Influence of temperature and water on subcritical crack growth parameters and long-term strength for igneous rocks

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SUMMARY
Understanding of time-dependent deformation and fracture propagation in rock is essential, since the knowledge of the long-term integrity of rock is required for many subsurface structures excavated in a rock mass. Time-dependent fracture propagation has been invoked as a potential key mechanism responsible for the increase in seismicity preceding earthquake ruptures and volcanic eruptions. In engineering projects, and in preventing natural hazards, the study of subcritical crack growth and the long-term strength of rock is necessary. Since the long-term strength is affected by the values of the subcritical crack growth parameters, it is important to know the influence of the surrounding environment on the subcritical crack growth parameters and long-term strength. The influence of the surrounding environment on the subcritical crack growth parameters, however, has not been completely clarified yet. In this study, the subcritical crack growth parameters were estimated under various environmental conditions on igneous rocks (andesite and granite) using the Double-Torsion method. Based on the results of subcritical crack growth parameters estimations, we calculated the long-term strength of rock. It was shown that the subcritical crack growth parameters were affected by the environmental conditions such as the temperature, humidity and existence of water. Especially, it was shown that the subcritical crack growth index in water was smaller than that in air. When the relative humidity of the air was higher, subcritical crack growth index tended to be smaller. The subcritical crack growth index at 90 per cent relative humidity was close to the value in water. By the calculation based on the results of our subcritical crack growth parameters estimation, it was shown that long-term strength decreased under the conditions of higher temperature, humidity in air and in water. It is concluded that the subcritical crack growth parameters and long-term strength are affected by the surrounding environment, that is, by the temperature, relative humidity and water. To ensure the long-term stability of rock, dry conditions, at low temperature, are most suitable.

Key words: Geomechanics; Defects; Fracture and flow; Fractures and faults; Atmospheric effects (volcano); Volcanic hazards and risks.

1 INTRODUCTION
Since it is essential to consider the long-term integrity of rock, estimation of the long-term strength for rock is necessary for designing and constructing subsurface structures in a rock mass, such as repositories for radioactive waste, caverns to store liquified natural gas or liquified petroleum gas, or underground power plants. For these purposes, a better understanding of time-dependent deformation and fracture propagation in rock is required. In addition, time-dependent fracture propagation has been invoked as a potential key mechanism responsible for the increase in seismicity preceding earthquake ruptures and volcanic eruptions (Main & Meredith 1991; Kilburn & Voight 1998; Main 1999, 2000; Heap et al. 2011). Chen & Jin (2011) suggested that dykes propagate subcritically. Therefore, the study of time-dependent fracturing is essential in both designing engineering projects and preventing natural hazards.

The study of crack propagation and its time-dependent behaviour is important, because rocks generally contain pre-existing cracks or...
flaws (e.g. Nishiyama & Kusuda 1994; Schöld et al. 2001; Nara et al. 2011a). Specifically, subcritical crack growth is considered to be one of the main mechanisms responsible for the time-dependent behaviour of rock in the brittle regime (Atkinson 1982, 1984; Atkinson & Meredith 1987a). It is known that subcritical crack growth in rock depends on both rock fabric and surrounding environment, as well as the level of applied stress. In particular, it has been shown that the preferred orientation of the pre-existing microcracks can be responsible for significant anisotropy of the crack velocity (Sano & Kudo 1992; Nara & Kaneko 2006; Nara et al. 2006). Waza et al. (1980), Meredith & Atkinson (1983), Sano & Kudo (1992) and Nara et al. (2009) showed that the crack velocity in water was higher than that in air. According to Atkinson & Meredith (1981), the type of chemical species also influences subcritical crack growth. In air, it has been clarified that the crack velocity in rock increased with increasing water vapour pressure (Meredith & Atkinson 1985; Nara & Kaneko 2005, 2006), relative humidity (Nara et al. 2010a,b, 2011b) and temperature (Nara et al. 2010a).

For studies of subcritical crack growth, the relation between the stress intensity factor $K_i$ and crack velocity $d_a/dt$ ($K_i$--$d_a/dt$ relation) has been investigated. The $K_i$--$d_a/dt$ relation is expressed as follows (Charles 1958; Wiederhorn & Bolz 1970)

$$\frac{d a}{d t} = A K_i^n$$

(1)

$$\frac{d a}{d t} = v_0 \exp \left( \frac{(-E^* + \beta K_i)}{RT} \right),$$

(2)

where $E^*$ is the stress free activation energy, $R$ is the gas constant, $T$ is the absolute temperature and $A$, $n$, $v_0$ and $\beta$ are constants. Specifically, $n$ is known as the ‘subcritical crack growth index’ (Atkinson 1984; Atkinson & Meredith 1987b) or the ‘stress corrosion index’ (Atkinson 1982; Sano 1988). Eq. (2) can be rewritten as follows:

$$\ln \left( \frac{v}{v_0} \right) = \alpha + \frac{\beta}{R T} K_i$$

$$\alpha = \ln v_0 - \frac{E^*}{R T}$$

(2’)

In this study, we call the constants $A$, $n$, $\alpha$ and $\beta$ as ‘subcritical crack growth parameters’.

Here $n$ and $\beta$ indicate the resistance to subcritical crack growth. Higher or lower values of $n$ and $\beta$ mean greater or less subcritical fracture resistance, respectively (Swanson 1984). The value of $n$ affects the long-term strength (Nara et al. 2010b). It is thus important to determine $n$ accurately for the precise estimation of the long-term strength.

