Joint Action Syntax in Japanese Martial Arts

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Abstract

Participation in interpersonal competitions, such as fencing or Japanese martial arts, requires players to make instantaneous decisions and execute appropriate motor behaviors in response to various situations. Such actions can be understood as complex phenomena emerging from simple principles. We examined the intentional switching dynamics associated with continuous movement during interpersonal competition in terms of their emergence from a simple syntax. Linear functions on return maps identified two attractors as well as the transitions between them. The effects of skill differences were evident in the second- and third-order state-transition diagrams for these two attractors. Our results suggest that abrupt switching between attractors is related to the diverse continuous movements resulting from quick responses to sudden changes in the environment. This abrupt-switching-quick-response behavior is characterized by a joint action syntax. The resulting hybrid dynamical system is composed of a higher module with discrete dynamics and a lower module with continuous dynamics. Our results suggest that intelligent human behavior and robust autonomy in real-life scenarios are based on this hybrid dynamical system, which connects interpersonal coordination and competition.

Introduction

Nonlinear dynamics has revealed that complex phenomena, from chemical reactions to the neural networks in the brain, emerge from simple principles. Examples of complexity theory include self-organization in thermodynamics theory [1], the slaving-principle in lasers [2], spatiotemporal chaos in fluid dynamics [3], and the synchronization of nonlinear-coupled oscillators [4–6]. Humans are considered to be complex systems as well. In response to various situations, people make instantaneous decisions and execute appropriate motor behaviors. Typical examples of this include the processes involved in interpersonal competition, such as fencing or Japanese martial arts originating from Samurai traditions.

The ability of humans to control complex cognitive processes, which is essential for what we recognize as intelligent behavior (e.g., decision making), depends on the prefrontal cortex (PFC) [7–9]. All goal-directed behaviors are learned and thus depend on a cognitive system that can grasp the rules of a game, the goals available, and the means used to achieve these goals. To this end, PFC activity exerts a top-down influence by providing excitation signals to bias other brain systems towards task-relevant information. This suggests that the PFC plays a role in the mapping of sensory inputs and internal states, such as the mapping between the current motivational state and memories or voluntary actions. PFC mapping can be described by the Hidden Markov Model (HMM), which holds that switches between states proceed according to conditional probabilities [10]. The HMM has been widely used to construct probabilistic language models in natural language processing and computational linguistics [11]; this model is regarded as an automation model in complex sciences. Human intentional dynamics and decision making have been modeled by neuropercolation based on the graph theory [12], which is a generalization of cellular automata [13,14] (i.e., PFC activity is considered to be a discrete dynamical system).

In contrast, complex human movements have been examined using the continuous model of a dynamical system. The Haken-Kelso-Bunz (HKB) model [15] was derived from the theory of nonlinear oscillators and synergetics [2,16,17], which was based on the observation of phase transitions for two-finger experiments [18,19]. These experiments have shown the abrupt change from one stable state to another for critical values as the movement frequency gradually increases. In a discrete dynamical system, this frequency would be a bifurcation parameter, and the phase differences of movements would be regarded as collective variables. The HKB model is based on the synchronization of nonlinear-coupled oscillators. If the system consists of two coupled oscillators, then the system has two stable states: in-phase synchronization and anti-phase synchronization. Pitchfork bifurcation describes the change from these two stable states to one stable state (i.e., from anti- to in-phase synchronization to in-phase synchronization). This model is commonly applied to interlimb coordination and/or perception-action coordination in human movements [20]. Interpersonal coordination has also been studied in terms of the coupling between two oscillators using...
visual information based on this model [21–23]. For example, synchronization among six people during the swinging of rocking chairs was examined using the Kuramoto order parameter [24,25]. Synchronized patterns among three people who communicated via perceptual information during sports activities was also confirmed based on the coupled-oscillators approach derived from symmetric Hopf bifurcation group theory [26,27].

