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Calculation method for computer-generated holograms considering various reflectance distributions based on microfacets with various surface roughnesses

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Computer-generated holograms are generated by three-dimensional technology, and they can be used to reconstruct natural and virtual objects by simulating light waves based on holography. This research was an improvement on previous work that took into consideration reflectance distributions from the various roughnesses of objects. The previous work generated roughness by using a simple model, so that only simple roughness was generated. The proposed method generated more complex roughness than that in the previous work, and the influence of roughness on the reflectance distributions was investigated. Computer simulations, which were compared with the reflectance distributions from the various roughnesses, were carried out. Moreover, computational and optical reconstructions were carried out as examples of reconstructions. As a result of the experiments, we confirmed that the various roughnesses actually influenced the reflectance distributions. © 2011 Optical Society of America

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1. Introduction

Computer-generated holograms (CGHs) involve the use of three-dimensional display technology to reconstruct three-dimensional objects [1]. CGHs are generated by hologram data by simulating the light-wave phenomena of the recording process for optical holography such as reflections, propagations, diffractions, and interferences. Therefore, CGHs can be used to reconstruct natural objects that satisfy the function of human eyes as well as optical holography, and virtual objects created by computers can also be reconstructed.

Rendering techniques for CGHs are a serious issue because the reality of reconstructed objects in CGHs is poorer than that in optical holography. We previously proposed a method that took into consideration reflectance distributions and reflected images for CGHs to improve the reality of reconstructed

objects [2]. Reflectance distributions involve a rendering technique, and these are needed to express various materials and roughnesses of objects. Objects with different materials can be reconstructed by taking into account reflectance distributions although the shapes of objects are the same. Reflected images also involve a rendering technique, and they occur as background reflections. Background reflections are necessary to express metallic objects because observers can see environmental scenes around the metallic objects through these. More metallic objects can be seen by taking into consideration the reflected images mapped on metallic objects surfaces.

Other rendering techniques for CGHs have been reported [3,4]. The polygon model is usually used for reconstructing three-dimensional objects. The technique of hidden surface removal, which only displays objects located at the front, gives us depth information on objects [3]. Matsushima reported the effects of shadow, which also gives us depth information [4]. However, as few researchers have reported reflectance distributions for CGHs, reflectance

distributions should be established to improve the reality of reconstructed objects [5–8].

This paper focuses on reflectance distributions and improvement to the previous work in which reflectance distributions were generated from the roughness of the object surface. Reflectance distributions according to roughness were actually derived by the previous method; however, roughness was derived as simple models of the surfaces of objects. We improved on the previous work that took into consideration more complex roughnesses of object surfaces than the previous method, and investigated reflectance distributions according to roughness. Computer simulations were carried out to investigate these. The intensity of light waves was generated from objects with various roughnesses, and compared with computer simulations. Computational and optical reconstructions were carried out as examples of reconstructions. As a result, we confirmed that the roughness of the object generated by this work influence the reflectance distributions and various reflectance distributions were generated.

This paper is organized as follows. First, Section 2 defines reflectance distributions and describes the algorithm from the previous work that generated reflectance distributions for CGHs. Then, Section 3 describes the problem in the previous work and experiments that compare the influence of reflectance distributions by using various parameters in the previous research. Section 4 provides the examples of computational and optical reconstructions. Finally, Section 5 summarizes this paper.

2. Previous Work

When light waves are incident to an object surface, reflected light has complex distributions. Reflectance distributions are given as these distributions, and they represent the materials and roughness of the object surface. Reflectance distributions are generally defined as having two reflectance components: the first is diffuse reflections, and the second is specular reflections. Figure 1 outlines both reflections: (a) has diffuse reflections, and (b) has specular reflections. Diffuse reflections represent a phenomenon on rough object surfaces such as materials made of paper and calcium sulfate. Incident light reflects from the surface in all directions, and the luminance of the directions is the same. Specular reflections are phenomena on flat object surfaces such as materials

Fig. 1. (Color online) Reflectance distributions: (a) diffuse reflection and (b) specular reflection.

made of mirrors and shining metal. Incident light reflects in a certain direction (specular direction) with strong luminance, and the reflected light has some spread angle. Various materials and roughnesses can be expressed by synthesizing these two reflections. In this paper, specular reflections for CGHs are generated from a rough surface. The specular reflections with various spread angles are given by controlling roughnesses of the surface.

