determined at e.g. point A or B. Further, the points A and B in all cases determine maximal and minimal values of parameters for the stability of self-complementarity. Therefore, the self-complementarity is stable in shaded regions of $124.0^\circ \lesssim |\rho| \lesssim 143.0^\circ$, $31.3^\circ \lesssim \theta_{12} \lesssim 37.5^\circ$, $7.0^\circ \lesssim \theta_{13} \lesssim 10.9^\circ$, and $\mathrm{Min}[\theta_{23}] \leq \theta_{23} \leq \mathrm{Max}[\theta_{23}]$ where $\mathrm{Max(Min)}[\theta_{ij}]$ is maximal(minimal) value of θ_{ij}. $\mathrm{Max(Min)}[\theta_{23}]$ is evaluated by $\mathrm{Max(Min)}[\theta_{23}] = 90^\circ - \mathrm{Min(Max)}[\theta_{12}] - \mathrm{Min(Max)}[\theta_{13}]$ due to (1). Note that value of one of mixing angle (e.g. θ_{23}) is not independently taken because of (1). In the case, we obtain $\mathrm{Max(Min)}[\theta_{23}] \simeq 51.7^\circ(41.6^\circ)$. The results are summarized in TAB. I.

	NH		
m_3	$\sqrt{\lvert\Delta m_{31}^2\rvert + \Delta m_{21}^2}$		0.2 eV
m_2	1.23×10^{-2} eV		0.194 eV
m_1	8.73×10^{-3} eV		0.194 eV
Phases	$\rho \neq 0,\ \delta = \sigma = 0$	$\sigma \neq 0,\ \delta = \rho = 0$	$\sigma \neq 0,\ \delta = \rho = 0$
Min[$\theta_{12}(\sin^2\theta_{12})$]	$31.3^\circ(0.27)$		
Max[$\theta_{12}(\sin^2\theta_{12})$]	$37.5^\circ(0.37)$		
Min[$\theta_{23}(\sin^2\theta_{23})$]	$41.6^\circ(0.44)$		
Max[$\theta_{23}(\sin^2\theta_{23})$]	$51.7^\circ(0.62)$		
Min[$\theta_{13}(\sin^2\theta_{13})$]	$7.0^\circ(0.015)$		
Max[$\theta_{13}(\sin^2\theta_{13})$]	$10.9^\circ(0.036)$		
Min[$\lvert\rho\rvert$ or $\lvert\sigma\rvert$]	$\lvert\rho\rvert = 124.0^\circ$	$\lvert\sigma\rvert = 124.9^\circ$	$\lvert\sigma\rvert = 101.9^\circ$
Max[$\lvert\rho\rvert$ or $\lvert\sigma\rvert$]	$\lvert\rho\rvert = 143.0^\circ$	$\lvert\sigma\rvert = 153.9^\circ(154.0^\circ)$	$\lvert\sigma\rvert = 104.6^\circ(105.2^\circ)$
Min[$\langle m\rangle_{ee}$] [meV]	6.08	6.33	97.3
Max[$\langle m\rangle_{ee}$] [meV]	9.13	9.42	72.7

TABLE I: Examples of solutions, and minimal and maximal values of neutrino parameters in the corresponding regions: Values of $|\sigma|$ in parentheses are maximal ones giving solutions of (5) but the solutions realized by these maximal values are not stable against running effects of CP-phases. The values of $|\rho|$ and $|\sigma|$ without parentheses are complete stable ones against the running effects of CP-phases.

FIG. 1 (b) shows a case of $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$ with ($\sigma \neq 0,\ \delta = \rho = 0$). In the case, we obtain a region of $|\sigma|$ as $124.9^\circ \lesssim |\sigma| \leq 154.0^\circ$, where the self-complementarity is satisfied, in a similar analysis to the previous case. Regions of mixing angles for the realization of the self-complementarity is the same as the previous case.

We have also analyzed a case of $m_3 = 0.2$ eV. Results are given in TAB. I. We cannot obtain any solutions of (5) in the case of ($\rho \neq 0,\ \delta = \sigma = 0$) but can do in one of ($\sigma \neq 0,\ \delta = \rho = 0$). In the case, values of $|\sigma|$ within $101.9^\circ \lesssim |\sigma| \lesssim 105.2^\circ$ can give the solutions. Allowed region of mixing angles are the same as ones in the $m_3 = \sqrt{|\Delta m_{31}^2|}$ case, i.e. contours of solutions can reach at the both points A and B like the case of $m_3 = \sqrt{|\Delta m_{31}^2|}$. For the Max(Min)[σ], it is determined by Min(Max)[θ_{23}] in contrast with the above two cases.