However, little is known about the dynamics underlying the continuous abrupt switching behavior observed in martial arts in which both quick decision making and execution are required. In this study, to clarify the intentional switching dynamics during interpersonal competition we observed a time series of the interpersonal distances (IPDs) between two players based on their moving trajectories during 24 matches of Japanese fencing or kendo from the viewpoint of a hybrid dynamical system [28,29]. Analogous to words and sentences in language, numerous complex behavioral patterns during interpersonal competition could be organized by syntactical rules that can be considered "action syntax" [30,31]. Grooming in rodents has been examined as "action syntax" and regarded as a Markov chain [32–34]; however, these stereotypical actions can be generated by a relatively simple feed-forward excitatory mechanism that cannot adapt to environmental changes, such as interpersonal competition. In response to various situations, very large numbers of movements can be generated in large strongly recurrent connected systems equipped with appropriate rules [31]. We first attempted to extend stereotypical "action syntax" to adaptive "joint action syntax" during complex interpersonal competition characterized by quick decision making and rapidly executed actions.

Methods

Participants

Twelve male members of the University of Tsukuba kendo club participated in the experiment. This club has won the kendo championship in the annual team competition for all Japanese universities three times since 2000. All participants were healthy. Six regular players on the team had expert status; their average age was 20.67±0.75 years, and they had an average of 14.17±1.77 years of kendo experience. Six substitute players held intermediate status; their average age was 21.17±1.57 years, and they had an average of 13.83±0.69 years of kendo experience. All participants provided written informed consent prior to the experiments. The participants in this study have given written informed consent for their photographs to be published, as outlined in the PLOS consent form. Procedures were approved by the Internal Review Board at the Research Center of Health, Fitness, and Sports at Nagoya University and conformed to the principles expressed in the Declaration of Helsinki.

Task

Each of the six players at both skill levels were matched against four different opponents of the same skill level. If one player had been matched against five other players in a round-robin system, a total of 15 matches would have been played. However, because no player would compete against one particular player, a total of 12 matches were played at each level. Following official kendo rules, each match lasted 5 min and was played on a square court with 11.00-m sides (Figure 1A–B, Figure S1A, and Video S1). Each match was judged by three referees. We observed 37.42±5.45 instances of striking and 2.33±1.25 points per match for experts and 37.42±10.11 instances of striking and 2.17±1.67 points per match for intermediate players.

Figure 1. Scene selection. (A) Kendo match. (B) Trajectories of two players during a kendo match over a 5-min period in a two-dimensional plane \((x,y)\). (C) Time series of interpersonal distance (IPD) for one match. (D) Time series of IPD for one sequence eliminating unrelated scenes. (E) Time series of IPD for one scene that begins with the contestants at the greatest distance from one another and ends with them coming together in a striking action. doi:10.1371/journal.pone.0072436.g001

Experimental Devices

Players’ movement trajectories were recorded using an optical motion capture system with eight different cameras (100 Hz, Oqus300, Qualysis, Inc.) and a Movie camera positioned at various locations near the court. Large reflective markers were attached to the back of each player’s head, the back of his waist, both ankles, the right knee, and the top of the shinai or fencing foil to detect movement (Figure S1B, S1C, and Video S1).

Scene Selection

First, an experimenter eliminated unrelated scenes in which the match was stopped by the referees. Additionally, scenes in which the reflective markers could not be seen because the players were outside the camera angles were also removed (Figure 1C). As a
result, the analyzed data averaged 4 min 19 s±11 s for experts and 4 min 26 s±22 s for intermediate players per match, although the matches lasted 5 min from start to finish. Each of the 12 matches was divided into 67 and 54 sequences for expert and intermediate competitors, respectively (Figure 1D). We divided each scene (which included only one striking action) because interpersonal competition in kendo is interrupted by the striking action, and movements after the striking action were considered transitions to the next competition. We detected positive and negative peaks in each sequence to identify the striking action. The positive peaks corresponded to moments of approaching movements between competitors; negative peaks corresponded to moments of detaching movements. A quick detaching movement was defined as a movement in which two adjacent peaks of IPD occurred at a frequency of 6 Hz. The time series for the Euclidean distance and flattened using a fourth-order Butterworth filter with a cutoff frequency of 6 Hz. The trajectories of the player’s head position were expressed as composite vectors of two time-dependent vectors calculated using the following equation:

