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Liquidation Costs and Endogenous Growth*

Hiroaki TAKASHIMA

In this paper, we examine the relation between the portfolio decisions that financial intermediaries make to avoid bank runs and the liquidation costs of capital investment. This relation also influences economic growth. This paper shows that if financial intermediaries avoid runs, there exist three regions of liquidation costs. Changes in liquidity costs in each region have different effects on economic growth in a simple endogenous growth model. We also examine portfolio decisions in an economy without financial intermediaries. This reveals the significance of financial sector for economic growth.

JEL Classification Numbers : G11, E13, D82, E44

Key Words : Liquidity Costs, Endogenous Growth, Financial Intermediaries

1. Introduction

Diamond and Dybvig (1983)1,2 shows that, in an economy where each individual faces an uncertain utility function, financial intermediaries (banks) can accomplish a more efficient allocation of intertemporal goods than that in the economy without financial intermediaries. In this paper, the key factor explaining why financial intermediaries exist in the economy is the uncertainty that exists from the point of view of each individual, however, uncertainty does not exist in the whole economy. Then, financial intermediaries can make portfolio decisions in a certain environment and give the higher expected utility to their depositors.

However, an important characteristics of financial intermediaries discussed in Diamond and Dybvig (1983) is the occurrence of bank runs caused by the pessimistic expectations of depositors toward the future situation of the

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* I would like to thank Professor Kazuo Uchida for helpful comments and suggestions. Responsibility for remaining errors is my own.

1) In Williamson (1987) and Tirole (1997), the reason why financial intermediaries exist in the economy is to economize monitoring cost to borrower.

2) Recent literatures on the financial intermediary of Diamond and Dybvig (1983) are, for example, Ping (1994) and Dutta and Kapur (1998).
intermediaries. In bank runs equilibrium, depositors suffer considerable utility loss.

Bencivenga and Smith (1991) studies how financial intermediaries affects the economic growth. In Bencivenga and Smith (1991), there are two investment assets, liquid investment that is less productive and is demanded for individuals who need liquidity early, and illiquid investment that is productive and is demanded for individuals who need liquidity later. This paper reveals the conditions for that illiquid investment occupies a higher proportion in portfolio of intermediaries than that in self-finance cases. When these conditions are satisfied, financial intermediaries promote economic growth relative to an economy without them. However, in this paper, financial intermediaries ignore the possibility of bank runs.

Cooper and Ross (1998) extends Diamond and Dybvig (1983) in two directions. One is to consider the characteristics of portfolio allocation of financial intermediaries when they recognize the possibility of bank runs in advance. The other is to combine the liquidity of investment capital with bank runs. For example, when the liquidity of investment is small, an intermediary whose portfolio investment occupies a large proportion is susceptible to bank runs by unanticipated withdrawals. Cooper and Ross (1998) shows that intermediaries decrease investment and increase reserve to avoid bank runs when the liquidity of investment is extremely small. This implies that financial intermediaries may adjust the composition of their portfolio depending on the liquidity of investment.

The purpose of our paper is to clearly show how financial intermediaries change the composition of their portfolio as the liquidity of illiquid investment is smaller and to reveal the relation between the liquidity of illiquid investment and the growth rate. In this paper, we particularly focus on the case where financial intermediaries avoid runs. This is because avoidance of runs is considered to be one of the plausible responses of financial intermediaries towards bank runs and we know simply that they affect economic growth in different ways, depending on their liquidity positions.

From the results of this paper, a decrease in the liquidity of investment has three effects on economic growth depending on the initial liquidity level. When the initial liquidity level is high, it has no effect on economic growth, when the initial liquidity level is intermediate, it increases economic growth.

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3) In most related literature, the reason why individuals have pessimistic expectations for the future of financial intermediaries is thought of as a sunspot. However, Postlewaite (1987) shows that bank runs occur as a result of an agent's optimization behavior under some pay-off matrix.

4) Cooper and Ross (1998) also considers the portfolio decision of intermediaries in the case that they maximize the expected utility of depositors under a given exogenous probability of runs. Then, similar results are also obtained.
and when the initial liquidity level is low, it decreases economic growth.

