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Intra-Industry Trade and FDI under Oligopoly Trade Patterns and Trade Policies

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This paper describes the patterns of trade in an oligopolistic international market. In a two country model, firms which share common technology can become either national or multinational. In the latter case, they require asymmetric plant-specific fixed costs. As the production costs and market sizes of the countries converge, intra-industry trade and foreign direct investment (FDI) occur. Then the supply logistics of firms in each country are determined by the level of trade costs and the ease with which FDI can be undertaken. The effects of several trade policies are examined within the above context and the applicability of the model is demonstrated with examples from the Japanese automobile industry.

JFL Classification Numbers: F12, F23
Key Words: Intra-Industry Trade, Multinational Firms, Production Cost Differentials, Market Size, Trade Costs.

1. Introduction

Over the last two decades, there have been great strides taken in the development of new trade theories, which model imperfect competition and increasing returns to scale, and are capable of explaining the extensive trade in similar products between developed countries that is intra-industry trade.[1] Furthermore, the new trade theory models have provided an apt framework for examining the patterns of foreign direct investment (FDI) and, in particular, the relationship between the activities of multinational firms and intra-industry trade.[2]

Rowthorn (1992) modelled an oligopolistic market to demonstrate that trade patterns are, in part, determined by trade costs and relative market sizes. His model implies that, as countries converge in market size and relative factor endowments, either intra-industry trade or FDI occurs given the level of trade costs. Similarly, Markusen and Venables (1998) use a model with a sin-

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[1] For example, Either (1982) observes that "the largest and fastest growing component of world trade since World War II has been the exchange of manufactures between industrialized countries".

[2] We define FDI as investment by a firm in a foreign means of production for the purpose of suppling the host country's market via local production.

gle factor of production, labor, to show that under certain conditions FDI will supplant intra-industry trade as economies converge. They extend their analysis to a model with two factors of production to show that convergence in market size may lead to growth in FDI (Markusen and Venables (2000)).

In Section 2, 3, and 4 we present a unified framework that is capable of explaining the existence of intra-industry trade and FDI. The model extends that of Markusen and Venables (1998) in order to describe the general patterns of trade without the use of numerical simulations.

We draw on examples from the Japanese Automobile Industry to show the applicability of some of the main results. Therefore, before describing the model a brief review of the Japanese Automobile Industry seems appropriate.

It is widely accepted that the automobile manufacturing industry has been one of the key engines of growth for Japan during the postwar years. Throughout its development, this industry has had to adapt to a number of developments in the market that have caused significant structural changes.

In 1955, The Japanese Ministry of International Trade and Industry (MITI) launched The National Car Plan and implemented a policy whereby subsidies were provided to the automobile industry for the purpose of reducing production costs. This helped to promote the expansion of the domestic automobile industry and stimulate employment. In addition, during this period the Japanese government moved to protect the domestic automobile industry by restricting foreign direct investment (FDI) and imposing high tariff rates on imported cars. For example tariffs of 40% were imposed on the smaller classes of vehicles. Around 1960, companies such as Suzuki, Honda and Matsuda, entered into the four-wheel automobile industry. During the 1960's, Japan enjoyed a period of rapid growth and the automobile industry expanded along with other industries. By the 1970's, the Japanese automobile industry had become strong enough to compete with manufacturers in the U.S. and began exporting. At this time U.S. barriers to trade targeting automobiles were low. However, in the 1980's a growing trade deficit with Japan and the precarious financial position of automobile manufacturers in the U.S.[3] forced a policy change and generated trade friction between the U.S. and Japan. As a result Japanese automobile manufacturers were encouraged to voluntarily restrict exports (VER) to the U.S. market in 1981. While a few Japanese firms had already invested in production facilities in the U.S. by this time, the VER created an incentive for more firms to follow suit and build factories in the U.S. during the 1980's.[4]

The purpose of this paper is three-fold. First, we show how trade patterns are determined by (1) trade costs, (2) market size, (3) production cost differentials, and (4) an index of barriers to FDI. If trade costs are small, we expect to

[3] For example, Chrysler, one of the "big three" automobile manufacturers, faced bankruptcy in 1980.