Light waves in CGHs are defined as complex amplitudes, and reflected light on an object surface is calculated by adding phase difference to incident light. Incident light reflects in the specular direction onto the object surface by adding a no-phase difference to the phase, and its reflectance distribution becomes a perfect specular reflection. This approach causes the edge of the object surface to become extremely bright and observers cannot see the whole object. Phase differences are generally generated as randomized numbers such as a white noise from 0 to 2π . Their reflectance distributions become perfectly diffuse reflections, and the whole object can be seen although there is some speckle noise. To reduce the speckle noise, Bräuer et al. [9] generated the phase difference by limiting the spread angle of reflected light, and its reflectance distribution became a nearly perfect diffuse reflection.

A few researchers have reported reflectance distributions for CGHs in other approaches [5–8]. Sakamoto et al. [5] and Nishi et al. [6] have been reported by using the Phong reflection model [10] in computer graphics, and reflected light from the object surface has been generated according to the Phong models. Sakamoto and Tsuruno [7] generated these by using the Cook-Torrance reflection model [11]. These methods have generated reflectance distributions according to their reflection models. Kim et al. [8] generated reflectance distributions by using an object divided into planar meshes. The maximum spread angle of the reflected light could be controlled according to the size of the mesh. Information on the angle of incident light was required to generate reflectance distributions in these methods. Therefore, they were not suited to take into consideration background reflections because light waves were incident to the object surface with various incident angles in the background reflections.

The reflectance distributions for CGHs in previous work [2] were generated according to the roughness on object surfaces by using Blinn and Torrance-Sparrow reflection models in computer graphics [12,13]. Figure 2(a) shows reflections on the object surface by using these reflection models. The object surfaces are divided into many microfacet-like mirrors, and each microfacet has some tilted angle. The roughness of the object surface is constructed with the tilted angles. When light waves are incident to a microfacet, they reflect from the microfacet in the specular direction, which is easily determined by the incident and tilted angles. Reflected light on the object surface is generated by synthesizing

Fig. 2. (Color online) Reflections: (a) on object surface, (b) on microfacet.

all the reflected light on each microfacet. Reflected light with various reflectance distributions is given by controlling the roughness.

Reflected light with reflectance distributions is calculated by adding phase difference according to the roughness of the object surface, which is constructed with many microfacet. Here, we describe how we calculated reflected light with reflectance distributions. We assumed that an object surface would be located on the object plane whose coordinates were (ξ, η) in Fig. 3. Reflected light on the object surface, whose complex amplitude is $g'(\xi, \eta)$, is calculated by

$$g'(\xi, \eta) = g(\xi, \eta) \exp[j\phi_{m,L}(\xi, \eta)], \quad (1)$$

where $g(\xi, \eta)$ is the complex amplitude of incident light on the object surface, j denotes an imaginary unit that satisfies $j = (-1)^{1/2}$, and $\phi_{m,L}(\xi, \eta)$ is the phase difference. The m and L are the parameters of phase difference $\phi_{m,L}(\xi, \eta)$, and it controls the roughness of the object surface. Figure 2(b) shows the reflection on a microfacet. The tilted angles of each microfacet are given by the Gaussian distribution function as

$$D(\theta) = \exp[-(\theta/m)^2], \quad (2)$$

where $D(\theta)$ is the probability, which is from 0 to 1. The θ is the tilted angle of a microfacet, and the tilted

