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# Effect of fiber-matrix interfacial bond degradation on monotonic and cyclic crack bridging laws in short fiber reinforced cementitious composites

# 短繊維補強複合材料の単調及び繰返し載荷下ひび割れ架橋則における繊維－母材間界面付着劣化の影響

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This paper presents a theoretical formulation of fiber bridging constitutive laws under monotonic and cyclic loading with the effect of interfacial bond degradation. A bilinear interfacial bond degradation function is introduced with accumulated crack opening displacement change. This parameter accounts for the damage history, which could be different at each location on the surface of a growing crack. With the interfacial bond degradation function, the analytical expressions of fiber bridging stress-crack opening displacement ( $\sigma_f$ - $\delta$ ) and fiber bridging stress amplitude -crack opening displacement amplitude ( $\Delta\sigma_f$ - $\Delta\delta$ ) relation are obtained. Two sets of comparisons lead to similar degradation functions, which supports the validity of the presented interfacial bond degradation function with accumulated crack opening displacement change.

***Key Words:*** *Bridging law, short fiber reinforced cementitious composite, cyclic loading, interfacial bond degradation*

## 1. Introduction

There is an increasing trend to use short fiber reinforced cementitious composites for the fatigue durability improvement of civil structures such as bridge steel deck overlay, bridge RC slab overlay or underlay, and railroad slab. In these structures, short fiber reinforced cementitious composites are expected to show cracking resistance, crack width control, and fatigue life improvement. It is well known that those beneficial composite properties are thanks to bridging fibers that transfer stresses across a crack. Therefore, it is important to understand a fiber bridging constitutive law, which is the relation between bridging stress and crack opening displacement, even under fatigue in order to evaluate those fatigue composite properties.

Fiber bridging constitutive law is a material property which is specific to each composite mix and process, expressed with micromechanical parameters such as fiber length, fiber diameter, fiber modulus, interfacial frictional bond strength, and so on[1)2)3)]. Fatigue damage is present in all of three material constituents of short fiber reinforced cementitious composites: not only in matrix, but also in fiber-matrix interface and fibers. The damages on interface and fibers are microscopic changes, but they are responsible for the fatigue life of short fiber reinforced cementitious composites, since these damages lead to degradation of crack bridging stress, which in turn controls fatigue crack growth behavior. The degradation of crack bridging stress is represented by the fiber bridging constitutive laws under monotonic and cyclic loading that take into account the damage on interface or fibers. Then, effects of the damage on interface or fibers can be included in the fatigue life model of short fiber reinforced cementitious composites.

This paper addresses on the fatigue damage on interface: interfacial bond degradation, while the past study by Matsumoto[4)] has addressed the fatigue damage on fibers: fiber fatigue rupture. First, bridging constitutive laws under monotonic and cyclic loading are reviewed. The degradation behavior of interfacial bond is discussed, and a bilinear bond degradation function is presented as a simple one. In addition, a new parameter, accumulated crack opening displacement change, is introduced in order to measure the damage history, which could be different at each location on the surface of a growing crack. This interfacial bond degradation function is taken into account, and the analytical expressions of fiber bridging stress-crack opening displacement ( $\sigma_f$ - $\delta$ ) and fiber bridging stress amplitude -crack opening displacement amplitude ( $\Delta\sigma_f$ - $\Delta\delta$ ) relation with interfacial bond degradation are obtained. Finally, the validation of accumulated crack opening displacement change is checked with experimental data.

## 2. Review of Fiber Bridging Laws under Monotonic and Cyclic Loading

### 2.1. Fiber Bridging Constitutive Law under Monotonic Loading

The essence of a monotonic fiber bridging constitutive law derived by Li is explained in this section[2]. The constitutive law relates the fiber bridging stress, $\sigma_f$, as a unique function of the crack opening displacement, $\delta$, under monotonic loading. The $\sigma_f(\delta)$ has been derived based on micromechanical modeling of fiber bridging with weak (friction controlled) fiber-matrix interface. The derivation starts from constructing the relation between fiber pull-out load, $P$, and crack opening displacement, $\delta$, of a single fiber embedded in the matrix. Then the pull-out load carried by individual fibers is integrated to construct the monotonic fiber bridging constitutive law, $\sigma_f$ ($\delta$). The integration accounts for the random distribution of location and orientation of short fibers at a designated crack plane. With the assumption of 3-D uniform randomness for the fiber centroidal distance, $z$, and orientation, $\phi$, at a designated crack plane, Li derived the constitutive law[1)2)].