[4] Honda, which had been producing motorcycles in the U.S. since 1979, expanded production in Ohio in 1982 and became the first Japanese automobile manufacturer to produce cars in the U.S. Two other major Japanese companies, Toyota and Nissan, started to invest in the U.S. during the mid 1980's.

observe intra-industry trade or one-way exports. If there are large barriers to trade and therefore high trade costs, we expect to observe tariff-jumping FDI (Rowthorn (1992)). Second, we explain the intra-industry trade and FDI that occurs between similar developed countries. For example, Japanese firms are exporting automobiles to the U.S. and at the same time supplying the market directly from local factories. This can be explained by Japan's relatively cheap unit costs and the relatively larger automobile market of the U.S. Third, we investigate the effects of several trade policies on intra-industry trade and FDI activities. For example, suppose a government implements a trade liberalization policy. Trade costs are lowered and restrictions on FDI are loosened. If the former is effective, intra-industry trade activities will occur. If the latter is effective, we will observe FDI.

This paper is organized as follows. In Section 2, we provide our basic model and show how it will be solved. In Section 3, we define the equilibrium and derive proposition 1, which describes the existence of an index of barriers to FDI that separates intra-industry trade and FDI equilibrium regions. In Section 4, we provide a description of intra-industry trade and FDI and explain when each occurs. In Section 5 we examine how the patterns of trade will be affected by four trade policies: (1) a production cost subsidy, (2) a market growth policy, (3) a restrictive trade policy, and (4) a FDI incentive policy. In Section 6, we re-examine trade patterns assuming equal production costs. In Section 7, we summarize our main results and suggest some possible extensions. Most of the proofs and mathematical calculations are supplied in the appendixes.

2. The Model

In this model there are two countries, Country 1 and Country 2, both of which produce agricultural and manufactured goods. Agricultural goods assume the role of a numeraire and are produced for supply to a perfectly competitive international market where trade can be undertaken with negligible costs. Firms in the manufacturing sector produce homogeneous goods X_i for supply to both domestic and foreign markets. If firms enter the market, which is oligopolistic in nature, they choose to supply the foreign market as either a national firm or a multinational firm. Next they select the profit-maximizing output for each country taking the output of rival firms as fixed in a Cournot fashion.

National and multinational firms respectively produce X_{ii}^n and X_{ii}^m to meet domestic demand. The marginal cost in Country i, w_i, is constant for all firms. However, as the endowments of factors of production may differ for each country, we allow marginal costs to differ between countries, i.e. $w_1 \neq w_2$.[5] National firms supply the foreign market with exports X_{ij}^n and in doing so incur trade costs $t = 1+\tau$, where $0 \leq \tau \leq 1$ represents the costs associated with transport and/or barriers to trade. Multinational firms supply foreign markets with

[5] In section 6, we discuss the case where $w_1 = w_2$.

output X_{ij}^{m} produced in a local plant.[6] As a result, multinational firms are able to avoid the trade costs incurred by national firms, but bear an additional fixed cost when investing in a second plant abroad. The profit functions for national firms and multinational firms in Country i are respectively

$$\pi_i^n = P_i X_{ii}^n + P_j X_{ij}^n - w_i X_{ii}^n - w_i (1+\tau) X_{ij}^n - w_i (F+G), \tag{1}$$
$$\pi_i^m = P_i X_{ii}^m + P_j X_{ij}^m - w_i X_{ii}^m - w_j X_{ij}^m - w_i (F+G) - w_j G', \tag{2}$$

where p_i and p_j denote the price of X in Country i and Country j. F and G denote firm-specific and plant-specific costs respectively. Both national and multinational firms incur these domestic fixed costs. However, as described above, multinational firms also incur an additional plant-specific fixed cost G' in the foreign country. We allow for the possibility that domestic plant investment and foreign plant investment may not be equal, i. e. $G \neq G'$.

While plant-specific fixed costs refer to the costs incurred when setting up production facilities, firm-specific costs refer to the fixed-costs incurred in producing headquarter services that are freely transferable to all production locations. These services include R&D, advertising and managerial expertise.

Firms segment the domestic and foreign markets when determining their optimal output levels. Assuming Cournot strategies, this implies that the price mark-up will equal the ratio of market share to price elasticity of demand.

$$P_i \left(1 - \frac{e_{ii}^n}{\varepsilon}\right) = w_i \tag{3}$$
$$P_j \left(1 - \frac{e_{ij}^n}{\varepsilon}\right) = tw_i \tag{4}$$
$$P_i \left(1 - \frac{e_{ii}^m}{\varepsilon}\right) = w_i \tag{5}$$
$$P_j \left(1 - \frac{e_{ij}^m}{\varepsilon}\right) = w_j \tag{6}$$

where e_{ij}^k denotes a firm's market share and ε denotes the price elasticity of demand. For analytical simplicity, we assume that consumer preferences are identical for both countries and described by a Cobb-Douglas utility function. Utility maximization provides the following demand function for manufacturing products.