The outline of this paper is as follows: Section 2 and 3 present a model and the market equilibrium, section 4 shows how financial intermediaries determine the composition of their portfolio depending on the liquidity of investment and its effect on economic growth.

2. Economic Environment

The economy consists of overlapping generations whose size is measured as one of identical, three-period-lived agents and financial intermediaries. It has two goods; consumption goods and capital goods, and no population growth.

Every agent is endowed with one unit of labor only in his young period which is supplied inelastically. No agents have any endowment in their middle and old age. There are two types of agents in the economy: $G_m$ type and $G_o$ type. $G_m$ type agents those occupy a proportion $\pi$ of a generation care only consumption in middle age and $G_o$ type agents those occupy a proportion $(1-\pi)$ care only old age consumption. Each agent does not know his consumption type until the beginning of middle period. However, $\pi$ is constant over time and known to the public, the realization of the consumption type of each agent is private information, therefore, not observable to the public. This setting of the agent’s preference is the same as in Diamond and Dybvig (1983). Therefore, as is the case there, financial intermediaries will be established to provide liquidity efficiently in the economy.

Figure 1 depicts the life schedule of agents born at period $t$. At the beginning of period $t$ (when young), he supplies labor endowment and earns wage $w_t$, $w_t$ denotes the wage rate at period $t$. He deposits his entire wage with financial intermediaries and the financial intermediaries spend the deposits on investments. The proceeds from investments are used for the payment to depositors contingent on the date of withdrawal. In period $t+1$, $G_m$ type agents

![Figure 1. Life Schedule of Agents Born at Period t](image-url)
make a withdrawal from financial intermediaries and consume it. In period $t + 2$, $G_0$ type agents also make a withdrawal and consume it.

The economy has two investment technologies available to agents: liquid investment and illiquid investment. Liquid investment yields one unit of consumption goods in period $t + 1$ per one unit of consumption good invested in period $t$. The liquid investment may be considered as the reserve of financial intermediaries. Illiquid investment yields one unit of capital goods in period $t + 2$ per one unit of consumption goods invested in period $t$. Capital goods are combined with labor to produce consumption goods. The production process of consumption goods will be discussed later. If it is stopped in period $t + 1$, then illiquid investment yields $(1 - \tau)$ units of consumption goods per unit, where $0 < \tau \leq 1$. $\tau$ denotes the liquidation costs of an immature illiquid investment.\(^5\) When $\tau$ is nearly one, the liquidation value of an immature illiquid investment is nearly zero. In contrast, when $\tau$ is nearly zero, the liquidation of one unit of immature illiquid investment yields nearly one unit of consumption goods.

In the economy, there is a continuum of firms whose size is one. Each firm uses capital goods and labor, producing consumption goods according to,

\[
y_t = A \left( k_t \right)^{\alpha} (l_t)^{1-\alpha}
\]

\[
A = B (\bar{k}_t)^{1-\alpha} \quad 0 < \alpha < 1.
\]

where $B$ is a positive number. $y_t$ is output per representative firm in period $t$. $k_t$, $l_t$, and $\bar{k}_t$ denote the capital stock, labor input and average capital stock per representative firm in period $t$, respectively. Each firm takes $A$ as given. In addition, we assume that the capital stock fully depreciates in the productive process.

3. Market Equilibrium

In each period $t$, each firm hires capital stock $k_t$ and labor $l_t$ in competitive factor markets. Thus, factors of production are paid their marginal product. Let $r_t$ denote the rental price of capital and $w_t$ denote the wage paid to a young agent in period $t$. Then,

\[
r_t = \alpha A \left( k_t \right)^{\alpha-1} (l_t)^{1-\alpha}
\]

\[
w_t = (1 - \alpha) A \left( k_t \right)^{\alpha} (l_t)^{-\alpha}
\]