According to Atkinson & Meredith (1987b), values of $n$ for rock range from 8 to 169. Even in the same rock type, different values have been obtained by different researchers. In the case of granite, for example, Sano & Kudo (1992) reported that $n$ of granite is 18–50. By contrast, according to Atkinson & Meredith (1987b), $n$ of granite can be 50–90. It is therefore obvious that $n$ of granite is highly variable. This difference may be due to the difference of the characteristics between rock samples used in different studies. As granite possesses preferred orientation of pre-existing cracks, the above difference may show the anisotropy of $n$.

Sano (1988) reported the experimental methodology to determine $n$ using a Double-Torsion (DT) test. He suggested that the friction and locking on the crack plane in rock were caused due to the roughness of the crack when the load-relaxation method of DT test was conducted, which could affect $n$. Care must be taken in applying loading in order to avoid unwanted effects of friction and locking. Therefore, the differences in the loading condition between the different studies may cause the difference of $n$ between each researcher. According to Nara & Kaneko (2006), $n$ of granite tends to be smaller when the crack propagates parallel to the rift plane where most pre-existing microcracks are distributed. Lajtai & Bielus (1986) determined $n$ in Lac du Bonnet granite using DT test in both air and water and suggested that $n$ tended to be smaller in water, although only one result of $n$ for each condition was shown. It is probable that the large differences in the measured $n$ in granite are due to the influences of surrounding environment and/or the anisotropy of granite. On the other hand, no clear dependence of $n$ was found for Oshima granite and Murata basalt according to Sano & Kudo (1992). Therefore, the influence of the surrounding environment is unclear.

Since the convenience of the integration and differentiation, the power law expressed as eq. (1) has mainly been used for the study of subcritical crack growth in rock (Atkinson 1984; Atkinson & Meredith 1987b). Exceptionally, Nara & Kaneko (2005, 2006) used the exponential law expressed as eq. (2) in order to understand how the applied tensile loading affected the decrease of the energy barrier for the subcritical crack growth. In the case of the exponential law, the influence of the surrounding environment on $\beta$ should be clarified.

In addition, it is essential to know the influence of the surrounding environment on other subcritical crack growth parameters ($A$ in the power law and $\alpha$ in the exponential law), because their evaluations also affect the long-term strength. Therefore, for the long-term integrity of rocks, it is essential to know the influence of the surrounding environment on subcritical crack growth parameters and long-term strength. In order to know the influence of environment, differences in the loading condition should be eradicated and all measurement should be conducted while controlling the surrounding environmental conditions, in order to avoid the possible causes of difference in $n$ described above. In addition, it is important to choose a sample, which provides reproducible results.

It is essential to conduct experiments in water if we want to consider the long-term stability of underground structures, because water is ubiquitous in the upper crust, and most rocks are saturated below a few hundred metres (Heap et al. 2009a). Nara et al. (2010b) indicated that $n$ may depend on the temperature in water. However, it has not been clarified enough, because they have measured $n$ for only one rock at two temperatures. It is also important to investigate other subcritical crack growth parameters ($A$ in the power law and $\beta$ and $\alpha$ in the exponential law) as well as $n$ in order to understand the influence of the surrounding environment on subcritical crack growth and long-term strength.

In this study, we investigate the subcritical crack growth parameters for igneous rock by measuring subcritical crack growth using DT test under uniform loading condition. In order to clarify the influence of temperature, relative humidity and existence of water, all measurements were conducted under controlled temperature and relative humidity. Because different environmental dependency of subcritical crack growth has been found between igneous rocks and sandstones (Nara et al. 2010a, 2011b), we used only igneous rocks as rock samples in this study.

2 Methodology

2.1 Experimental methodology of Double-Torsion test

In order to estimate the subcritical crack growth parameters, it is necessary to determine the $K_i$--$d_a/dt$ relation. To achieve this, we used a DT test, and specifically the load-relaxation (RLX) method. Fig. 1 shows a schematic illustration of the specimen and loading...
configuration for the DT test. The details of this method have been summarized by Williams & Evans (1973) and Sano & Kudo (1992).

All experiments were conducted following the same procedure and loading conditions, with same apparatus as that described in Nara et al. (2009, 2010a).

At first, pre-cracking was conducted. In this procedure, we observed the crack length (using a digital microscope during loading) reach 25 mm, which is the minimum length defined by Trantina (1977). By observing the crack length during pre-cracking, we can make the initial crack length uniform in all experiments. The temperature and relative humidity in the testing room were then set and kept constant, and the apparatus and the DT specimen were exposed to the testing environment for more than 20 hr. Finally, the measurement of subcritical crack growth was conducted with the DT-RLX method in order to obtain $K_{IC} = da/dt$ relation. The details of the loading conditions are provided in Nara et al. (2010a).

2.2 Calculation methodology of long-term strength evaluation

For the evaluation of the long-term strength, we considered a situation where an infinite plate containing a single crack with the length of $2a$ is subjected to an uniform tensile stress $\sigma$. In this case, the stress intensity factor is expressed as follows:

$$K_1 = \sigma (\pi a)^{1/2}. \tag{3}$$

If we use the power law of subcritical crack growth expressed with eq. (1), the following equation can be obtained:

$$\frac{da}{dt} = \pi^{n/2} A \sigma^n a^{n/2}. \tag{4}$$

From eq. (4), the following equation can be obtained:

$$\int a^{-n/2} da = \int \pi^{n/2} A \sigma^n dt. \tag{5}$$

The general solution of the above equation is expressed as follows:

$$\frac{1}{1 - n/2} a^{1-n/2} = \pi^{n/2} A \sigma^n t + c. \tag{6}$$

where $c$ is a constant of integration. Assuming that $a = a_0$ when $t = 0$, the constant $c$ can be determined as follows:

$$\frac{2}{2-n} a_0^{2(n-2)/n} = c. \tag{7}$$

Substituting eq. (7) into eq. (6), the following equation can be obtained:

$$a^{2(n-2)/n} = \frac{2}{2-n} \pi^{n/2} A \sigma^n t + a_0^{2(n-2)/n}. \tag{8}$$