Fig. 3. Coordinate systems for calculations.

angles along the ξ and η axes are calculated according to the equation. The tilted angle is controlled by m ; thus, the roughness of the object surface is controlled by it. The sizes of microfacets are given by L , and it is assumed that the sizes of microfacets are the same about the ξ and η axes. Therefore, phase difference on a microfacet, $\phi'_{m,L}(\xi', \eta')$, is calculated by

$$\phi'_{m,L}(\xi', \eta') = 2d(\xi', \eta') \frac{2\pi}{\lambda}, \quad (3)$$

where (ξ', η') is the local coordinate on a microfacet, $d(\xi', \eta')$ is the difference in distance between a microfacet and the object plane, and λ is the wavelength of light-waves. The difference, $d(\xi', \eta')$, is calculated by $d(\xi', \eta') = \xi' \tan \theta$, where θ is given according to Eq. (2). The phase difference according to the roughness of the object surface is generated by synthesizing all the phase difference on each microfacet. Reflected light is calculated by adding the phase difference to the incident light. Then, hologram data are generated as interference patterns with reference light and object light, which are calculated by propagating the reflected light to the hologram plane from the object plane.

3. Comparison of Reflectance Distributions

As mentioned in the previous section, reflectance distributions are changed by parameters m and L . In computer graphics, luminance of reflected light-beams is calculated according to a viewpoint by using the reflection models. On the other hand, the proposed method calculates reflected light waves from a surface that is constructed by using the reflection models. The reflected light for CGHs has wave properties although the construction of the surface is the same as the computer graphics' reflection models with the parameters m and L . Therefore, the patterns of the reflectance distributions in the method do not agree with that in the computer graphics' reflection models.

In the previous work, we compared with intensity patterns generated by using various parameters m to confirm spread angles of the patterns with m . If m is large, tilted angles whose range is wide are generated, and a rough surface is constructed. The reflected light spreads in various directions and its reflectance distributions become nearly perfect diffuse reflections. However, if m is small, tilted angles whose range is narrow are generated, and a flat surface is constructed. The reflected light only travels a certain direction and its reflectance distributions become nearly perfect specular reflections. Specular reflections between perfectly diffuse and specular reflections are generated by controlling m .

The sizes of microfacets also influence reflectance distributions; however, it was assumed that L was a constant value in the previous work. When light waves diffract on an aperture, the diffraction angle is determined by the size of the aperture. Diffraction angle θ is given by $\sin \theta = \lambda/L$, where L is the size of

the aperture. If L is large, the diffraction angle becomes narrow. If L is a small, on the other hand, it becomes wide. The angle of reflected light on a microfacet is given by the size of the microfacet as well as the diffraction angle on the aperture. Therefore, even if m is a constant value, reflectance distributions can be changed according to the size of the microfacet.

We investigated how parameters m and L influenced reflectance distributions.

A. Computer Simulation

We carried out one-dimensional computer simulations to compare reflectance distributions with various reflectance factors m and sizes of microfacets L . Light-waves are generally calculated on a two-dimensional surface in reconstructions; however, the spread angle of the reflected light about the x - and y -axis directions become almost the same patterns. Therefore, the intensity patterns from one-dimensional surfaces are outputted in order to confirm the difference of the spread angle of the specular reflections with various parameters clearly.

The parameters in computer simulations are listed in Table 1. The object surface was 1.22 mm and it was located on the object plane, and the hologram plane was located 17 mm from the object plane as shown in Fig. 4. Incident light was a plane wave in this experiment, which was parallel to the hologram plane, and light waves were incident to the object surface from the hologram plane. The intensity of object light on the hologram plane was calculated from the reflected light. The object surface was constructed by using various parameters m and L , and reflected light was determined. Then, the reflected light was propagated from the object plane to the hologram plane, and the object light was determined. The intensity of object light versus the angle that was determined according to the origin of the object plane and the coordinate of the hologram plane was output. Object light was propagated with the shifted angular spectrum method [14] as the method of propagation because the range of the angle was narrow in the experiments in the previous work.