\(^5\) As Cooper and Ross (1998) pointed out, it is natural that the liquidity cost $\tau$ is endogenously determined in the economy. However, we do not think of $\tau$ as an endogenous variable to briefly investigate the relation between $\tau$ and investment.
Since the size of a generation and firm is equal, each firm employs one unit of labor in equilibrium, that is, $l = 1$. In addition, $k_t = \bar{k}$, in equilibrium. From these relation and the formulation of $A$, $r_t$ and $w_t$ are given as,

$$r_t = \alpha B$$
$$w_t = (1 - \alpha)Bk$$

(3.2)

Thus, $r_t$ is constant over time and henceforth we will omit time subscripts, $r_t = r$ for all $t$. The wage rate in period $t$, $w_t$, is proportional to the capital stock per a firm. Hence the rate of return on illiquid investment is equal to $r$. We assume that $r > 1$. Thus, the illiquid investment has a higher rate of return in two periods than the liquid investment but has a lower rate of return in one period than the liquid investment. The difference of one period return rate between liquid and illiquid investment depends on the value of $\tau$.

4. Portfolio Decisions of Financial Intermediaries and Liquidation Costs of Illiquid Investment

In the economy considered here, each agent is uncertain as to the timing of consumption but there is no uncertainty in the whole economy. In such an economy, as Diamond and Dybvig (1983) shows, financial intermediaries can improve the expected utility of depositors.

However, in these circumstances, bank runs due to unanticipated withdrawal by $G_o$ type depositors driven by pessimistic expectations may occur. Bank runs is a situation where financial intermediaries (banks) don’t have sufficient goods to pay for withdrawal claims of depositors. We now consider bank runs following Cooper and Ross (1998).

Financial intermediaries allocate deposits in the form of liquid investment and illiquid investment in each period $t$. Let $C_{m_t+1}$ and $C_{o_t+2}$ denote the consumption of a $G_m$ type agent in period $t+1$ and a $G_o$ type agent in period $t+2$, respectively, that correspond to the payments of financial intermediaries contingent on the date of withdrawal. We assume that there are a large number of financial intermediaries and they compete with each other for deposits. Then, $C_{m_t+1}$ and $C_{o_t+2}$ are expressed in the following equations.

$$C_{m_t+1} = \frac{s_t - s_t}{\pi}$$

(4.1)

$$C_{o_t+2} = \frac{r + s_t}{1 - \pi}$$

(4.2)

$$D_t = i_t + s_t$$

(4.3)

$D_t$, $i_t$, and $s_t$ represent the amounts of the deposits, illiquid investment
and liquid investment in period $t$ per depositor, respectively. $s_t$ denotes the amounts of the liquid investment used for payments to $C_{o,t+2}$. Of course, $s_t > s_t$. If $G_o$ type agents withdraw in middle age (they consume in period $t+2$ using liquid investment technology), financial intermediaries pay this by $s_t$ and liquidating $i_t$ before maturity. When they can not pay for all of the claims of withdrawal, bank runs occur. Namely, when deposit contracts satisfy (4.4), the possibility of runs exists.

$$(1 - \tau)i_t + s_t < (1 - \pi)C_{m,t+1} \quad (4.4)$$

The right-hand side of (4.4) is the quantity of consumption goods per depositor which financial intermediaries must pay according to deposit contracts when all $G_o$ type agents misrepresent their consumption type. The left hand side of (4.4) is the quantity of consumption goods per depositor that financial intermediaries can obtain by liquidating illiquid investment.

Equation (4.4) shows that liquidation cost $\tau$ is an important factor for the occurrence of bank runs. In the case that $\tau$ is large, financial intermediaries that have a higher proportion of illiquid investment in their portfolio are more vulnerable. In contrast, if $\tau$ is small, the importance of $s$ decreases because financial intermediaries can pay for the unanticipated withdrawal by liquidating illiquid investment. This implies that financial intermediaries may adjust the composition of their portfolio depending on liquidity cost $\tau$. The purpose of this paper is to show the relation between the portfolio of financial intermediaries and $\tau$ if they consider a risk of bank runs in advance.