$X(t) = \sqrt{(\text{normalized } X_{IPD}(t))^2 + (\text{normalized } V_{IPD}(t))^2}$

(5)

Return Map and State Transition Analysis
The peak detection of $X_{IPD}(t)$ in each scene was calculated using a second-order Savitzky-Golay smoothing filter with nine points [33]. The mean intervals for scenes were 11.2 s ± 5.67 s for experts and 12.2 s ± 5.43 s for intermediate players. The peaks in each scene can be visualized using a plot, which is a type of a return map. Such a map plots the present peak $X_n$ versus the next peak $X_{n+1}$. For each scene, we plotted the observed data as the present peak $X_n$ versus the next peak $X_{n+1}$ using the amplitude of $X(t)$ at the peaks of $X_{IPD}(t)$ as a discrete dynamical system (Figure 2C–D), referred to as a return map analysis. Periodicities are revealed on such a plot as intersections with the line of identity $X_n = X_{n+1}$ [36, 37]. These intersections are known as an attractive fixed point and repellers or saddle points. These attractive fixed points are deterministically approached from a direction called the stable direction or manifold, and the repellers are diverged from these attractive fixed points along the unstable direction or manifold as a linear function. Theoretically, we postulated the linear function, $X_{n+1} = aX_n + b$. The intersections can be classified into two properties depending on the absolute value of $a$. When $a$ is less than 1, $|a| < 1$, then the intersection is considered to be an attractive fixed point (i.e., an “attractor”). When the absolute value of $a$ is more than 1, $|a| > 1$, then the intersection is referred to as a repellent fixed point (i.e., a “repeller”). An attractor can be further classified into two types. When $0 < a < 1$ (Figure 3a), the trajectories asymptotically close to the attractor, that is, the IPDs decrease gradually (Figure 3A). When $-1 < a < 0$, the trajectories rotationally close towards the attractor (Figure 3b); that is, the IPDs decrease by alternately moving a step towards and a step away from the attractor (Figure 3B). A repellor also has two types of trajectories: $1 < a$, and $a < -1$, corresponding to asymptotical and rotational trajectories, respectively, as shown in Figure 3c, d and 3C, D. Trajectories also approach and diverge from points that do not cross the line $X_n = X_{n+1}$. We postulated that these functions, an exponential function, $X_{n+1} = b \exp(aX_n)$ (Figure 3c, e), and a logarithmic function, $X_{n+1} = a \log X_n + b$ (Figure 3f, g), represent intermittency.

A total of 346 scenes with more than five peaks of $X(t)$ were fitted to three types of functions: $X_{n+1} = aX_n + b$, $X_{n+1} = b \exp(aX_n)$, and $X_{n+1} = a \log X_n + b$. The number of fitted points was altered from three to six points on the return map using moving windows from the beginning of the data to the end of each scene:

3 points : \{ (X_n, X_{n+1}), (X_{n+1}, X_{n+2}), (X_{n+2}, X_{n+3}) \}
Calculations for function fitting, threshold determination, and transition probabilities were performed by programs written in the C-programming language, with several source files provided by “Numerical Recipes in C” [38].

Results

Return Map Analysis

For each scene, we plotted a return map of the time series of the observed data, $X_n$ versus $X_{n+1}$, using the amplitude of $X(t)$ at the peaks of $X_{IPD}(t)$. Figure 3a–f shows examples of the fitting of six functions to the series of points on the return maps (Video S2–S4). We found 291 scenes that could be fit by the candidate functions: 162 of these scenes included expert competitors, and 129 scenes included intermediate competitors. In total, 485 series of points in these 291 scenes were well fit to the functions; 284 trajectories were revealed as attractors, and 146 trajectories were fitted as repellents; 55 trajectories were identified as intermittency (Table 1). All six types of candidate functions could be found using 3-, 4-, and 5-point fitting; 16 trajectories were fitted using 6 points for four types of functions. We found that 121 scenes were switched among two to nine different functions in each scene; 80 scenes switched between two functions, 22 scenes among three, 11 scenes among four, six scenes among five, and one scene each among seven and nine functions (Figure 4A–B, Table 2, Figure S4, and Video S5). These results suggest that complex movements occurring during the interpersonal competition of a kendo match could be generated by simple rules that attract toward or repel from fixed attractive and/or repellent points (Figure 5A–B).