Of course, other parameters (CP-phases and neutrino masses) contributing to mixing angles evolve under the RGEs. First, we comment on running effects of CP-phases. It has been seen that the Majorana phases are important for the stability of self-complementarity. One may worry about running effects of phases on low energy solutions for the stability, i.e. whether such effects spoil the solutions at a high energy scale or not. We have approximated the running effects of δ, ρ, and σ from the seesaw scale on solutions by using leading-log estimation in the RGEs. These running effects $(\Delta\delta, \Delta\rho, \Delta\sigma)$ are $(\Delta\delta, \Delta\rho, \Delta\sigma) \sim (\mathcal{O}(0.1^\circ), \mathcal{O}(0.1^\circ), \mathcal{O}(0.1^\circ))$ and $(\mathcal{O}(1^\circ), \mathcal{O}(1^\circ), \mathcal{O}(1^\circ))$ for the cases of $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$ and 0.2 eV, respectively. Therefore, most regions of Majorana phases are stable against such small running effects up to the seesaw scale because of $\text{Max}[\sigma] - \text{Min}[\sigma] > \Delta\sigma$. In fact, we show complete stable values of $|\rho|$ and $|\sigma|$, which are described by values without parentheses in TAB. I, including the above running effects from CP-phases. Further, small (but non-vanishing) running effect of δ does not affect the stability of self-complementarity. Then one must remember that our analyses are only for the Majorana neutrino case.

Next, we consider running effects of neutrino masses. We have also evaluated the effects from neutrino masses by the use of leading-log approximation in corresponding RGEs for the neutrino masses, which are also given in [6]. These running effects from the seesaw scale to the electroweak one are

$$(\Delta m_1^{eff}, \Delta m_2^{eff}, \Delta m_3^{eff}) \sim \begin{cases} (\mathcal{O}(10^{-8}), \mathcal{O}(10^{-7}), \mathcal{O}(10^{-6}))\ \text{eV} \\ (\mathcal{O}(10^{-6}), \mathcal{O}(10^{-6}), \mathcal{O}(10^{-6}))\ \text{eV} \end{cases}, \tag{9}$$

for the cases of $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$ and 0.2 eV, respectively, where Δm_i^{eff} affects only evolutions of mixing angles and CP-phases not absolute values of neutrino mass eigenvalues, i.e. overall (flavor mixing independent) contributions from running of Yukawa couplings (top Yukawa gives dominant contribution) are omitted. Even with these running effects of neutrino masses, the solutions given in the TAB. I are stable, i.e. we can also obtain solutions of (5) within the almost same range of $|\sigma|$ as ones in TAB. I. We have also numerically checked evolutions of mixing angles and their sum in order to make sure that the one of solutions in the NH is stable. In the calculation, we take

$$m_1 = 8.73 \times 10^{-3}\ \text{eV}, \tag{10}$$

$$m_2 = 1.23 \times 10^{-2}\ \text{eV}, \tag{11}$$

$$m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}, \tag{12}$$

$$\delta = \sigma = 0,\ \rho = 133.5^\circ, \tag{13}$$

at low energy as an example and

$$\theta_{12} = 33.5^\circ,\ \theta_{23} = 47.7^\circ,\ \theta_{13} = 8.8^\circ, \tag{14}$$

as low energy boundary conditions for the RGEs. The running effects of the mixing angles from the seesaw scale to the electroweak one are

$$(\Delta\theta_{12}, \Delta\theta_{23}, \Delta\theta_{13}) \sim \\ (-\mathcal{O}(10^{-4}), \mathcal{O}(10^{-3}), -\mathcal{O}(10^{-4}))\ [\text{degree}]. \tag{15}$$

Since the deviation of the leptonic self-complementarity from 90° is $\mathcal{O}(10^{-4})$ degree, the self-complementarity relation can be still stable.