In this paper, we particularly consider the situation where financial intermediaries provide deposit contracts that have no possibility of runs.\(^7\) The reason is that the avoidance of bank runs is considered as one of the plausible responses of financial intermediaries who face a risk of runs and that, in this case, an increase in $\tau$ influences the amounts of liquid and illiquid investment of financial intermediaries in different manners according to the value of $\tau$.

The utility function of agents $U(C)$ is,

$$\frac{1}{1 - \sigma} (C)^{1 - \sigma} \quad 0 < \sigma < 1$$

$$\log(C) \quad \sigma = 1$$

where $\sigma$ represents the degree of risk aversion. Financial intermediaries deter-

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\(^6\) Financial intermediaries cannot reject the withdrawals in middle age by $G_o$ type agents because the preference type of each agent is unknown to them.

\(^7\) Even if financial intermediaries are subject to runs avoiding condition, they can give the higher utility to depositors.
mine \( i_t \) and \( s_t \) by solving the following maximization problem.\(^8\)

\[
\begin{align*}
\max_{i_t, s_t} & \quad \pi U(C_{m,t+1}) + (1 - \pi) U(C_{o,t+2}) \\
\text{s.t} & \quad C_{m,t+1} = \frac{w_t - i_t - s_t}{\pi} \\
& \quad C_{o,t+2} = \frac{ni_t + s_t}{1 - \pi} \\
& \quad w_t - i_t \tau - C_{m,t+1} \geq 0 \\
& \quad i_t \geq 0 \\
& \quad s_t \geq 0
\end{align*}
\]

Constraint (4.8) is the condition for no possibility of runs. By substituting (4.6) into (4.8), we can verify that constraint (4.8) coincides with the condition for no bank runs. By solving the maximization problem, financial intermediaries can determine \( i_t \) and \( s_t \) depending on \( \tau \) as in Proposition 1.

Proposition 1

\( i_t \) and \( s_t \) are determined depending on \( \tau \) as following. \( \tau^* \) is the value such that

\[
\tau^* = 1 - r \left( 1 + \frac{r - 1}{\pi \tau} \right)^{-\frac{1}{\tau}}
\]

By a simple calculation, we find that \( \tau^* \) exists uniquely in the range of \((1 - r \frac{s - 1}{\tau}, 1)\).

\[
\begin{align*}
(\text{I}) \quad & 0 < \tau \leq 1 - r \frac{s - 1}{\tau} \quad \Rightarrow \quad i_t = \frac{(1 - \pi)r - \frac{r - 1}{\tau}}{\pi + (1 - \pi)r} w_t, \quad s_t = 0 \\
(\text{II}) \quad & 1 - r \frac{s - 1}{\tau} < \tau \leq \tau^* \quad \Rightarrow \quad i_t = \frac{(1 - \pi)w_t}{(1 - \pi)} \quad s_t = 0 \\
(\text{III}) \quad & \tau^* < \tau \leq 1 \quad \Rightarrow \quad i_t = \frac{Z - 1}{\sqrt{1 - \pi}} w_t, \quad s_t = \frac{r + (\tau - 1)Z}{\sqrt{r + Z}} w_t
\end{align*}
\]

\[
\begin{align*}
\tau^* & = \frac{r - 1 + \pi \tau}{1 - \pi} \quad Z = \left( 1 + \frac{r - 1}{\pi \tau} \right)^{\frac{1}{\tau}}
\end{align*}
\]

The proof of proposition 1 is available on your request. Proposition 1 shows that the portfolio of financial intermediaries change depending on the liquidation cost when they prevent runs. One of the interesting results of

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8) From the first order condition, deposit contract \( (C_{m,t+1}, C_{o,t+2}) \) satisfies \( U'(C_{m,t+1}) \leq U'(C_{o,t+2}) \). This implies that \( C_{m,t+1} < C_{o,t+2} \) from \( U' < 0 \). Therefore, \( \sigma_t \) type agents have no incentive to misrepresent their consumption type except the case that they expect that bank runs will occur in the next period. This means that the self-selection constraint is satisfied.
proposition 1 is that when \(1 - \frac{s^1 - r}{s} < \tau < \tau^*\), a rise in \(\tau\) increases illiquid investment, but when \(\tau^* < \tau \leq 1\), it decreases illiquid investment. In addition, from proposition 1, we can see that financial intermediaries have excess liquid investment only when \(\tau^* < \tau \leq 1\). This result is consistent with that of Cooper and Ross (1998).