State Transitions

We identified two discrete states in each histogram of return maps using the threshold as a minimum value for each match: the “farthest apart” high-velocity state (F), and the “nearest (closest) together” low-velocity state (N) (Figure 5A–D, and Figure S3). Thus, we identified four trajectories, $\{X_n=F, X_{n+1}=F\}$,
Figure 5: Joint Action Syntax in Japanese Martial Arts

Discussion

In this study, the return map analysis revealed that continuous interpersonal competition, which may appear to be quite complex, could be expressed in terms of a number of discrete dynamics represented by simple linear functions. The state transition...
analysis revealed second-order transition probabilities between two states: the “farthest apart” high-velocity state (F) and “the nearest (close) together” low-velocity state (N). These two states have a hierarchical structure that depends on the previous state. Third-order transition probabilities also revealed differences between expert and intermediate competitors. This result suggests that intentional switching dynamics is embedded in complex continuous interpersonal competition (such as a martial arts competition) and is thus better described as a hybrid dynamical system consisting of higher discrete and lower continuous modules connected via a feedback loop [28,29]. This switching dynamic allows for very complex, diverse, continuous human movements.

Switching dynamics have been studied theoretically [39], numerically [40], and behaviorally [41] as a continuous dynamical system excited by a temporal input. This model accounts for the dynamics of switching among some attractors as fractal transitions within finite time intervals, as expressed in the following ordinary differential equation:

$$\dot{x} = f(x, I(t))$$  \hspace{1cm} (6)

where $x \in \mathbb{R}^N$ is the state of the system, and $I(t) \in \mathbb{R}^N$ is the temporal input. This model has been extended to a hybrid dynamical system composed of a higher module with discrete dynamics and a lower module with continuous dynamics [28,29]. The higher module selects the switching input $I_l(t)$ at the interval $T_l$ based on the following:

$$I_l(t), l = 1, \ldots, L$$  \hspace{1cm} (7)

where $I_l(t) \in \mathbb{R}^N$, and $t$ correspond to the $l$th input for the lower module, the number of inputs, and the time, respectively. The lower module can be described as a set of continuous non-autonomous dynamical systems [39], defined by the following ordinary differential equation:

$$\dot{x} = f(x, I_l(t))$$  \hspace{1cm} (8)

where $x \in \mathbb{R}^N$ is the state of the lower module. The two modules are connected by a feedback system, in which the higher module switches at regular intervals in response to the states of the lower module. This system converges into various switching attractors that correspond to infinite switching manifolds; this defines the feedback control rule at the switching point. The feedback system could be considered to be an automaton that generates various sequences from the fractal set by choosing the typical switching feedback control rule at the switching point. The feedback system could be considered a macroscopic model in which the discrete module corresponds to the brain function as decision making, and the continuous module corresponds to the motor function as human.

### Table 1. Numbers of well-fitted series of points by function fitting using three to six points from the series in a scene.

<table>
<thead>
<tr>
<th>Function</th>
<th>Aa</th>
<th>Ar</th>
<th>Ra</th>
<th>Rr</th>
<th>Exp</th>
<th>Log</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitting points</td>
<td>E</td>
<td>I</td>
<td>E</td>
<td>I</td>
<td>E</td>
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<td>E</td>
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<td>3</td>
<td>61</td>
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<td>17</td>
<td>11</td>
<td>2</td>
<td>–</td>
<td>4</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>3</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>3</td>
<td>–</td>
</tr>
<tr>
<td>Sum</td>
<td>184</td>
<td>100</td>
<td>74</td>
<td>44</td>
<td>48</td>
<td>48</td>
<td>29</td>
</tr>
</tbody>
</table>

Aa and Ar are asymptotical, $0 < a < 1$, and rotational, $-1 < a < 0$, attractors. Ra and Rr are asymptotical, $1 < a$, and rotational, $a < -1$, repellers. Exp and Log are intermittencies of exponential and logarithmic functions, respectively. E and I denote expert and intermediate competitors, respectively.