The bridging stress, $\sigma_f$, is related to the crack opening displacement, $\delta$, through the integration of the load carried by individual bridging fibers at different stages of debonding and sliding:

$$\tilde{\sigma}_f = \frac{8}{\pi\tau(L_f/d_f)d_f^2}\int_{\phi=0}^{\frac{\pi}{2}}\int_{z=0}^{\frac{L_f}{2}\cos\phi} P(\delta)e^{f\phi}p(\phi)p(z)\,dz\,d\phi \qquad (1)$$

where $\tilde{\sigma}_f = \sigma_f / \sigma_o$, $\sigma_o = V_f\,\tau\,(L_f / d_f) / 2$, $V_f$ = fiber volume fraction, $\tau$ = interfacial frictional bond strength, $L_f$ = fiber length, $d_f$ = fiber diameter, $f$ = snubbing coefficient, $p(\phi) = \sin\phi$, $p(z) = 2 / L_f$. The factor $e^{f\phi}$ in (1) refers to a snubbing effect which describes the mechanical interactions between a loaded inclined fiber and the matrix material[4)].

Substituting $P(\delta)$ into (1) yields explicit equations for the constitutive law. For pre-peak bridging stress,

$$\tilde{\sigma}_f\Big|_{prepeak} = g\left[2\left(\frac{\tilde{\delta}}{\tilde{\delta}^*}\right)^{\frac{1}{2}} - \left(\frac{\tilde{\delta}}{\tilde{\delta}^*}\right)\right] \quad for\ \ 0 \le \tilde{\delta} \le \tilde{\delta}^* \qquad (2)$$

where $g = 2 / (4 + f^2)(1 + e^{\pi f/2})$, $\tilde{\delta} = \delta / (L_f / 2)$, and $\tilde{\delta}^* = \delta^* / (L_f / 2)$. $\delta^* = (\tau L_f^2) / (E_f d_f)$, at which all fibers have completed debonding, and $E_f$ = fiber modulus. For post-peak bridging stress,

$$\tilde{\sigma}_f\Big|_{postpeak} = g\left[1 - \left(\tilde{\delta} - \tilde{\delta}^*\right)\right]^2 \qquad for\ \ \tilde{\delta}^* < \tilde{\delta} \le 1\,. \qquad (3)$$

The whole picture of the bridging stress-crack opening displacement relation ($\sigma_f$- $\delta$ relation) for a composite with $\tilde{\delta}^*$ = 0.002 is shown in Fig. 1 and 2.

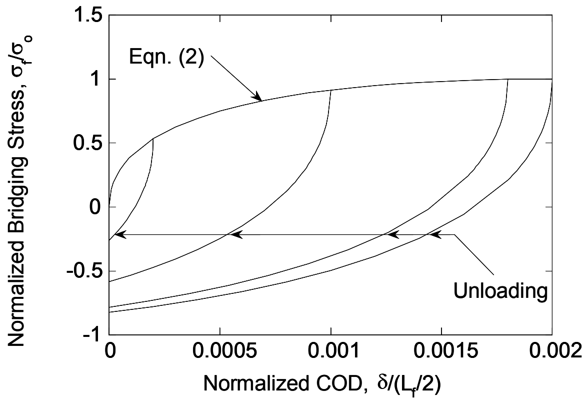



Fig. 1 Loading-unloading curves for pre-peak bridging stress[3)].

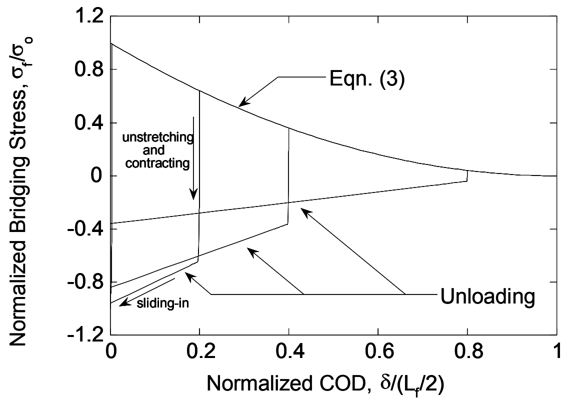



Fig. 2 Loading-unloading curves for post-peak bridging stress[3)].