The reason of the above result is as follows. When the illiquid investment technology is sufficiently liquid, financial intermediaries need not care for any possibility of runs because they can pay for unanticipated withdrawals by liquidating illiquid investment. Therefore, they choose the first-best portfolio regardless of the value of \(\tau\). This is shown in Proposition 1.

On the other hand, when the illiquid investment is sufficiently illiquid, the first-best portfolio allocation generates a possibility of runs. Therefore, financial intermediaries must change the composition of their portfolio. Then, there are two ways to avoid bank runs; one is to increase the illiquid investment in exchange for \(C_m\), and the other is to decrease the illiquid investment and to increase \(s\). The former method corresponds to (II) and the latter one corresponds to (III) of Proposition 1. From (4.4), both methods can eliminate the possibility of runs.

Financial intermediaries choose the method that has a less negative effect on the utility of depositors. Now, we imagine that financial intermediaries change the illiquid investment by \(\Delta h_i\) from that determined in (I) \([\Delta h_i\text{ is positive in (II)}\text{ and is negative in (III)}\] to avoid bank runs. Then, using Taylor expansion, the harm to depositors when financial intermediaries choose portfolio (II) and (III) is approximately expressed as,

\[
-\pi U'(C_{m,t+1})\Delta h_i + r(1-\pi) U'(C_{s,t+2})\Delta h_i,
\]

\[
(1-\pi)(r-1) U'(C_{s,t+2})\Delta h_i
\]

where \(C_{m,t+1}\) and \(C_{s,t+2}\) represent the amount of consumption in first-best allocation (I). Equation (4.11) expresses the harm to a depositor by (II) and (4.12) expresses that by (III). When \(1 - \frac{s^1 - r}{s} < \tau \leq \tau^*\), financial intermediaries can eliminate a possibility of runs by a small change in \(\Delta h_i\) from (I). Then, equation (4.11) and the envelope theorem imply that the harm to depositor by (II) is almost zero. While that, equation (4.12) takes a negative value. Hence, when \(1 - \frac{s^1 - r}{s} < \tau \leq \tau^*\) financial intermediaries choose portfolio (II).

However, if, when \(\tau^* < \tau \leq 1\), they choose portfolio (II), then financial intermediaries must replace a large amount of \(C_{m,t+1}\) with illiquid investment to avoid runs. This generates a considerable utility loss in middle age consumption, but does not increase the utility from old consumption so much. Therefore, consumption smoothing by allocation (III) is chosen when \(\tau^* < \tau \leq 1\). Proposi-
tion 1 states that portfolio decisions of financial intermediaries depending on their liquidity position may cause more complex economic dynamics. In the next section, we will consider the influences of portfolio adjustment on economic growth using a simple endogenous growth model.

5. The Rate of Economic Growth and the Liquidity of Investment

In this section, we consider the relation between the growth rate and liquidation cost \( \tau \) in the economy with financial intermediaries who avoid runs. The capital stock of each firm in period \( t + 1 \) is given as (5.1) in equilibrium,

\[
k_{t+1} = i_t = \phi w_t = \phi (1-\alpha)Bk_t,
\]

where \( \phi \) is the ratio of illiquid investment to deposits. Since \( y_{t+1} = Bk_{t+1} \) in our model, the growth rate is given as (5.2).

\[
\frac{y_{t+1}}{y_t} = \frac{Bk_{t+1}}{Bk_t} = (1-\alpha)B\phi
\]

The value of \( \phi \) is given in Proposition 1. Therefore, we can report the growth rate in the economy with financial intermediaries as Proposition 2.