From this equation, the following equation can be obtained:

$$t = \frac{2}{(n - 2) \pi^{n/2} A} \frac{a_0^{2(n-2)/n}}{\sigma^n}. \tag{9}$$

Even though the crack propagates statically, the manner of the crack propagation will change from static to dynamic as time goes by. This can be expressed as follows:

$$t_i = \frac{2}{(n - 2) \pi^{n/2} A} \frac{a_0^{2(n-2)/n}}{\sigma^n}. \tag{10}$$

where $t_i$ is a time-to-failure. Considering the situation that a material reaches failure in $x$ years under a constant stress, this stress is defined as ‘long-term strength’, $S(x)$. Because the time-to-failure is $x$ years ($3.15 \times 10^7$ s) under this stress, the following equation can be obtained from eq. (10):

$$[S(x)]^n = \frac{1}{3.15 \times 10^7} \frac{2}{(n - 2) \pi^{n/2} A} a_0^{2(n-2)/n}. \tag{11}$$

Assuming that the tensile strength and the fracture toughness of a material are $S_i$ and $K_{IC}$, respectively, when the crack length is $a_0$, the relationship between $S_i$ and $K_{IC}$ can be expressed as follows:

$$K_{IC} = S_i (\pi a_0)^{1/2}. \tag{12}$$

From eqs (11) and (12), we can obtain the following equation:

$$S_i = \left\{ \frac{1}{3.15 \times 10^7} \frac{2}{(n - 2) \pi A} \left[ K_{IC} \left( \frac{2(n-2)}{n} \right) \right]^{(n-2)/n} \right\}^{1/n}. \tag{13}$$

Based on the power law, the long-term strength can be estimated by using eq. (13) (Nara et al. 2010b).

Next, we explain how to estimate long-term strength and time-to-failure of a material using the exponential law of subcritical crack growth expressed with eq. (2). If the $K_{IC} = da/dt$ relation is expressed with eq. (2), substituting eq. (3) into eq. (2), the following equation can be obtained:

$$\frac{da}{dt} = v_1 \exp \left( \beta' \sqrt{\alpha} \right)\tag{14}$$

From eq. (14), the following equation can be obtained:

$$\int_{a_0}^{a} \exp \left( -\beta' \sqrt{\alpha} \right) da = \int_{0}^{v_1} dt. \tag{15}$$

If we assume that $a^{1/2} = \xi$, the left-hand side of eq. (15) can be arranged as follows:

$$\int_{a_0}^{a} \exp \left( -\beta' \sqrt{\alpha} \right) da = 2 \int_{\pi}^{\pi} \xi \exp \left( -\beta' \xi \right) d\xi$$

where $c \equiv \beta/\beta'$, the following equation can be obtained:

$$\frac{2}{\beta'} \left( \beta' \sqrt{\alpha_0} + 1 \right) \exp \left( -\beta' \sqrt{\alpha_0} \right)$$

$$- \frac{2}{\beta^2} \left( \beta' \sqrt{\alpha} + 1 \right) \exp \left( -\beta' \sqrt{\alpha} \right). \tag{16}$$
From eqs (15) and (16), we can obtain the following equation:

\[
\frac{2}{\beta^3} \left( \beta \sqrt{a_0} + 1 \right) \exp \left( -\beta \sqrt{a_0} \right) 
- \frac{2}{\beta^3} \left( \beta' \sqrt{a} + 1 \right) \exp \left( -\beta' \sqrt{a} \right) = v_i t. \quad (17)
\]

In eq. (17), the first term on the left hand side is a constant. The second term converges on 0 with increasing \( a \). Therefore, the time \( t \) converges on a constant value with increasing crack length \( a \). In this case, \( a \) increases with elapsed time, and then diverges at a given \( t \). For convenience of calculation, a solution about \( t \) is desirable. Based on this consideration, the following equation is obtained from eq. (17):

\[
t = \frac{2}{v_i \beta^3} \left( \beta' \sqrt{a_0} + 1 \right) \exp \left( -\beta' \sqrt{a_0} \right) 
\times \left[ 1 - \frac{\left( \beta' \sqrt{a} + 1 \right)}{\left( \beta' \sqrt{a_0} + 1 \right)} \exp \left( -\beta' \left( \sqrt{a} - \sqrt{a_0} \right) \right) \right]. \quad (18)
\]

The time-to-failure \( t_i \) is obtained by the following equation assuming that \( a \) diverges in eq. (18):

\[
t_i = \frac{2}{v_i \beta^3} \left( \beta' \sqrt{a_0} + 1 \right) \exp \left( -\beta' \sqrt{a_0} \right). \quad (19)
\]

This equation can be rewritten as follows:

\[
t_i = \frac{2}{(\exp \alpha x \beta / RT) \alpha^2} \left( \beta \frac{RT}{\sigma} \sqrt{\pi a_0} + 1 \right) 
\times \exp \left( -\frac{\beta}{RT} \sigma \sqrt{\pi a_0} \right). \quad (20)
\]

Assuming that the tensile strength and the fracture toughness of a material are \( S_i \) and \( K_i \), respectively, when the crack length is \( a_0 \), the relation between \( S_i \) and \( K_i \) can be expressed as follows:

\[
K_i = S_i \sqrt{\pi a_0}. \quad (12)
\]

Since the time-to-failure is \( x \) years \((3.15 \times 10^7 x \text{ s})\) under this stress, the following equation can be obtained from eq. (20):

\[
3.15 \times 10^7 x = \frac{2}{(\exp \alpha x \beta / RT) \alpha^2} \left( \frac{\beta}{RT} S_i(x) \right) 
\times \left[ \frac{\beta}{RT} S_i(x) K_i + 1 \right] \exp \left( -\frac{\beta}{RT} S_i(x) K_i \right). \quad (21)
\]

By using this equation, we can estimate the long-term strength and time-to-failure of a material based on the exponential law.