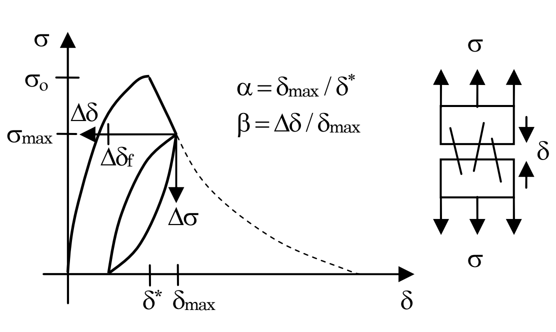



Fig. 3 Definition of parameters.

### 2.2. Fiber Bridging Constitutive Law under Cyclic Loading

A cyclic fiber bridging constitutive law has been derived by Matsumoto. The details of the law are found elsewhere[3]. Here, the definitions of parameters are introduced, and a brief explanation is given.

Fig. 3 shows the bridging stress-crack opening displacement behavior of a cracked composite. Under monotonic loading, the bridging stress increases with crack opening displacement until it reaches a peak value, $\sigma_o$, at the corresponding crack opening displacement, $\delta^*$, and it decays to zero together with fiber pull-out. For cyclic loading, we consider unloading and reloading. The unloading starts from a point on the monotonic relation, and the coordinates of the point are denoted by $\sigma_{max}$ and $\delta_{max}$. The fiber bridging constitutive law is defined with this point the origin. Namely, bridging stress amplitude, $\Delta\sigma$, and crack opening displacement amplitude, $\Delta\delta$, are measured from this point, shown in Fig. 3. The cyclic fiber bridging constitutive law is derived as a function of $\alpha$, $\beta$, and $\delta^*$.

Unloading curves of fiber bridging constitutive law are shown in Fig. 1 and 2. In Fig. 1, the unloading curves are solely due to unstretching and contracting of fibers, since fibers are still in interfacial debonding at ($\delta_{max}$, $\sigma_{max}$). On the other hand, in Fig. 2, all the fibers have achieved full interfacial debonding, and they are in the middle of sliding-out at ($\delta_{max}$, $\sigma_{max}$). Therefore, the unloading curves are exerted by unstretching and contracting of fibers first and by sliding-in of fibers next.

A good agreement has been obtained in the comparison with the experimental data of a steel fiber reinforced concrete under tensile cyclic loading[3]. However, it should be noted that the unloading curves towards full crack closure is derived under the assumption that no fiber buckling takes place. Therefore, further model improvement is necessary to analyze the unloading curves, especially in compression range, of buckling-prone fibers such as polymeric fibers.

## 3. Interfacial Bond Degradation

### 3.1. Interfacial Bond under Monotonic Loading

Fiber-matrix interface behavior in fiber composites is important to understand and improve post-cracking properties such as composite strengths, ductility, and fracture energy. The underlying mechanism is that fiber-matrix interface undergoes debonding, as fibers are pulled out upon cracking, and that the debonded interface undergoes sliding, giving rise to frictional resistance to the fiber pull-out force. The frictional stress at the interface is termed interfacial bond strength in this study. This crack closure action works on matrix cracks, resulting in the improvement of post-cracking properties.

Under monotonic loading, the interfacial frictional bond strength of fiber composites has been studied extensively. Starting from the constant bond strength model[2], which successfully explained post-cracking stress-displacement relation and fracture energy for steel and polymeric fiber reinforced cementitious composites, it is now understood with experimental evidences that the interfacial frictional bond strength is dependent on sliding distance[6)7)8)]. Based on the slip-dependent interface, crack bridging constitutive laws have been modeled for aligned continuous[9] and randomly distributed discontinuous fiber reinforced composites[10].

The slip-dependent interface can work in either way: slip-hardening or softening. Slip-softening interface has been observed in steel fibers in cement matrix, and this is attributed to the compaction of interfacial microstructure, which reduces the actual contact area at the interface, resulting in the decreased frictional resistance to the fiber pull-out force[6)11)]. Slip-hardening interface has been observed in polymeric fibers in cement matrix, and fiber surface abrasion is most likely the cause here. Fiber surface abrasion, which is caused by the slippage of the fibers with low hardness against the surrounding matrix with high hardness, creates fiber fibrils in the debonded interface, resulting in the increased frictional resistance to the fiber pull-out force.

### 3.2. Interfacial Bond under Cyclic Loading

Under cyclic loading, there is increasing evidence to show that the interfacial frictional bond strength changes under a large number of cyclic loads.