Proposition 2

Let \( g^{f,i} \) be the growth rate in the economy with financial intermediaries who avoid bank runs. Then \( g^{f,i} \) is given as follows depending on \( \tau \). Here, we define that \( \nu \equiv (1-\alpha)B \).

\[
(I) \quad 0 < \tau \leq 1 - \frac{r-1}{\pi} \quad g^{f,i} = \frac{(1-\pi)}{\pi + (1-\pi)r^{-\frac{r-1}{\pi}}\nu}
\]

\[
(II) \quad 1 - \frac{r-1}{\pi} < \tau \leq \tau^* \quad g^{f,i} = \frac{(1-\pi)}{(1-\pi)\nu}
\]

\[
(III) \quad \tau^* < \tau \leq 1 \quad g^{f,i} = \frac{\left(1 + \frac{r-1}{\pi\nu}\right)^{\frac{1}{r-1}} - 1}{r - 1 + \frac{r-1}{\pi\nu} + \left(1 + \frac{r-1}{\pi\nu}\right)^{\frac{r-1}{\pi}}}\nu
\]

Since it is obvious from (5.2) and Proposition 1, we omit the proof of Proposition 2 in this paper. From Proposition 2, when \( 1 - \frac{r-1}{\pi} < \tau \leq \tau^* \), the growth rate is increasing with \( \tau \) and when \( \tau^* < \tau \leq 1 \) the growth rate is decreasing with \( \tau \). \(^9\)

Figure 2 depicts this relation between the growth rate \( g^{f,i} \) and the

\(^9\) In our model, the growth rate in (I) corresponds to the growth rate which Bencivenga and Smith (1991) shows.
In general, whether the growth rate is greater or less than one depends on the value of the parameter. The change of the parameter which increases the value of $\nu$ or $\phi$ raises growth rate.

6. Portfolio Decisions and Liquidation Costs in an Economy with No Financial Intermediaries

In this section, we examine the relation between liquidation costs $\tau$ and portfolio decisions in an economy with no financial intermediaries in order to understand the significance of them.

In this case, there is no opportunity for young agents to insure against preference uncertainty. All agents hold investments directly in each period $t$. In period $t+1$, $G_m$ type agents liquidate their illiquid investment and consume. $G_e$ type agents hold their liquid and illiquid investments in two periods and consume the proceeds from these investments in their old age. This implies that agents do not hold excess liquid investment in their portfolio because $G_m$ type agents consume all their liquid investment.

Then, young agents born at period $t$ allocate their wage between illiquid investment $i_t$ and liquid investment $s_t^*$ by solving the following problem.

$$\max_{s_t} \quad \frac{1}{1-\sigma}(s_t^* + i_t (1-\tau))^{1-\sigma} + \frac{1}{1-\sigma}(\nu i_t + s_t^*)^{1-\sigma}$$  \hspace{1cm} (6.1)

---

10) When the degree of relative risk aversion is more than one, the quantity of middle age consumption in the first best allocation is more than initial wage $w$. Hence, in this case, portfolio (I) is eliminated, portfolio (II) is chosen when $1 - \frac{\tau^{\frac{1}{2}}}{2} < \tau \leq \tau^*$ and portfolio (III) is chosen when $\tau^* < \tau \leq 1$. 

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Figure 2. The Rate of Economic Growth and Liquidation Costs in the Economy with Financial Intermediaries

![Graph showing the relationship between liquidation costs and growth rate]
We summarize the solution to this problem as in proposition 3.