In this study, we used the Brazilian tensile strength for \( S_i \) and \( S_i(x) \).

### 3 ROCK SAMPLES

Kumamoto andesite (KA) was selected as a rock sample, because this provided reproducible results (Nara & Kaneko 2005; Nara et al. 2009, 2010a,b). Because granite rock masses have been used for various geomechanical and engineering purposes, we also used Oshima granite (OG) and Inada granite (IG).

KA consists of plagioclase (about 50 per cent), with hornblende and augite (2–3 per cent) as phenocrysts in a fine-grained groundmass (Jeong et al. 2007). Some phenocrysts of about 1–2 mm in length were distributed in the groundmass in KA. In Table 1, P-wave velocities measured in three orthogonal directions are summarized as well as OG and IG. From P-wave velocity measurement, we considered that KA is essentially isotropic. Young’s modulus and Poisson’s ratio measured in air were 31.9 and 0.27 MPa, respectively (Jeong 2003).

OG comprises quartz (36 per cent), plagioclase (37 per cent), K-feldspar (22 per cent), biotite (4 per cent) and hornblende (less than 1 per cent; Sano et al. 1981). IG comprises quartz (36 per cent), plagioclase (32 per cent), K-feldspar (28 per cent), biotite (4 per cent), and small amount of accessory minerals such as allanite, zircon, apatite, ilmenite, etc. (Lin 2002). In OG and IG, orthorhombic elasticity and anisotropy of the crack velocity have been observed (Sano & Kudo 1992; Sano et al. 1992; Nara & Kaneko 2006). Nara & Kaneko (2006) named three principal axes as axis-1, axis-2 and axis-3 in the order of P-wave velocity propagating parallel to these axes. Axis-3 is therefore normal to the Riff plane. The values of P-wave velocity propagating parallel to these axes in OG and IG are summarized in Table 1. In Tables 2 and 3, the orthorhombic elastic compliance of OG and IG are summarized, respectively (Nara & Kaneko 2006). Sano & Kudo (1992), Nara & Kaneko (2006) and Nara et al. (2006) prepared granite DT specimens taking the crack propagation and opening directions into account, as shown in Fig. 2. For example, the crack propagation and opening directions are parallel to axis-2 and axis-3, respectively, in specimen-2-3. Nara et al. (2006) reported that the opening direction of the crack controlled the relationship between the crack velocity and the stress intensity factor for DT test. This means that the properties of the tensile fracturing in granite are controlled by the preferred orientation of pre-existing cracks and their propagation direction. In this study, we used specimen-2-3, specimen-3-2 and specimen-2-1 for OG. For IG, we used specimen-2-1.

### Table 1. P-wave velocity in Kumamoto andesite, Oshima granite and Inada granite.

<table>
<thead>
<tr>
<th>Rock samples</th>
<th>P-wave velocities (km s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KA</td>
<td>4.8 (in three orthogonal directions)</td>
</tr>
<tr>
<td>OG</td>
<td>4.9 (in axis-1), 4.6 (in axis-2), 4.5 (in axis-3)</td>
</tr>
<tr>
<td>IG</td>
<td>4.7 (in axis-1), 4.3 (in axis-2), 4.1 (in axis-3)</td>
</tr>
</tbody>
</table>

### Table 2. Elastic compliance of Oshima granite (after Nara & Kaneko (2006)).

<table>
<thead>
<tr>
<th>Effective compliance ( s_{ij} ) (×10⁻¹² Pa⁻¹)</th>
<th>( j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 16.7 -3.28 -3.28 0 0 0</td>
</tr>
<tr>
<td>2</td>
<td>2 -3.28 18.9 -3.28 0 0 0</td>
</tr>
<tr>
<td>3</td>
<td>3 -3.28 -3.28 19.7 0 0 0</td>
</tr>
<tr>
<td>i</td>
<td>4 0 0 0 0 46.0 0 0</td>
</tr>
<tr>
<td>5</td>
<td>5 0 0 0 0 43.4 0 0</td>
</tr>
<tr>
<td>6</td>
<td>6 0 0 0 0 42.4 0 0</td>
</tr>
</tbody>
</table>

### Table 3. Elastic compliance of Inada granite (after Nara & Kaneko (2006)).

<table>
<thead>
<tr>
<th>Effective compliance ( s_{ij} ) (×10⁻¹² Pa⁻¹)</th>
<th>( j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 18.1 -3.32 -3.32 0 0 0</td>
</tr>
<tr>
<td>2</td>
<td>2 -3.32 21.1 -3.32 0 0 0</td>
</tr>
<tr>
<td>3</td>
<td>3 -3.32 -3.28 23.9 0 0 0</td>
</tr>
<tr>
<td>i</td>
<td>4 0 0 0 0 52.9 0 0</td>
</tr>
<tr>
<td>5</td>
<td>5 0 0 0 0 49.1 0 0</td>
</tr>
<tr>
<td>6</td>
<td>6 0 0 0 0 46.1 0 0</td>
</tr>
</tbody>
</table>
Figure 2. Schematic view of specimen orientations in granite (after Nara et al. (2006)). Fewest pre-existing microcracks are distributed parallel to plane-1, and most parallel to plane-3, respectively.