The change of the interfacial frictional bond strength can be estimated from the hysteresis loops of stress-displacement relation observed experimentally under fatigue loading, if a micromechanical model is available for the cyclic stress-displacement relation. Assuming that there is no fiber fatigue rupture during fatigue loading, softening hysteresis loops are attributed to the degradation of the interfacial frictional bond strength. Softening hysteresis loops have been observed in aligned continuous fiber reinforced ceramic matrix composites such as SiC/CAS[12)13)14)] and SiC/SiC[15)16)]. The degradation of the interfacial frictional bond strength is significant. The interfacial bond strength was observed to decrease rapidly from 20 to 5 MPa within 100 cycles by Evans et al.[13] and Zok et al.[14] and from 50 to 10 MPa within 1,000 cycles by Evans[12]. After the rapid degradation, the interfacial bond strength was observed to take a constant value by the researchers.

Another way to estimate the change of the interfacial frictional bond strength has been developed and applied to SiC/CAS[17)18)]. This new approach is based on the measurement of the temperature increase, which is caused by frictional heating at the debonded fiber-matrix interface, and the temperature increase is related to the interfacial frictional bond strength. The interfacial bond strength degraded in a similar manner, and it was observed to decrease rapidly from over 15 to approximately 5 MPa within 25,000 cycles.

As for fiber reinforced cementitious composites, two studies refer to the effects of cyclic loading on the interfacial frictional bond strength. First, fatigue tensile loading tests with a focus on the long-term change of hysteresis loops have been conducted by Zhang et al[19]. Hysteresis loops of bridging stress-displacement relation have been measured for steel fiber reinforced concrete under fatigue loading with constant displacement amplitude. For various maximum displacements, the crack bridging stress was observed to follow two stages. It drops rapidly to 80 ~ 90 % of the original value within approximately 100 cycles and decreases slowly to as low as 50 % at 100,000 cycles. Since no fiber or aggregate rupture was found in the tests, the decreased bridging stress is attributed to the degradation of the interfacial frictional bond strength. Second, single fiber pull-out tests have been carried out to directly measure the interfacial frictional bond strength of polyethylene fiber reinforced cement[21]. The interfacial bond strength was measured with and without cyclic loads applied before the fiber pull-out tests. Contrary to the case of the steel fiber reinforced concrete[19], the measured interfacial bond strength was observed to increase by 22 % after 10 cycles of loads, and the toughness, which is the area under a load-displacement curve, also by 21 %. The results of these two studies are parallel to the observations of slip-dependent interface under monotonic loading. Hard steel fibers cause damage on the matrix side, whereas soft polymeric fibers are vulnerable to surface abrasion damage.

The presence of interfacial bond degradation has been shown with ample evidence, and, in this paper, we will restrict our focus on the more common case of interfacial bond degradation rather than that of fiber fatigue rupture. The interfacial bond degradation is implemented into the bridging constitutive laws under monotonic and cyclic loading. Namely, the interfacial bond, $\tau$, is reduced according to the number of cycles and the magnitude of each cycle, so the bridging constitutive laws are dependent on the damage history of the interface. Hence, the monotonic and cyclic constitutive laws can be represented by

$$\tilde{\sigma}_f = function\left(\tilde{\delta}, \tilde{\delta}^*, \tau(damage\ history)\right) \tag{4}$$

and

$$\Delta\tilde{\sigma}_f = function\left(\Delta\tilde{\delta}, \tilde{\delta}_{\max}, \tilde{\delta}^*, \tau(damage\ history)\right). \tag{5}$$

A new parameter, which is called accumulated crack opening displacement change, is introduced in the current study. This new parameter is simple, but meaningful enough to analyze a growing crack with non-uniform crack profile (i.e. $\Delta\delta$ varying along the crack face). Details on the bridging constitutive laws with the new parameter are discussed in the next chapter.

## 4. Effect of Interfacial Bond Degradation on Monotonic and Cyclic Constitutive Laws

### 4.1. Accumulated Crack Opening Displacement Change as a Parameter for Interface Damage

Interfacial bond degradation is a phenomenon where interfacial frictional bond strength decreases as a large number of load cycles is applied to a cracked fiber composite. Under cyclic loading of a cracked fiber composite, bridging fibers also undergo unloading and reloading. The fibers slide back and forth against the matrix at the debonded interface, leading to the interfacial wearing. As the number of load cycles increases, this causes gradual wearing-out of the fiber-matrix interface which results in lower frictional bond strength.