Proposition 3

\( i_t \) and \( s_t^* \) are determined as follows depending on \( \tau \) in an economy with no financial intermediaries. We define that \( \bar{\tau} = (1-\pi)(r-1)\pi^{-1} \) and \( \bar{\tau} \) is defined as \( \tau \) such that \( \tau (1-\tau)^{-\bar{\tau}} = \bar{\tau} r^{-\bar{\tau}} \). \( \bar{\tau} \) exists uniquely in the range of \((0, 1)\) and \( \bar{\tau} \) exists also uniquely in the range of \((0, \infty)\). From a simple calculation, we can easily see that \( \bar{\tau} \leq \bar{\tau} \).

\[
\begin{align*}
\text{(i)} & \quad 0 < \tau \leq \bar{\tau} \quad i_t = w_t, \quad s_t^* = 0 \\
\text{(ii)} & \quad \bar{\tau} < \tau < 1 \quad i_t = \frac{\left(\frac{\bar{\tau}}{\tau}\right)^{\frac{1}{\bar{\tau}}}-1}{(r-1) + \left(\frac{\bar{\tau}}{\tau}\right)^{\frac{1}{\bar{\tau}}}} w_t, \quad s_t^* = \frac{r - (1-\tau) \left(\frac{\bar{\tau}}{\tau}\right)^{\frac{1}{\bar{\tau}}}}{(r-1) + \left(\frac{\bar{\tau}}{\tau}\right)^{\frac{1}{\bar{\tau}}}} w_t \\
\text{(iii)} & \quad \bar{\tau} \leq \tau \leq 1 \quad i_t = 0, \quad s_t^* = w_t
\end{align*}
\]

If \( \bar{\tau} > 1 \), portfolio (iii) vanishes and agents choose portfolio (ii) in the range of \( \bar{\tau} < \tau \leq 1 \). The proof of Proposition 3 is omitted here but is available on your request.

As shown in Proposition 3, changes in liquidation costs \( \tau \) influence the portfolio decisions of young agents only when \( \bar{\tau} < \tau \leq \bar{\tau} \). Then, an increase in \( \tau \) decreases the ratio of illiquid investment to wage \( \phi \) and increases the ratio of liquid investment to wage. In other ranges of \( \tau \), changes of \( \tau \) do not influence portfolio decisions. When \( \tau \) is sufficiently low \((0 < \tau \leq \bar{\tau})\), young agents allocate their entire income to illiquid investment. In the next period, \( G_m \) type agents liquidate illiquid investment and consume it. \( G_o \) type agents hold illiquid investment for two periods and consume the proceeds in their old age. When \( \tau \) is sufficiently high \((\bar{\tau} \leq \tau \leq 1)\), young agents allocate their entire income to liquid investment. \( G_m \) type agents and \( G_o \) type agents hold liquid investment until their middle age and old age, consuming the proceeds from it.

Of course, such portfolio allocations of agents also influences economic growth. In our model, the growth rate is given by \((1-\pi)\nu \phi \). Note that a proportion of \( \pi \) of each generation liquidates illiquid investment. Therefore, capital stock per firm becomes \((1-\pi)i_t \) in equilibrium. Figure 3 depicts the relation between the growth rate and \( \tau \).
7. Concluding Remarks

This paper investigates how financial intermediaries who avoid bank runs decide the composition of their portfolio depending on liquidation costs of the illiquid investment and the effects on economic growth, using a simple endogenous growth model. The results of this paper show that the composition of a financial intermediary's portfolio changes considerably depending on liquidation costs. Of course, it also effects economic growth and implies that the effects of economic policies may depend on the liquidity position of financial intermediaries. For example, from the result of this paper, taxation on the liquidation of illiquid investment increases the growth rate when it is good and decreases the growth rate when it is bad. This paper can also be used to consider the influences of monetary policies by replacing liquid investment with money. In that case, the effects of monetary policy will also change according to the liquidity position of financial intermediaries.

However, the results of this paper are limited to the case where financial intermediaries avoid runs. If they choose other principles in making portfolio decisions, the results change. Especially, portfolio decisions seems important in the case that they maximize the expected utility of depositors including that in the bank runs equilibrium as in Cooper and Ross (1998).

Finally, it is worth noting that non-monotone effects of liquidation costs on illiquid investment is particular to an economy with financial intermediaries. Without them, an increase in liquidation costs simply leads to a decrease of illiquid investment and an increase of liquid investment. This implies that fi-
Financial intermediaries promote economic growth by their efficient allocation of goods as well as generate a complex growth pattern.

References