Finally, we evaluate effective mass term of neutrino-less double beta decay ($0\nu\beta\beta$), $\langle m\rangle_{ee} \equiv |\sum_{i=1}^{3}(V_{\text{PMNS}})^2_{ei} m_i|$, in our parameter space. It is written down as

$$\langle m\rangle_{ee} = \left|m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2\right|, \tag{16}$$

in our notation. The phenomenon of $0\nu\beta\beta$ can distinguish whether neutrinos are Dirac or Majorana particles. The results at benchmarks given in Tab. I are also presented in the table.

The magnitude of $\langle m\rangle_{ee}$ strongly depends on the scale of m_1 or m_2 rather than mixing angles and CP-phases in the cases. In the NH with $m_3 = \sqrt{|\Delta m_{31}^2|}$ and $\sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$, dominant contribution comes from the second term of RHS of (16) because of the small s_{13}^2 and vanishing m_1. We predict 6.08 meV$\lesssim \langle m\rangle_{ee} \lesssim 97.3$ meV for the NH within the parameter space to make the self-complementarity stable. The Heidelberg-Moscow experiment [10] for $0\nu\beta\beta$ is giving the most severe bound on $\langle m\rangle_{ee}$, which is $\langle m\rangle_{ee} \lesssim 210$ meV. The CUORE experiment [11] is expected to reach $\langle m\rangle_{ee} = (24-93)$ meV in the future. Therefore, a part of our predicting region may be checked in future experiments.

We have investigated a stability of leptonic self-complementarity relation in the PMNS sector against quantum corrections by considering RGEs in the MSSM. The current experimental data of neutrino oscillation indicates that that the self-complementarity can be satisfied at 3σ ranges of each mixing angle. This motivates us to study the self-complementarity and its stability as a key to find a physics behind the SM. As the results of analyses, we have found solutions stabilizing the self-complementarity by using low energy data of neutrino oscillation experiments. It has been seen that the Majorana play an important role to give the solutions. The self-complementarity relation can be satisfied up to an arbitrary high energy scale if neutrino parameters are correlatively within $31.3^\circ \lesssim \theta_{12} \lesssim 37.5^\circ$, $7.0^\circ \lesssim \theta_{13} \lesssim 10.9^\circ$, and Max(Min)$[\theta_{23}] \simeq 51.7^\circ(41.6^\circ)$ with $124.0^\circ \lesssim |\rho| \lesssim 143.0^\circ$ or $124.9^\circ \lesssim |\sigma| \lesssim 153.9^\circ$ for $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$, and $101.9^\circ \lesssim |\sigma| \lesssim 104.6^\circ$ for $m_3 = 0.2$ eV of NH at a low energy. These solutions and leptonic self-complementarity relation are stable against running effects of CP-phases and neutrino masses. Regarding with the $0\nu\beta\beta$, the effective neutrino mass can be predicted as 6.08 meV$\lesssim \langle m\rangle_{ee} \lesssim 97.3$ meV for the stable solution in NH case of $m_3 = \sqrt{|\Delta m_{31}^2|}$.

Acknowledgment: This work is partially supported by Scientific Grant by Ministry of Education and Science, Nos. 22011005, 24540272, 20244028, and 21244036. The works of K.K. and R.T. are supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists.

[1] K. Abe *et al.* [T2K Collaboration], Phys. Rev. Lett. **107** (2011) 041801; P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. Lett. **107** (2011) 181802; Y. Abe *et al.* [DOUBLE-CHOOZ Collaboration], Phys. Rev. Lett. **108** (2012) 131801; F. P. An *et al.* [DAYA-BAY Collaboration], Phys. Rev. Lett. **108** (2012) 171803; J. K. Ahn *et al.* [RENO collaboration], Phys. Rev. Lett. **108** (2012) 191802.

[2] Z. -z. Xing, Phys. Lett. B **696** (2011) 232; Y. -j. Zheng and B. -Q. Ma, Eur. Phys. J. Plus **127** (2012) 7.

[3] S. Luo and Z. -z. Xing, arXiv:1203.3118 [hep-ph].

[4] H. Fritzsch and Z. -z. Xing, Phys. Lett. B **517** (2001) 363.

[5] P. H. Chankowski and Z. Pluciennik, Phys. Lett. B **316** (1993) 312; K. S. Babu *et al.*, Phys. Lett. B **319** (1993) 191.

[6] S. Luo *et al.*, Phys. Rev. D **72** (2005) 053014.