The presence of interfacial bond degradation in a fiber composite has been shown with ample evidence, as discussed in the previous chapter. These experimental observations come to show a common behavior of interfacial bond degradation in a fiber composite. Namely, the interfacial bond strength was observed to drop very rapidly within a small number of cycles and, afterwards, take a constant value or decrease very slowly. For example, for CAS/SiC, this drop is observed to happen from 20 to 5 MPa within 30 cycle by Evans et al.[13] and Zok et al.[14], from 50 to 10 MPa within 1,000 cycle by Evans[12], and from over 15 to approximately 5 MPa within 25,000 cycle by Cho et al.[17] and Holmes and Cho[18], and, for steel FRC, bridging stress is observed to drop rapidly to 80 ~ 90 % of the initial value within 100 cycles and, afterwards, slowly to 50 % by 100,000 cycle by Zhang et al.[19]

The experimental observations indicate two features. First, the fiber-matrix interface degrades rapidly with the number of load cycles. Second, the degradation seems to cease or proceed at a very slow rate. Presumably, the rough interface is made smooth at first, then the smooth interface is subjected to frictional wearing. In mechanical modeling, the degradation can be represented by the decaying value of the interfacial frictional bond strength. This concept is incorporated into the monotonic and cyclic constitutive laws mentioned in chapter 2 by the following procedure.

Based on the experimental observations, degradation of interfacial frictional bond strength is expressed as a bilinear function. When the interfacial frictional bond strength, $\tau$, is assumed to degrade from an initial value, $\tau_i$, to a final steady state value, $\tau_f$, we have

$$\frac{\tau}{\tau_i} = max\ of \begin{cases} 1.0 + r_1 X \\ b \end{cases} \tag{6}$$

where $r_1$ = degradation coefficient and $b = \tau_f / \tau_i$. The degradation coefficient, $r_1$, is a coefficient which is negative for degradation and is expected to be an interfacial parameter specific to both a given material system and processing details. The coefficient, $r_1$, is probably related to the hardness of and the radial residual stress at the interface. $X$ is a parameter which measures the degree of accumulated interface damage and can be the number of load cycles, $N$, one disadvantage of using $N$ is that the same value of $N$ does not necessarily mean the same degree of accumulated damage. Namely, for two different load amplitudes, the same $N$ does not necessarily indicate the same accumulated damage.

A more fundamental parameter is introduced for $X$ in this paper. In the current model, $X$ is represented by accumulated crack opening displacement change,

$$\sum_{i=1}^{N} \Delta\delta_i \quad . \tag{7}$$

Namely,

$$\frac{\tau}{\tau_i} = max\ of \begin{cases} 1.0 + r_1 \sum_{i=1}^{N} \Delta\delta_i \\ b + r_2 \sum_{i=1}^{N} \Delta\delta_i \end{cases} . \tag{8}$$

where $r_1$ = degradation coefficient for the early trend (negative for degradation), $\Delta\delta_i$ = crack opening displacement change at $i$-th cycle, $b$ = intercept for the long trend, and $r_2$ = degradation coefficient for the long trend (negative for degradation) (Fig. 4). This accumulated crack opening displacement change measures the physical distance the mouth of the fiber embedded hole experiences.

The idea behind this approach is the following. The fiber remains intact during repeated crack opening and closing, whereas the matrix or interfacial zone wears out gradually resulting in lower interfacial frictional bond strength. This is not necessarily the case in reality, because both fiber and matrix degrade with the relative slip distance from their original position and the interfacial frictional bond strength is determined by the history of both fiber and matrix degradation

However, this rigorous approach requires more precise modeling on the interface mechanics. The current approach focuses on the effect of the averaged value of frictional bond strength along the debonded interface, and assumes that fiber

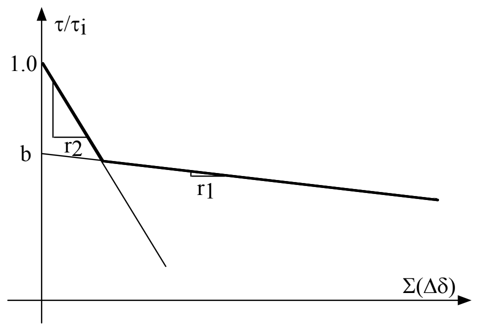



Fig. 4 Assumed bilinear interfacial bond degradation function.

rupture is negligible.

For a growing crack with non-uniform crack profile, this is even more advantageous, in this case, (8) becomes

$$\frac{\tau}{\tau_i} = max\ of \begin{cases} 1.0 + r_1 \sum_{i=1}^{N} \Delta\delta_i(x) \\ b + r_2 \sum_{i=1}^{N} \Delta\delta_i(x) \end{cases} . \tag{9}$$

Here, the interfacial bond degradation is measured with

$$\sum_{i=1}^{N} \Delta\delta_i(x) = \text{accumulated crack opening displacement change at } x. \tag{10}$$

where $x$ is the position on the crack surface. This parameter takes a value in response to the number of cycles and the crack opening displacement change experienced at each point on the bridged crack surface. For a growing crack with non-uniform crack profile, the value is the maximum value near the crack mouth and the minimum at the crack tip, meaning that crack bridging degradation is the most severe near the crack mouth and minimal at the crack tip.