In Table 4, Brazilian tensile strength and fracture toughness of OG and IG are summarized considering the crack propagation direction, which are necessary to calculate the long-term strength. The values for KA are also summarized in Table 4.

4 RESULTS

4.1 Evaluation of subcritical crack growth parameters

We show the $K_a$–$da/dt$ relation obtained by the DT-RLX method. At first, the influences of water and water vapour on the $K_a$–$da/dt$ relation are shown. In Fig. 3, we show the $K_a$–$da/dt$ relations for KA (see Fig. 3a), OG (specimens-2-3 and -2-1, see Figs b and c) and IG (specimen-2-1, see Fig. 3c) obtained under various humidity conditions or in distilled water. Fig. 3 shows how the crack velocity changes as a function of the stress intensity factor. We see that the crack velocity increases as the stress intensity factor increases. From Fig. 3, it is clear that the crack velocity in distilled water is higher than that in air. It is also clear that the crack velocity increases with increasing relative humidity. If the environment changed from air to water, the crack velocity increased by more than 3 orders of magnitude for the same stress intensity factor.

Next, we show the influence of the temperature on the $K_a$–$da/dt$ relation. Fig. 4 illustrates the $K_a$–$da/dt$ relations for KA (see Fig. 4a) and OG (specimen-2-3, see Figs 4b and c) obtained at various temperatures in air and in distilled water. It is shown that the crack velocity increases with increasing temperature. These results are consistent with the concept of stress corrosion in silicate materials: that the thermally activated chemical reaction between the siloxane bond at the crack tip and water under tension leads to the weakening of the siloxane bond (Anderson & Grew 1977).

By fitting eqs (1) and (2) to $K_a$–$da/dt$ relations shown in Figs 3 and 4 with the least-square method, we can evaluate the values of $n$ and $\beta$. In Fig. 5, we show the plots of $n$ and $\beta$ for igneous rocks in air as a function of the relative humidity. Figs 5(a)–(d) were obtained from the results in this study (see Figs 3b and c). Figs 5(e) and (f) were prepared by analysing the results of Nara et al. (2010a) using eqs (1) and (2) in order to show the dependence of $n$ and $\beta$ on the relative humidity for KA as well as OG. Since Nara et al. (2010a) did not use eq. (2), the values of $\beta$ in Fig. 5(f) are therefore the new results obtained in this study. Fig. 5 shows that $n$ and $\beta$ tend to decrease with increasing the relative humidity in air.

In Fig. 6, the plots of $n$ and $\beta$ as a function of the temperature are shown. It is indicated that $n$ and $\beta$ tend to decrease with increasing the temperature. Only for OG in air, $\beta$ has no dependency on the temperature. From Figs 6(a) and (b), it is clear that the values of $n$ and $\beta$ in air are higher than those in water. On the other hand, Figs 6(c) and (d) indicate that the values of $n$ and $\beta$ in air with high humidity are similar to those in water.

In Tables 5 and 6, we summarize the values of subcritical crack growth parameters. In these tables, we provide the average and standard deviation from two to three samples. From Tables 5 and 6, we see that average values of $n$ and $\beta$ tend to decrease with increasing relative humidity in air. In addition, it is also shown that $n$ and $\beta$ in air tends to be higher than that in water when the relative humidity is low, but becomes similar when the relative humidity reaches around 90% per cent. Tables 5 and 6 also show that the values of $\log A$ and $\alpha$ tend to be higher in air with higher relative humidity and in water. The values of $n$ and $\beta$ tend to decrease with increasing the temperature, and those of $\log A$ and $\alpha$ tend to increase with increasing the temperature.

4.2 Evaluation of long-term strength

Here, we show the results of the long-term strength estimation. In Fig. 7, the relations between the time-to-failure and long-term strength of igneous rocks in air and in water at a similar temperature for KA (Figs 7a and b), OG (Figs 7c and d) and IG (Figs 7e and f) are shown. Figs 7(a), (c) and (e) are obtained from the calculation using eq. (13) based on the power law. On the other hand, Figs 7(b), (d) and (f) are obtained from eq. (21) which is based on the exponential law. From these figures, it is obvious that the long-term strength in water is lower than that in air. Figs 7(c) and (d) shows that the long-term strength decreases with increasing the relative humidity.

In Fig. 8, the relations between the long-term strength and the time-to-failure for igneous rocks in air at a similar relative humidity with various temperatures (Figs 8a and b) and in water with various temperatures (Figs 8c, d, e and f) are shown. Figs 8(a), (c) and (e) are obtained from the calculation using eq. (13) based on the power law. On the other hand, Figs 8(b), (d) and (f) are obtained from eq. (21) based on the exponential law. From these figures, it is clear

<table>
<thead>
<tr>
<th>Rock samples</th>
<th>Fracturing direction</th>
<th>Brazilian tensile strength (MPa)</th>
<th>Fracture toughness (MN m$^{-3/2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KA</td>
<td>Parallel to plane-3</td>
<td>9.50</td>
<td>2.10 (after Nara &amp; Kaneko (2005))</td>
</tr>
<tr>
<td>OG</td>
<td>Parallel to plane-3</td>
<td>6.14</td>
<td>2.15 (after Nara et al. (2012))</td>
</tr>
<tr>
<td>IG</td>
<td>Parallel to plane-1</td>
<td>10.04</td>
<td>1.89</td>
</tr>
</tbody>
</table>
that the long-term strength decreases with increasing temperature. Heap et al. (2009b) reported the increase of the creep strain rate in rock when the temperature increased. This indicates that numerous cracks grow, interact and coalesce in bulk rock samples with increasing temperature, and is in accordance with results in this study.