Table 1. Parameters for Computer Simulation

Object Plane	
Number of pixels	8192 [pixels]
Pixel pitch	0.30 [μm]
Size	2.43 [mm]
Propagation distance z_0	17 [mm]
Wavelength	632 [nm]
Hologram Planes	
Number of pixels	81920 [pixels]
Pixel pitch	0.30 [μm]
Size	24.32 [mm]
Object Surface	
Number of pixels	4096 [pixels]
Pixel pitch	0.30 [μm]
Size	1.22 [mm]

Fig. 4. Setup for computer simulation.

Here, speckle noise influenced the intensity of reflected light for enabling it to be compared with various m and L . The output intensity was averaged from 10,000 reflected light calculations to reduce the influence of speckle noise. In each calculation, a surface is constructed on each time by using randomized numbers, so the various roughnesses of the surface are constructed with the same parameters. The output intensity corresponds to the statistical averaged value according to the parameters. For reconstructions, the calculation in the above steps is generally a one-time calculation. Although the intensity patterns in reconstructions will not agree with the simulated intensity patterns completely, it is necessary to estimate the spread angles of specular reflections and the luminance of reconstructed objects with various parameters.

B. Intensity Distributions

Figure 5 shows the results from computer simulations by using the conventional method. Figure 5(a) has the results obtained by adding white noise to the phase of incident light, and object light was spread widely. Its reflectance distribution becomes a nearly perfect diffuse reflection. Figure 5(b) has the results obtained by adding no-phase to the phase, and the pattern is the same as the pattern diffracted from an aperture. Its reflectance distribution becomes a perfectly specular reflection.

Figure 6 shows the results obtained by using the proposed method with various m and L . Figure 6(a) is $L = 1$ pixel, (b) is $L = 32$ pixels, and (c) is $L = 4096$ pixels. The spread angle of the object light is controlled by m in each graph. It is wide when m is a large value, and it is narrow when m is a small value. The spread angle is also controlled by L . Compared with Figs. 6(a) and (b), it has spread widely from the object surface constructed by L , which is a large value, although m is a constant value. However, compared with Figs. 6(b) and (c), it is almost the same although the pattern in (c) is not smooth. This is caused by L being too large, i.e., the object surface is constructed with a few microfacets. There are few specular directions on each microfacet, so intensity is given as a nonsmooth pattern. In the case of $L = 4096$ pixels, the surface is constructed by only a

Fig. 5. (Color online) Intensity versus angle with conventional methods: (a) is random phase and (b) is no-phase.

microfacet, and the surface becomes a single planer mirror. The microfacet has various tilted angles in simulations so that the intensity patterns have some spread angle according to the parameters and a non-smooth spread pattern when m is a large value.

The patterns of each intensity are different due to L . Figure 7 also shows the results obtained by using the proposed method with various m and L . Figure 7(a) is $m = 0.50$, (b) is $m = 0.20$, (c) is $m = 0.10$, (d) is $m = 0.05$, and (e) is $m = 0.01$. The reflectance distribution in Fig. 7(a) is a nearly perfect diffuse reflection, that in (e) is a nearly perfect specular reflection, and these in (b–d) are specular reflections with various spread angles. In Figs. 7(a) and (e), the spread angles are almost the same by using various L , although the pattern in (a) is not smooth. The spread angles in (b–d) are controlled by L . The results from these graphs indicate that when the reflectance distributions are nearly perfectly diffuse or specular reflections in Figs. 7(a) and (e), the spread angle of the intensity is almost the same regardless of L . The spread angle is controlled by L when it is a specular reflection with some spread angle.