The cyclic constitutive law derived in this study helps determine the crack opening displacement change for each cycle of a given load amplitude for prescribed composite parameters. This will be shown later.

### 4.2. Monotonic and Cyclic Bridging Constitutive Laws with Interfacial Bond Degradation

Bridging stress degradation induced by interfacial bond degradation during fatigue loading is evaluated under monotonic and cyclic loading with the use of the constitutive laws derived in chapter 2. Fatigue loading can be defined by only one parameter: accumulated crack opening displacement change under any load sequences (e.g. constant load amplitude, constant crack opening displacement amplitude, or variable load amplitude). Under a given fatigue loading of accumulated crack opening displacement change, interfacial bond degradation is specific to both a given material system and processing details, and it is given by a bilinear function of accumulated crack opening displacement change with two degradation coefficients, $r_1$ and $r_2$. Therefore, monotonic and cyclic constitutive laws are simply given by the original equations with (9).

Fig. 5 shows how the normalized bridging stress, $\tilde{\sigma}_f = \sigma_f / \sigma_o$, changes with the normalized crack opening displacement, $\tilde{\delta} = \delta / (L_f / 2)$, under monotonic and cyclic loading, for three cases of interfacial bond strength ratio and three cases of maximum crack opening displacement, $\tilde{\delta}_{max} = \delta_{max} / (L_f / 2)$. Note that $\tilde{\delta}^* = 0.002$ for no damage, $\tilde{\delta}^* = 0.001$ for $\tau / \tau_i = 0.5$, and $\tilde{\delta}^* = 0.0002$ for $\tau / \tau_i = 0.1$. One consequence of interfacial bond degradation is the decreased bridging strength as seen in Fig. 5.

This is simply because the maximum bridging strength,

$$g\sigma_o = \frac{gV_f \tau \left(L_f / d_f\right)}{2}, \tag{11}$$

is scaled down due to the interfacial bond degradation, and accordingly the bridging stress amplitude under unloading is also scaled down. Another consequence is that the normalized crack opening displacement at the maximum bridging stress,

$$\tilde{\delta}^* = \frac{2\tau L_f}{E_f d_f}, \tag{12}$$

is decreased due to the interfacial bond degradation. However, the crack opening displacement at which the bridging stress vanishes do not change, because all the fibers are pulled out with none of them ruptured.

## 5. Validation of the Concept of Accumulated Crack Opening Displacement Change

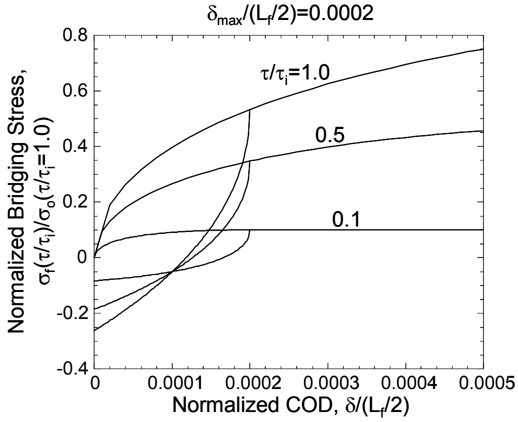
The theoretical cyclic fiber bridging constitutive law with interfacial bond degradation is compared with the data of fatigue loading experiment conducted by Zhang et al[19]. Fiber parameters used for the theoretical cyclic bridging constitutive law are shown in Table 1, matrix parameters in Table 2, and interface parameters in Table 3. The deduced values in Table 3 for monotonic loading fall within reported values[6], but for cyclic loading they are lower. This apparently lower frictional bond strength is presumably because the normal pressure acting on the fiber is reduced near the crack surface as the surrounding matrix is destressed with the fiber pullout, when compared to around the embedded fiber end.