5 DISCUSSION

5.1 Value of subcritical crack growth index \( n \)

To study subcritical crack growth in rock, most researchers have used the power law (Atkinson & Meredith 1987b) and few researchers have used the exponential law (Nara & Kaneko 2005, 2006) because of the convenience for integration and differentiation. Especially, it is important to discuss the value of \( n \) because various researchers determined \( n \).

Even though several researchers have reported the values of \( n \), the reported values vary even if the same rock sample and same experimental method are used. For example, using the DT-RLX method for Westerly granite, Atkinson & Rawlings (1981) reported that \( n \) was 39 in air at 293 K and 30 per cent relative humidity. Meredith & Atkinson (1985) reported that \( n \) for Westerly granite was 53–56 in air at 293 K with 0.6 kPa water vapour pressure (corresponding to 25 per cent relative humidity). Swanson (1984) reported that \( n \) for Westerly granite was 69 in air at 293 K and 40–50 per cent relative humidity. For Inada granite in air at room temperature, \( n \)
was determined as 18–41 by Sano & Kudo (1992) and around 100 by Kodama et al. (2003). It is useful to consider the reasons for these differences.

Sano (1988) suggested that hysteresis of the $K_1$ vs $da/dt$ relation was observed if the influence of surface friction and locking on the crack plane was different in the DT-RLX method. Since the concept of the DT method does not consider the influence of friction and locking on the crack surface, the obtained $K_1$ vs $da/dt$ relation can be affected if the friction and locking have a significant influence. The value of $n$ will be higher if the influence of friction and locking is larger, because the load-relaxation is suppressed. Sano (1988) also indicated that the measured $n$ was large when the initial load was small and the thickness of the DT specimen was large. Kodama et al. (2003) used a DT specimen of Inada granite with 4 mm of thickness $d_n$ (see Fig. 1), which is more than twice that of Sano & Kudo (1992). This larger thickness can cause a stronger influence of friction and locking, and can cause quite a large value of $n$.

In measurements of $n$ in Westerly granite, Atkinson & Rawlings (1981) and Meredith & Atkinson (1985) used a DT specimen with $d_n = 2$ mm. The thickness $d_n$ in Swanson (1984) was 1.5 mm. Considering that $n$ by Atkinson & Rawlings (1981) was the smallest, the difference in $n$ between these studies was not caused by the friction and locking due to the thickness of the specimen. According to Pletka et al. (1979), $n$ decreases if background relaxation (load relaxation from experimental apparatus) occurs. This may be the

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**Figure 4.** $K_1$ vs $da/dt$ relation obtained under different temperature conditions. (a) Kumamoto andesite in distilled water, (b) Oshima granite (specimen-2) in air with 86–91 per cent relative humidity and (c) Oshima granite (specimen-2) in distilled water.
Figure 5. Plot of $n$ or $\beta$ for igneous rocks in air as a function of relative humidity. (a) $n$ for Oshima granite (specimen-2-3) at 327–330 K, (b) $\beta$ for Oshima granite (specimen-2-3) at 327–330 K, (c) $n$ for Oshima granite (specimen-2-1) at 330 K, (d) $\beta$ for Oshima granite (specimen-2-1) at 330 K, (e) $n$ for Kumamoto andesite at 290–293 K and (f) $\beta$ for Kumamoto andesite at 290–293 K.
Subcritical crack growth parameters for rocks

Figure 6. Plot of $n$ or $\beta$ for igneous rocks as a function of temperature. (a) $n$ for Kumamoto andesite, (b) $\beta$ for Kumamoto andesite, (c) $n$ for Oshima granite (specimen-2·3) and (d) $\beta$ for Oshima granite (specimen-2·3).

Table 5. Summary of subcritical crack growth parameters for Kumamoto andesite, Oshima granite and Inada granite in air with different relative humidities and in distilled water at constant temperature.

<table>
<thead>
<tr>
<th>Rock samples</th>
<th>Temperature (K)</th>
<th>Condition</th>
<th>Humidity or pH</th>
<th>$n$</th>
<th>$\log A$</th>
<th>$\beta$ (m$^{5/2}$ mol$^{-1}$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KA</td>
<td>327–329</td>
<td>Air</td>
<td>49–52 per cent</td>
<td>47 ± 2</td>
<td>−12.9 ± 0.3</td>
<td>0.089 ± 0.005</td>
<td>−59.6 ± 1.6</td>
</tr>
<tr>
<td></td>
<td>344–348</td>
<td>Water</td>
<td>pH = 5</td>
<td>28 ± 2</td>
<td>−5.8 ± 0.5</td>
<td>0.072 ± 0.006</td>
<td>−39.8 ± 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Air</td>
<td>50 per cent</td>
<td>40 ± 2</td>
<td>−11.2 ± 0.3</td>
<td>0.083 ± 0.004</td>
<td>−11.2 ± 0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water</td>
<td>pH = 5</td>
<td>22 ± 3</td>
<td>−5.6 ± 1.1</td>
<td>0.061 ± 0.003</td>
<td>−34.2 ± 3.5</td>
</tr>
<tr>
<td>OG specimen-2·3</td>
<td>327–330</td>
<td>Air</td>
<td>2 per cent</td>
<td>69 ± 4</td>
<td>−22.6 ± 1.2</td>
<td>0.108 ± 0.013</td>
<td>−82.2 ± 4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>71 per cent</td>
<td>49 ± 12</td>
<td>−16.0 ± 3.3</td>
<td>0.082 ± 0.018</td>
<td>−61.7 ± 12.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>86 per cent</td>
<td>41 ± 8</td>
<td>−11.4 ± 1.0</td>
<td>0.080 ± 0.015</td>
<td>−53.2 ± 7.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water</td>
<td>pH = 5–7</td>
<td>36 ± 3</td>
<td>−9.4 ± 0.7</td>
<td>0.076 ± 0.005</td>
<td>−48.0 ± 2.8</td>
</tr>
<tr>
<td>OG specimen-2·1</td>
<td>330</td>
<td>Air</td>
<td>2 per cent</td>
<td>80 ± 22</td>
<td>−27.7 ± 6.9</td>
<td>0.116 ± 0.028</td>
<td>−93.2 ± 21.6</td>
</tr>
<tr>
<td>IG specimen-2·1</td>
<td>293</td>
<td>Air</td>
<td>71 per cent</td>
<td>61 ± 10</td>
<td>−21.6 ± 2.3</td>
<td>0.091 ± 0.016</td>
<td>−73.8 ± 10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water</td>
<td>pH = 7–8</td>
<td>59 ± 14</td>
<td>−14.1 ± 2.7</td>
<td>0.103 ± 0.033</td>
<td>−71.2 ± 19.4</td>
</tr>
</tbody>
</table>