C. Half-Value Angles

Half-value angles were calculated from the intensity of each m and L to compare the spread angles of object light. A half-value angle means an angle when the intensity is 0.5, and it versus m and L was plotted as a graph. Figure 8 shows the results obtained by using the proposed method. This graph is plotted

as the half-value angles versus m with some kinds of L . The range of m in Fig. 8(a) is from 0 to 1.0, and that in (b) is from 0.01 to 0.1. The half-value angles increase with increasing m because the reflectance distribution with large m becomes a nearly perfect diffuse reflection. Compared with the half-value angles in Fig. 8(a), the patterns that are given by $L = 1, 2$, and 4 pixels have almost the same pattern. However, the patterns that are given by $L = 256$, and 4096 pixels have patterns with non-smooth shapes. This is also caused by L being too large. As the patterns of intensity with large L have nonsmooth patterns, the half-value angles that are given by the patterns also have nonsmooth patterns. Compared with the half-value angles in Fig. 8(b), the half-value angles increase with increasing L .

Figure 9 plots the half-value angles versus L with various kinds of m . When the reflectance distribution becomes a nearly perfect specular reflection whose m is 0.01, the half-value angle is almost the same regardless of L . The half-value angles of $m = 0.05$ and 0.10, whose reflectance distributions become specular reflections with various spread angles, increase with increasing L . Then, the half-value angles increase until some angles. The results from the graphs in Figs. 8 and 9 indicate that when the reflectance distribution is nearly a perfect specular reflection, the half-value angle of the intensity is almost the same regardless of L . When it is a specular reflection with some spread angle, the spread angle is controlled by L .

Fig. 6. (Color online) Intensity versus angle with proposed method: (a) is $L = 1$, (b) is $L = 32$, and (c) is $L = 4096$. Unit of L is pixels.

Fig. 7. (Color online) Intensity versus angle with proposed method: (a) is $m = 0.50$, (b) is $m = 0.20$, (c) is $m = 0.10$, (d) is $m = 0.05$, and (e) is $m = 0.01$. Unit of L is pixels.

Finally, we summarized the relation of the half-value angle versus m versus L in the graph in Fig. 10. Figure 10(a) is output as a three-dimensional view and (b) is output as a top view of (a).

D. Discussion About Reflectance Distributions

The results in the graphs in Figs. 5–10 confirmed that both m and L influence the reflectance distributions. By changing m and L , the spread angle of specular reflection is actually controlled; however, the intensity pattern may not be smooth. When the reflectance distribution becomes a nearly perfect diffuse reflection, L should not be a large value because it influences the smoothness of the intensity pattern. However, when the reflectance distributions become a nearly perfect specular reflection, L does not influence these. When it becomes specular reflec-

tion with some spread angle, various spread angles are derived by changing m and L , and smooth intensity patterns are derived.

However, L should be a small value by taking into consideration the smoothness of the intensity patterns. The intensity pattern generated by various L can also be generated by controlling m , and it is difficult to determine where the limit is between smooth or nonsmooth intensity patterns. In addition, when computational or optical reconstructions are carried out with large L , the rough surface of models are seen, and the luminance of the reconstructed objects are not smooth. The nonsmooth intensity patterns seems to be suited in some cases such as rough surfaces, which are constructed by a large size of facets or meshes. Otherwise, the smooth intensity patterns from a small size of facets are suited for

Fig. 8. (Color online) Half-value angle versus m with proposed method: range of m in (a) is from 0 to 1 and in (b) it is from 0.01 to 0.1. Unit of L is pixels.

Fig. 9. (Color online) Half-value angle versus L with proposed method. Unit of L is pixels.

expression of a flat or smooth surface. As described in Section 2, [8] controlled the reflectance distributions by only varying the size of the mesh. Excessively large meshes may cause the above problems as well as the proposed method. As summarized above, this is an easy and suitable way of controlling the reflectance distributions with various m and small L , i.e., the reflectance distributions should only be controlled by m with small L such as $L = 1$ pixel.