Theoretical cyclic hysteresis loops shown in Fig. 6 are obtained by the superposition of aggregate and fiber bridging stress. The aggregate bridging stress under monotonic loading is given by an empirical equation proposed by Stang[20]. The aggregate bridging stress, $\sigma_m$, as a function of the crack opening displacement, $\delta$, is given by

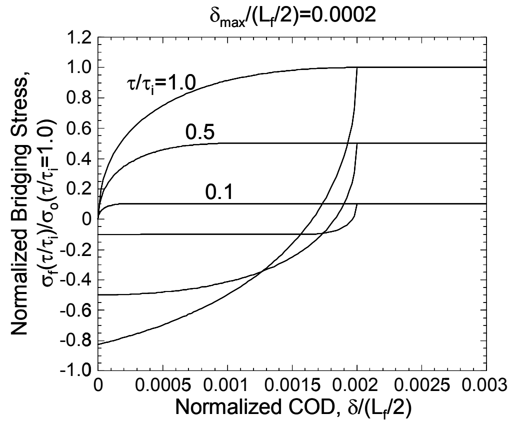
$$\sigma_m(\delta) = \frac{\sigma_m^u}{1 + \left(\dfrac{\delta}{\delta_{mo}}\right)^p} \tag{13}$$

where $\sigma_m^u$ = maximum aggregate bridging stress at $\delta = 0$, $\delta_{mo}$ = crack opening displacement which corresponds to the half of $\sigma_m^u$, and $p$ describes the shape of the bridging curve. The aggregate bridging stress under cyclic loading is given by

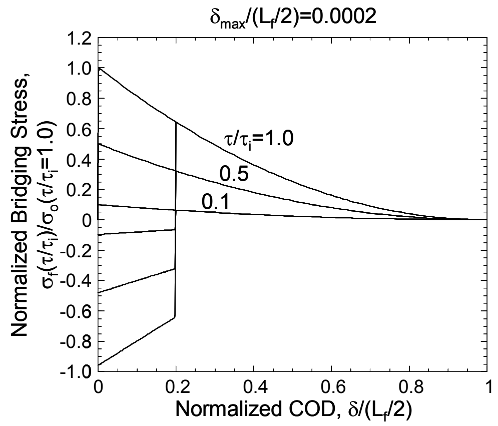
$$\Delta\sigma_m(\Delta\delta) = \frac{\sigma_{m\max}}{\delta_{\max} - \delta_{\min}} \Delta\delta \tag{14}$$




(a) $\tilde{\delta}_{\max}$ =0.0002




(b) $\tilde{\delta}_{\max}$ =0.002




(c) $\tilde{\delta}_{\max}$ =0.2

Fig. 5 Bridging stress degradation due to interfacial bond degradation.

Table 1 Fiber parameters[19].

| Type | $L_f$(mm) | $E_f$(GPa) | $d_f$(μm) | $V_f$(%) |
|---|---|---|---|---|
| smooth steel | 25 | 210 | 400 | 1 |

Table 2 Matrix parameters[19].

| $E_m$ (GPa) | $\sigma_m^u$ (MPa) | $p$ | $\delta_{mo}$ (mm) |
|---|---|---|---|
| 35 | 5.42 | 1.2 | 0.015 |

Table 3 Interface parameters for fatigue hysteresis loops.

| $\tau_{mono}$ (MPa) | $\tau_{cyc}$ (MPa) | $f$ |
|---|---|---|
| 6.0 | 6.0 x 0.6 | 0.8 |

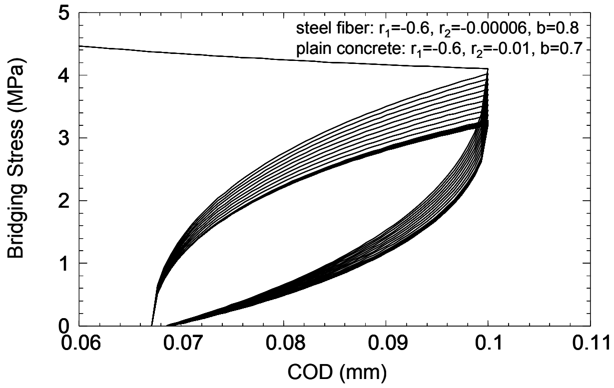



Fig. 6 Theoretical hysteresis loops of a straight steel FRC.

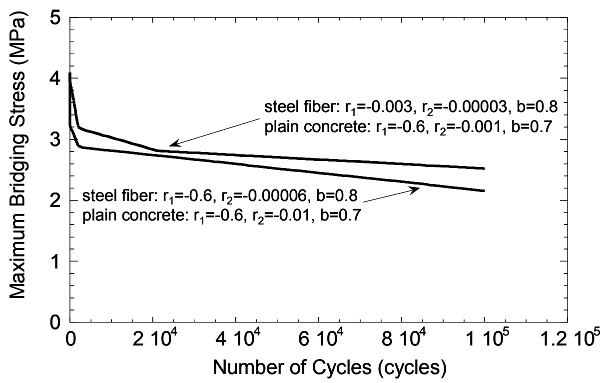



Fig. 7 Degradation of maximum bridging stress of a straight steel FRC.

