Linear and nonlinear stability analyses of shallow open-channel flow with lateral velocity gradients

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Linear and nonlinear stability analyses of shallow open-channel flow with lateral velocity gradients
(流速勾配を有する浅い開水路流れの線形・非線形安定解析)

The presence of vegetation is commonly observed in both natural and rectified watercourses. Vegetation in watercourses is desirable in some cases as it prevents bank erosion and provides habitat and food for numerous species. On the other hand, vegetation causes serious problems in other cases as it increases channel resistance and reduces channel capacity for the draining of flood water. Vegetation in a part of a channel produces transverse shear flow, which may lead to flow instability and the generation of large scale horizontal vortices. These horizontal vortices have a strong influence on the velocity distribution and the amount of discharge conveyed by a channel without overflow, and enhance the lateral mixing of not only the flow itself, but also the substances transported by the flow both inside and outside the vegetated area. Therefore, it is important to determine the conditions under which instability occurs, and the characteristics of the horizontal vortices from both an engineering and an environmental points of view.

In this study, we perform temporal linear and nonlinear stability analyses of flow in an open channel partially covered with vegetation. By not employing the rigid-lid and the inviscid flow assumptions, we could study the effects of the Froude number and the kinematic eddy viscosity, respectively, on the growth rate of perturbations. We employ the St. Venant shallow water equations with the Reynolds stress included to reproduce the velocity gradient due to the differential drag between the regions with and without vegetation. The temporal and spatial variations of the flow vanish except for the lateral variation of the streamwise velocity in the base state, which is used as a starting point of the stability analysis. This base state flow field is not, however, just a temporal average of flow affected by fully-developed horizontal vortices, but the flow undisturbed by the vortices. We thus employ a kinematic eddy viscosity representing turbulence with a length scale smaller than the flow depth. This eddy viscosity is estimated for the flow unaffected by the large-scale horizontal vortices.

We impose perturbations on the base state flow velocities and flow depth, and study how various hydraulic parameters affect the time development of the perturbations. In the scheme of linear analysis, the amplitude of the perturbations is assumed to be infinitesimally small, such that terms including the second order of the amplitude parameter vanish. The perturbed equations are solved by employing a spectral collocation method with the Chebyshev polynomials. The conditions required for perturbations grow in time were studied over a wide range of (1) Froude number, (2) normalized non-vegetated zone width, and three other dimensionless parameters which represent the relative effect of (3) bed friction, (4) vegetation drag and (5) sub-depth eddy viscosity. Our results indicate that, while the base state flow field is unstable in the range of typical, moderate values of the hydraulic parameters, stability is retained in the range of sufficiently small and large vegetation densities, small widths of
the vegetated zone, large bed shear effect, large sub-depth eddy viscosity effect, and moderate Froude numbers where the flow is stable to both the transverse mixing and the gravity. The growth rate of perturbations could be evaluated for Froude numbers far from zero because the rigid-lid assumption was not used. The use of a theoretical sub-depth kinematic eddy viscosity unaffected by the lateral motions permitted a consistent estimation of the growth rate of perturbations. Assuming that the characteristic wavenumber and frequency of perturbations associated with maximum perturbation growth rate correspond to those of vortices realized in experiments, we compared predicted and observed vortex shedding frequencies. The analysis was shown to be capable of predicting the order of magnitude of the vortex shedding frequencies, yet there is a systematic discrepancy in the predicted frequencies when compared to the observed frequencies. This discrepancy, typically in the range of a factor of approximately two, may be caused by the limitation of linear stability analysis.

The predictive theory of the present study also included a weakly nonlinear stability analysis. By using the term weakly, it means that the time variation of the amplitude is much slower than the time variation of the wavelike part of the disturbance. We made use of the multiple scale analysis, a perturbation technique where the neutral instability, i.e., the condition where the growth rate of the perturbations is zero, is deviated towards the unstable region, i.e., where the perturbation grows with time. The ratio between the minimum and maximum velocities in the channel was taken as the parameter for expansion in the non-linear analysis. The perturbed equations in the nonlinear analysis are solved employing a spectral method with the Chebyshev polynomials, as in the linear analysis. The resulting linear system is manipulated and we obtain the Landau equation. From the Landau equation, we can determine the bifurcation pattern, which was found to be typically supercritical, i.e., where the absolute value of the amplitude grows up to an upper bound as the time tends to infinity. The theoretical results of the nonlinear analysis were validated through a comparison with experimental measurements from previous studies. In these experiments, vortices were generated in a channel featuring a lateral array of pile dikes. We predicted supercritical bifurcation for 13 out of the 16 experimental runs evaluated, although an equilibrium amplitude was observed in all runs. The three cases where we could not predict supercritical bifurcation had in common a very small aspect ratio, such that the shallow water hypothesis may no longer be valid.

The nonlinear model was able to predict the shear layer width and the maximum friction velocity reasonably well. The gradient of the time-averaged velocity was also well represented by the model. The eddy viscosity as a function of the perturbed velocity could be derived and it was found to be relatively coherent with the empirical constant values from previous studies. Therefore, the present model was shown to be capable of predicting the large scale eddy viscosity from a formulation which included a sub-depth eddy viscosity completely unaffected by the large scale lateral motions.