Subcritical crack growth parameters for rocks
reason for the smaller value in Atkinson & Rawlings (1981). It is also possible that the initial load or displacement of the loading point for Swanson (1984) was small, and friction and locking on the crack plane occurred.

Since differences in specimen thickness and experimental conditions can cause differences in $n$, it is difficult to understand the characteristics of $n$ by comparing the values of $n$ between different experimental configurations. In order to learn the characteristics of $n$, we have conducted DT-RLX method experiments in a fixed loading condition following Nara & Kaneko (2005) for KA and Nara & Kaneko (2006) for OG and IG. We set the displacement of the loading point so that the initial load for the DT-RLX test corresponds to the fracture toughness, to decrease the influence of friction and locking as much as possible. We put the experimental apparatus in a room where the temperature can be kept constant during experiment using air conditioning equipment. Therefore, we can avoid background relaxation due to the expansion or contraction of experimental apparatus caused by changes of temperature. Before conducting the DT-RLX test, we checked that there was no load relaxation by applying a large load to rigid stainless steel. It is thus considered that $n$ in this study can be compared directly.

### 5.2 Influence of surrounding environment on the subcritical crack growth index $n$ and $\beta$

From the results in this study, it is found that $n$ and $\beta$ for igneous rocks depend on the temperature, relative humidity and presence of water. $n$ and $\beta$ tend to decrease with increasing temperature and relative humidity. In addition, $n$ and $\beta$ in water are generally smaller than that in air. Subcritical crack growth in igneous rock is affected both by mechanical effects such as the tensile stress, and chemical effects such as stress corrosion. If $n$ and $\beta$ are infinity, subcritical crack growth does not occur. In the case that $n$ and $\beta$ are close to 0, the crack propagation occurs mainly by chemical effects and stress has little influence. In rock, this kind of phenomena is observed for carbonate rock in regions of low stress intensity factor and crack velocity, where crack growth is governed by dissolution (Henry et al. 1977).

On the other hand, the region where $n$ and $\beta$ are close to 0 has not been found yet for igneous rocks. In this study, only the decrease of $n$ and $\beta$ was found under high temperature, high humidity conditions and in water. Taking the tendency of the change of $n$ and $\beta$ for igneous rocks into account, it may be appropriate to consider that stress corrosion affects the values of $n$ and $\beta$, because the stress corrosion process is also highly activated under high temperature, high humidity conditions and in water. In the region where the stress intensity factor and crack velocity are high, the mechanical influence on subcritical crack growth is dominant and chemical influence is relatively small. By contrast, chemical influence becomes dominant in the region where the stress intensity factor and crack velocity are low. In this region, the crack velocity at a constant stress intensity factor will increase when the chemical components such as temperature, humidity and amount of water increase. The values of $n$ and $\beta$ in igneous rocks therefore decrease under higher temperature and humidity conditions or in water as explained in Fig. 9 schematically.

### 5.3 Importance of subcritical crack growth parameters to evaluate the long-term strength

All results in Figs 7 and 8 show the importance of subcritical crack growth parameters on the estimation of the long-term strength. It is clarified that the long-term strength is lower in the environments where $n$ and $\beta$ are smaller and $\log A$ and $\alpha$ are larger. It seems that lower temperature and lower humidity or dry conditions should be important for the long-term integrity of igneous rocks and structures in an igneous rock mass.

The values of $n$ and $\beta$ affect the slope of the relationship between the long-term strength and time-to-failure shown in Figs 7 and 8. If we can make the values of $n$ and $\beta$ higher, the time-to-failure of the material under a constant stress and the long-term strength of the material with a constant time-to-failure will increase.

The values of $\log A$ and $\alpha$ are also important for the long-term strength evaluation. In this study, however, we have evaluated the long-term strength of igneous rocks using $K_I-da/dt$ relations obtained by DT test. The values of $A$ and $\exp(\alpha)$ are corresponding to the values of the crack velocities at $K_I = 1$ [MN/m$^{1/2}$] in the power law (see eq. 1)) and $K_I = 0$ [MN/m$^{1/2}$] in the exponential law (see eq. 2'), respectively. In the case of this study, therefore, the values of $A$ and $\exp(\alpha)$ can be obtained by the extrapolation of $K_I-da/dt$ relation which is obtained from the actual measurement. It is obvious that the values of $A$ and $\exp(\alpha)$ are varied remarkably if the evaluation of $n$ and $\beta$ are changed. Therefore, in the case that the long-term strength is evaluated based on $K_I-da/dt$ relation obtained from DT test, the evaluation of $n$ and $\beta$ are important, because they are evaluated from the actual measurement points. We consider that detailed discussion about the evaluation of $A$ and $\alpha$ should be conducted using some different methodology such as Ko & Kemeny (2011).