4. Computational and Optical Reconstructions

To compare the reflectance distributions with various m and L , computational and optical reconstructions were carried out. Computational reconstructed images were output by simulating them with a virtual lens in computers from the object light calculated by using the proposed method with various m and L . Objects in these computational and optical reconstructions were two-dimensional squares of 1024×1024 pixels (9.73×9.73 mm) and pixel pitches of $9.5 \times 9.5 \mu\text{m}$, and they were located on the object plane. The hologram plane was located at 292 mm from the object plane, and the parameters are listed in Table 2 and the object image is shown in Fig. 11. The incident light is a plane wave and is incident to the object surface vertically from the positive to the negative z direction. The hologram data in the optical reconstructions were output on a reflective LCD (4K2K LCD, Victor Company of Japan),

Table 2. Parameters for Computational and Optical Reconstructions

Object and Hologram Planes	
Number of pixels	2048×2048 [pixels]
Pixel pitch	$9.5 \times 9.5 \mu\text{m}$
Size	19.5×19.5 [mm]
Propagation distance z_0	300 [mm]
Wavelength	632 [nm]
Object Surface	
Number of pixels	1024×1024 [pixels]
Pixel pitch	$9.5 \times 9.5 \mu\text{m}$
Size	9.73×9.73 [mm]
4K2KLCD (In Optical Reconstructions)	
Number of pixels	4096×2160 [pixels]
Pixel pitch	$9.5 \times 9.5 \mu\text{m}$
Size	38.9×20.5 [mm]

Fig. 11. Object for computational and optical reconstructions.

and a red LED was used as the reference light. Speckle noise was reduced because the LED was an incoherent light source.

The computational reconstructed images are shown in Fig. 12, and the optical reconstructed images are shown in Fig. 13. Figures 12(a) and 13(a) are the computational and optical reconstructed images produced by adding white noise to the phase. Figures 12(b) and 13(b) are those produced by using the proposed method with $m = 0.10$ and $L = 1$ pixel. Figures 12(c) and 13(c) are also those produced by using the proposed method with $m = 0.10$ and

Fig. 10. (Color online) Half-value angle versus m versus L with proposed method: (b) is top-view of (a).

Fig. 12. Computational reconstructed images generated by (a) adding random phase, (b) proposed method with $m = 0.10$ and $L = 1$ pixel, and (c) proposed method with $m = 0.10$ and $L = 8$ pixels.

Fig. 13. (Color online) Optical reconstructed images captured by digital camera. These images are generated by (a) adding random phase, (b) proposed method with $m = 0.10$ and $L = 1$ pixel, and (c) proposed method with $m = 0.10$ and $L = 8$ pixels.

$L = 8$ pixels. Compared with these figures, the proposed method $L = 1$ generates smooth luminance as well as the conventional method, which adds random phase. Thus, a small L is suited to achieve smooth luminance in reconstructed images. The resolution of the LCD in the experiments was not enough to see the reconstructed objects at various angles, so that the problems caused by L being too large were not confirmed at various angles. In future work, we will try to compare the reflectance distributions of the reconstructed images with various m and L at various angles.

5. Conclusion

We improved the previous work that took into consideration the influence of reflectance distributions with various roughnesses. The object surface was divided into many microfacets, and roughness was constructed by synthesizing the microfacets. The roughness in the previous work was only generated by the tilted angles of each microfacet, so that the proposed method improved on the previous research by taking into consideration the size of microfacets. Various roughnesses were generated by taking into consideration the size of microfacets. We investigated what influence the sizes of microfacets had on the

reflectance distributions by using computer simulations. We found the sizes of microfacets actually influenced the reflectance distributions, and varying these and tilted angles generated various reflectance distributions. They were suited to control the reflectance distributions by only varying the tilted angles with small microfacets. We confirmed that smooth luminance was obtained by small microfacets as a result of carrying out computational and optical reconstructions.

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