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**Figure 4.** Switching among three different functions. (A) Time series of normalized $X_{IPD}(t)$ and state variable, $X(t)$, and circles show peaks of normalized $X_{IPD}(t)$. (B) Return map for this time series that demonstrates switching among three different functions. At first, from the peak of $X_9$ to $X_5$, the observed series of points have approached to attractive fixed point asymptotically with $X_{n+1} = 0.557 X_n + 0.574$. From the peak of $X_5$ to $X_{13}$, the observed points diverged from a repellant fixed point rotationally with $X_{n+1} = -1.107 X_n + 1.600$. Finally, from the peak of $X_9$ to $X_{13}$, the observed points approached the other attractive fixed point asymptotically with $X_{n+1} = 0.182 X_n + 0.574$. doi:10.1371/journal.pone.0072436.g004
Table 2. Well fitted scenes switching between different functions in each scene.

<table>
<thead>
<tr>
<th>Number of functions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert</td>
<td>100</td>
<td>39</td>
<td>14</td>
<td>4</td>
<td>3</td>
<td>–</td>
<td>1</td>
<td>–</td>
<td>1</td>
<td>162</td>
</tr>
<tr>
<td>Intermediate</td>
<td>70</td>
<td>41</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>129</td>
</tr>
<tr>
<td>Sum</td>
<td>170</td>
<td>80</td>
<td>22</td>
<td>11</td>
<td>6</td>
<td>–</td>
<td>1</td>
<td>–</td>
<td>1</td>
<td>291</td>
</tr>
</tbody>
</table>

The numbers show the different functions in each scene.

doi:10.1371/journal.pone.0072436.t002

Figure 5. State transition diagrams. (A, B) Return maps were plotted using observed points as four different linear functions of an attractor (red) and a repeller (blue) for expert and intermediate competitors respectively. The circles show crossing points with the line of identity, $x_n = x_{n+1}$. (C, D) Red, blue, and black lines show histograms of crossing points for an attractor, a repeller, and the sum of these respectively. (E, F) Second-order state transition diagrams with the conditional probabilities consisted of the “farthest apart” high velocity state (F) and the “nearest together” low velocity state (N) for expert and intermediate competitors, respectively. (G, H) The third-order state transition diagrams comprised four second-order sub-states.

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movements. Thus, the loop from the discrete to the continuous module can be regarded as an efferent pathway, and that from the continuous to the discrete module as an afferent pathway.

This idea is similar to neural syntax and could facilitate progress in defining cell assemblies, identifying their neuronal organization, and revealing causal relationships between assembly organization and behavior [31]. In general, syntax (grammar) is a set of principles that govern the transformation and temporal progression of discrete elements (e.g., letters and musical notes) into ordered and hierarchical relations (e.g., words, phrases, sentences, phrases, chord progressions, and keys) that allow for a congruous interpretation of the meaning of language and music by the brain. In addition to its contribution to language and music, the grouping or chunking of fundamentals by syntax allows for the generation of a virtually infinite number of combinations from a finite number of lexical elements using a minimal number of rules in sign, body, artificial and computer languages, and mathematical logic. “Action syntax” [30] has been examined as a Markov chain in grooming behavior in rodents [32–34]. However, this behavior can be regarded as a stereotypical action that was generated by a relatively simple feed-forward excitatory mechanism. In contrast, interpersonal competition does not exploit this mechanism, because the environment changes abruptly and unpredictably. In response to various situations, very large numbers of movements can be generated in large strongly recurrently connected systems equipped with appropriate syntax [31]. The hybrid dynamical system [28,29] in this study can be considered as a valid model for “joint action syntax”, which can generate various movements in interpersonal competition, such as martial arts.