### Table 6. Summary of estimation of subcritical crack growth by power law for Kumamoto andesite and Oshima granite with different temperatures.

<table>
<thead>
<tr>
<th>Rock samples</th>
<th>Condition</th>
<th>Humidity or pH</th>
<th>Temperature (K)</th>
<th>$n$</th>
<th>$\log A$</th>
<th>$\beta$ (m$^{3/2}$/mol$^{-1}$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KA</td>
<td>Air</td>
<td>49–55 per cent</td>
<td>293</td>
<td>54 ± 3</td>
<td>–14.6 ± 0.6</td>
<td>0.087 ± 0.004</td>
<td>–65.7 ± 2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>328–329</td>
<td>47 ± 2</td>
<td>–12.9 ± 0.3</td>
<td>0.089 ± 0.005</td>
<td>–59.6 ± 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>348</td>
<td>40 ± 2</td>
<td>–11.2 ± 0.3</td>
<td>0.083 ± 0.004</td>
<td>–112 ± 0.3</td>
</tr>
<tr>
<td>Water</td>
<td>pH = 5–7</td>
<td>284–286</td>
<td>38 ± 2</td>
<td>–7.5 ± 0.4</td>
<td>0.079 ± 0.007</td>
<td>–50.5 ± 2.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>327–328</td>
<td>28 ± 2</td>
<td>–5.8 ± 0.5</td>
<td>0.072 ± 0.006</td>
<td>–39.8 ± 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>344–346</td>
<td>22 ± 3</td>
<td>–5.6 ± 1.1</td>
<td>0.061 ± 0.003</td>
<td>–34.2 ± 3.5</td>
</tr>
<tr>
<td>OG specimen-2</td>
<td>3</td>
<td>Air</td>
<td>86–91 per cent</td>
<td>293</td>
<td>50 ± 21</td>
<td>–14.5 ± 3.3</td>
<td>0.080 ± 0.035</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>330</td>
<td>41 ± 8</td>
<td>–11.4 ± 1.0</td>
<td>0.080 ± 0.015</td>
<td>–53.2 ± 7.1</td>
</tr>
<tr>
<td>Water</td>
<td>pH = 5–7</td>
<td>284</td>
<td>54 ± 6</td>
<td>–12.9 ± 0.2</td>
<td>0.093 ± 0.014</td>
<td>–66.8 ± 6.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>328</td>
<td>36 ± 3</td>
<td>–9.4 ± 0.7</td>
<td>0.076 ± 0.005</td>
<td>–48.0 ± 2.8</td>
</tr>
</tbody>
</table>
Figure 7. Relations between long-term strength and time-to-failure for igneous rocks in air and distilled water. (a) Kumamoto andesite at 344–348 K evaluated by power law, (b) Kumamoto andesite at 344–348 K evaluated by exponential law, (c) Oshima granite fracturing parallel to plane-3 at 327–330 K evaluated by power law, (d) Oshima granite fracturing parallel to plane-3 at 327–330 K evaluated by exponential law, (e) Inada granite fracturing parallel to plane-1 at 293 K evaluated by power law and (f) Inada granite fracturing parallel to plane-1 at 293 K evaluated by exponential law.
Figure 8. Relations between long-term strength and time-to-failure for igneous rocks under various temperature conditions. (a) Oshima granite fracturing parallel to plane-3 in air with 86–91% relative humidity evaluated by power law, (b) Oshima granite fracturing parallel to plane-3 in air with 86–91% relative humidity evaluated by exponential law, (c) Kumamoto andesite in distilled water evaluated from power law, (d) Kumamoto andesite in distilled water evaluated from exponential law, (e) Oshima granite fracturing parallel to plane-3 in distilled water evaluated by power law, (f) Oshima granite fracturing parallel to plane-3 in distilled water evaluated by exponential law.
6 CONCLUSION

In this study, subcritical crack growth in three igneous rocks was measured and the values of subcritical crack growth parameters were estimated using the Double-Torsion test with the load relaxation method. The influences of the temperature, relative humidity and existence of water on the subcritical crack growth parameters were investigated. It was shown that the values of subcritical crack growth index \( n \) and \( \beta \) were smaller, and \( \log A \) and \( \alpha \) became higher in water. Increasing the temperature in both air and water and increasing the relative humidity in air resulted in decreases of the subcritical crack growth index \( n \) and \( \beta \) and increases of \( \log A \) and \( \alpha \). These changes in the subcritical crack growth parameters resulted in a decrease of the long-term strength.

We conclude that information about the influence of the surrounding environment on subcritical crack growth, and hence long-term strength, for igneous rocks is essential for both designing geotechnical projects excavating igneous rock mass and predicting fracturing in rock mass related to earthquake and volcanic hazards. Specifically, from the perspective of lifetime prediction of igneous rock conducted in this study, dry conditions with low temperatures are the most suitable for avoiding time-dependent weakening of igneous rocks and extending the lifetime of structures in the subsurface rock mass.

Considering actual conditions of volcanoes, however, they are essentially massive aquifers. In addition, the temperature of rocks in volcanoes is hot. Similar situation will be found in a geothermal reservoir. Therefore, it is essential to understand the remarkable influence of temperature, humidity and existence of water on long-term integrity of igneous rocks.

ACKNOWLEDGMENTS

The authors would like to thank Prof Tetsuro Yoneda, Prof Toshifumi Igarashi and Prof Naoki Hiroyoshi at Hokkaido University for help and advice with the experimental programme. We also appreciate Dr Ben Lishman at University College London for the useful comments on this paper.

REFERENCES


Figure 9. Schematic illustration of change for slope of \( K_1 \)-da/dt relation (n or \( \beta/RT \)) under different mechanical and chemical influences.