[7] J. A. Casas *et al.*, Nucl. Phys. B **573** (2000) 652; N. Haba *et al.*, Prog. Theor. Phys. **103** (2000) 367; J. R. Ellis and S. Lola, Phys. Lett. B **458** (1999) 310; N. Haba *et al.*, Eur. Phys. J. C **10** (1999) 677; Prog. Theor. Phys. **103** (2000) 145; N. Haba and N. Okamura, Eur. Phys. J. C **14** (2000) 347. N. Haba *et al.*, Prog. Theor. Phys. **103** (2000) 807; Eur. Phys. J. C **17** (2000) 513; N. Haba *et al.*, Phys. Lett. B **489** (2000) 184; S. Antusch *et al.*, Nucl. Phys. B **674** (2003) 401; S. Antusch *et al.*, JHEP **0503** (2005) 024; J. -w. Mei, Phys. Rev. D **71** (2005) 073012 J. -w. Mei and Z. -z. Xing, Phys. Lett. B **623** (2005) 227; S. Luo and Z. -z. Xing, arXiv:1203.3118 [hep-ph]; N. Haba and R. Takahashi, arXiv:1206.2793 [hep-ph].

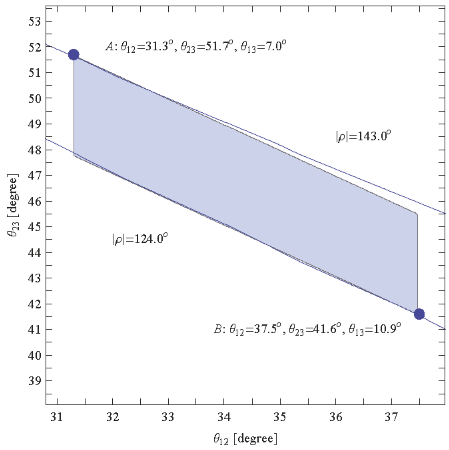
[8] D. V. Forero *et al.* arXiv:1205.4018 [hep-ph].

[9] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192** (2011) 18.

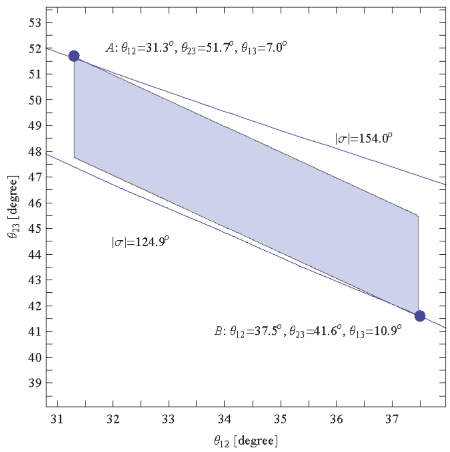
[10] H. V. Klapdor-Kleingrothaus *et al.*, Eur. Phys. J. A **12** (2001) 147.

[11] C. Arnaboldi *et al.* [CUORE Collaboration], Nucl. Instrum. Meth. A **518** (2004) 775; M. Sisti [CUORE Collaboration], J. Phys. Conf. Ser. **203** (2010) 012069; F. Bellini *et. al.*, Astropart. Phys. **33** (2010) 169.

(a) $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$ and $(\rho \neq 0,\ \delta = \sigma = 0)$



(b) $m_3 = \sqrt{|\Delta m_{31}^2| + \Delta m_{21}^2}$ and $(\sigma \neq 0,\ \rho = \sigma = 0)$



(c) $m_3 = 0.2$ eV and $(\sigma \neq 0,\ \rho = \sigma = 0)$

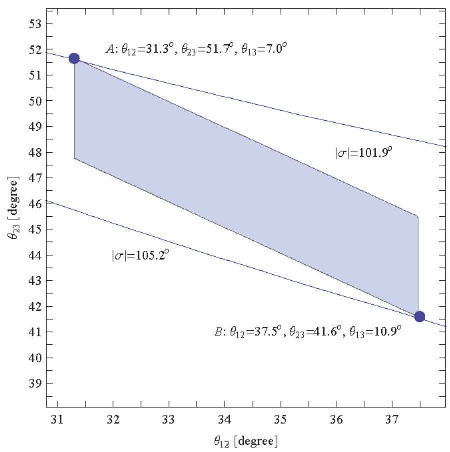


FIG. 1: Examples of solutions of (5)