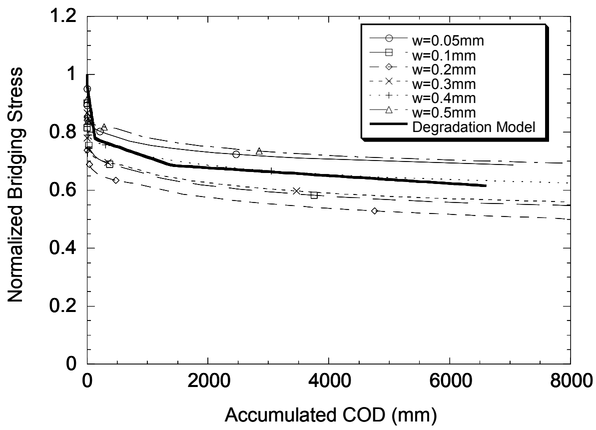



Fig. 8 Comparison between the experimental data of crack bridging degradation[23] and the degradation model assumed in this study.

where ($\delta_{max}$, $\sigma_{m\,max}$) is the point at which unloading occurs and ($\delta_{min}$, 0) is the point at which the FRC is fully unloaded to zero load level. These two points are taken from the experimental plot for each unloading-reloading branch.

Both aggregate and fiber bridging stress are degraded with accumulated crack opening displacement change, but with different sets of degradation coefficients. Degradation coefficients for fiber and aggregate bridging, which yields the best fit to the experimental data, are shown in Fig. 6, and, in this analysis, the interfacial bond strength is reduced with the accumulated crack opening displacement change after unloading and before reloading.

From the hysteresis loops, bridging stress degradation can be obtained by picking up the maximum point of the loops. This is shown as a lower line in Fig. 7.

Fig. 7 also shows another bridging stress degradation as an upper line. This life is drawn with the degradation coefficients for fiber and aggregate bridging that have been obtained through the fatigue life analysis of FRCs[22]. Theoretical results show that a single set of interfacial degradation coefficients explains a wide range of experimental S-N data of FRCs in flexural fatigue[22].

Furthermore, this set of coefficients is compared with the data of crack bridging degradation tests conducted by Zhang et al[23]. Fig. 8 shows crack bridging degradation curves determined by fitting to the experimental data for various initial crack opening displacement values ($w$ = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5 mm)[23] and also the aforementioned degradation model obtained through the fatigue life analysis of FRCs[22]. The crack bridging degradation curves were obtained for a smooth steel 1 % fiber reinforced concrete tested under uniaxial fatigue loading with the minimum to maximum stress ratio, $R$, = 0.0, therefore crack bridging is exerted by aggregates and fibers. Accordingly, theoretical crack bridging stress degradation is also obtained by the superposition of aggregate and fiber bridging degradation function. The degradation model obtained through the fatigue life analysis falls within the experimentally determined curves.

The two lines shown in Fig. 7 do not differ from each other, and each explains the hysteresis loops and fatigue life of FRCs, respectively. Although further refinement is needed, this supports the validity of accumulated crack opening displacement change as a governing parameter in bridging stress degradation phenomenon.

## 6. Concluding Remarks

This paper presented a theoretical formulation of fiber bridging constitutive laws under monotonic and cyclic loading with the effect of interfacial bond degradation.

First, bridging constitutive laws under monotonic ($\sigma_f$ - $\delta$) and cyclic loading ($\Delta\sigma_f$ - $\Delta\delta$) were reviewed. Also, the notations of laws were introduced.

The degradation behavior of interfacial bond was discussed. A bilinear interfacial bond degradation function was introduced with accumulated crack opening displacement change. This parameter accounts for the damage history, which could be different at each location on the surface of a growing crack.

The interfacial bond degradation function was introduced, and the analytical expressions of fiber bridging stress-crack opening displacement ( $\sigma_f$ - $\delta$ ) and fiber bridging stress amplitude -crack opening displacement amplitude ( $\Delta\sigma_f$ - $\Delta\delta$ ) relation with interfacial bond degradation were obtained.

Finally, the validation of accumulated crack opening displacement change was compared with experimental data. Two sets of comparisons led to similar degradation functions, which supports the validity of the presented interfacial bond degradation function with accumulated crack opening displacement change.

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