In kendo, experts competed at greater distances and with higher velocities compared with intermediate competitors (Figure 5C–D). A point (ippon) in a kendo competition is achieved when an accurate strike is made on the opponent with the uppermost third of the shinai; (i.e., the top 0.30–0.40 m of its total length, 1.20 m). The average time of a strike movement was 0.36±0.08 s for the experts and 0.38±0.09 s for the intermediate players from an average interpersonal distance of 2.37±0.18 m for the experts and 2.42±0.18 m for the intermediate players. A split-second offensive or defensive maneuver may decide the outcome of a match. Thus, contestants must carefully maintain and change their interpersonal distance to balance the gain/loss of offensive and defensive maneuvers. This critical interpersonal distance, which induces the offensive and defensive maneuver may decide the outcome of a match. Thus, the intentional switching dynamics in interpersonal competition characterized by quick decision making and rapidly executed actions remain poorly understood. Higher-level cognitive brain functions generate seemingly homogeneous spatiotemporal sequences of neural activity to produce meaningful neural words and sentences in response to diverse environments. We can postulate a hybrid dynamical system that simulates decision making and/or intelligence [13,14,31]. Additionally, human movements are self-organized with robust autonomy not only in individuals but also in populations [20–22,25,27]. Joint action syntax, derived from hybrid dynamical system, is common and essential in nature from the level of neuronal activity to that of the activities of daily living. Furthermore, this model can be used to incorporate intentionality and robust decision making in the movement of artificial systems.

Supporting Information

Figure S1 Experimental setting and interpersonal distance (IPD). A Experimental setting. The black triangles correspond to cameras (a total of eight). B Reflective markers attached to the back of the player’s head, back of his waist, both ankles, right knee, and the top of the shinai to detect movement. C Top view captured by an optical motion capture system. The bar shows the IPD between two markers attached to the back of the player’s head. (EPS)

Figure S2 Variances of $V_{IPD}$ and correlation coefficients between $X_{IPD}$ and $V_{IPD}$ for each $\tau$. (TIF)

Figure S3 Histograms of each match and its return map. A and C, Examples of histograms of well-fitted peaks of each match. The thresholds were determined from the minimum frequency value of each match. The higher values of the peaks were regarded as “farthest apart” high-velocity states, denoted as “F”; the lower values of the peaks were regarded as “nearest (closest) together” low-velocity states, denoted as “N”. The threshold of A was 0.56, and the threshold of C was 0.6. B and D, Return maps corresponding to the histograms A and C, respectively, divided into four second-order sub-states: FF: Far-Far; FN: Far-Near; NN: Near-Near; and NF: Near-Far. (TIF)

Figure S4 Examples of return maps and switching function from a series of points in a scene. Red lines show attractors, and blue lines show repellers. Cyan lines show intermittency in all panels. A Nine functions, B seven functions, and C–H five functions were switched sequentially. (EPS)

Video S1 Experimental setting and motion capture data. Animated clip showing an example of the experimental setting and an example of the motion capture data from side and top of views. (MP4)

Video S2 Attractor on the return maps. Animated clip showing examples of the pattern of attractors on the return maps, which include the time series of $X(t)$ and $X_{IPD}(t)$, velocities of step-toward and step-away of each player, and 2D movement from top of view, and approaching to the attractive fixed point asymptotically and rotationally. (MOV)

Video S3 Repeller on the return maps. Animated clip showing examples of the pattern of repellers on the return maps, which include the time series of $X(t)$ and $X_{IPD}(t)$, velocities of step-toward and step-away of each player, and 2D movement from top of view, and diverging from the repellent fixed point asymptotically and rotationally. (MOV)

Video S4 Intermittency on the return maps. Animated clip showing examples of the pattern of intermittencies on the return maps, which include the time series of $X(t)$ and $X_{IPD}(t)$, velocities of step-toward and step-away of each player, and 2D movement from top of view, and approaching and diverging
around the attractive fixed point and/or repellent fixed point exponentially and logarithmically.

(MOV)

Video S5 Switching among several functions in one scene. Animated clip showing examples of the pattern of switching among several functions on the return maps, which include the time series of $X(t)$ and $X_{PP}(t)$, velocities of step-toward and step-away of each player, and 2D movement from top of view.

(MOV)

References


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Author Contributions

Conceived and designed the experiments: MO YY. Performed the experiments: MO AK KK KY YY. Analyzed the data: YY MO AK. Contributed reagents/materials/analysis tools: KY KG YY. Wrote the paper: YY KG.