$$X_i = \frac{\beta M_i}{P_i}, \tag{7}$$

where M_i stands for GDP in Country i, and β is the share of expenditure on

[6] Throughout this paper the superscripts (k=n, m) will be used to denote whether a particular variable is associated with a national or multinational firm. In addition, the subscripts ii and ij will denote whether a particular variable is associated with output, produced by a firm based in Country i, that is bound for the domestic market i or the foreign market j, $(i \neq j)$.

manufactured goods. Given this demand function the price elasticity of demand for any particular manufactured good equals 1 reducing the price mark-up to its market share.

$$e_{ij}^{k} = \frac{X_{ij}^{k}}{X_i} \tag{8}$$

The total supply of manufactured goods for Country i is

$$X_i = n_i X_{ii}^{n} + n_j X_{ji}^{n} + m_i X_{ii}^{m} + m_j X_{ji}^{m}, \tag{9}$$

where n and m respectively denote the number of national and multinational firms in Country i or j.

Next, we use zero-profit conditions to determine the number of each type of firm that survives in the market. Substituting equations (3) and (4) into equation (1) and equations (5) and (6) into equation (2), these can be written as

$$\begin{aligned} P_i e_{ii}^{n} X_{ii}^{n} + P_j e_{ij}^{n} X_{ij}^{n} &= w_i (F+G) \\ P_i e_{ii}^{m} X_{ii}^{m} + P_j e_{ij}^{m} X_{ij}^{m} &= w_i (F+G) + w_j G'. \end{aligned} \tag{10}$$

Given w_i and M_i this model has ten endogenous variables: X_{ii}^{k}, X_{ij}^{k}, P_i, n_i and m_i. There are ten independent equations, equations (3) through (6), (7), (9) and (10). Therefore, this system can be solved.

Substituting equations (7) into equations (3), (5) and (6), we find that the following relationship holds.[7]

$$X_{ii}^{n} = X_{ii}^{m} = X_{ji}^{m} \tag{11}$$

Furthermore, we derive from equations (3) and (4) the following relationship.

$$\frac{X_{ji}^{n}}{X_i} = 1 - \frac{tw_j}{w_i}\left(1 - \frac{X_{ii}^{n}}{X_i}\right) \tag{12}$$

For notational simplicity it is convenient to redefine the market shares as

$$x_{ij}^{k} = e_{ij}^{k} = \frac{X_{ij}^{k}}{X_i}.$$

This allows us to rewrite equations (11) and (12) as equations (13) and (14) respectively.

$$x_{ii}^{n} = x_{ii}^{m} = x_{ji}^{m} \tag{13}$$

$$x_{ji} = 1 - tw_{ji}(1 - x_{ii}), \tag{14}$$

where $w_{ji} = \frac{w_j}{w_i}$. Furthermore, we drop the superscript n in equation (14). With-

[7] From equations (3), (5) and (6) we have $P_i(1-\frac{X_{ii}^{n}}{X_i}) = P_i(1-\frac{X_{ii}^{m}}{X_i}) = P_i(1-\frac{X_{ji}^{m}}{X_i})$. This reduces to equation (11).

out loss of generality, we assume that $F+G=1$ and rewrite the zero-profit conditions in (10) as equations (15) and (16) using price mark-up equations (3) through (6).

$$(x_{ii})^2 S_i + w_{ji}(x_{ij})^2 S_j = 1 \tag{15}$$
$$(x_{ii})^2 S_i + w_{ji}(x_{ij})^2 S_j = 1 + \rho w_{ji}, \tag{16}$$

where $S_i + \frac{\beta M_i}{w_i}$ and $\rho = \frac{G'}{F+G}$. S_i expresses the market size of manufactured goods in terms of commodity units[8] and ρ is an index of barriers to FDI.

The total supply of manufactured goods, equation (9), is also rewritten as

$$n_i x_{ii} + n_j x_{ji} + m_i x_{ii} + m_j x_{ji} = 1. \tag{17}$$

While we allow for the possibility that multinational firms may incur relatively high plant-specific costs when undertaking FDI, $G'>G$, we exclude the extreme case where the foreign investment G' exceeds total domestic investment $F+G$ and assume that

$$F + G - G' > 0. \tag{18}$$

This provides Result 1.

Result 1
There is no possibility of an autarky occurring in the market equilibrium.

As shown in equation (13) it is possible for a multinational firm to produce an output equivalent to the total output of two autarkies. Under condition (18) a multinational firm's total fixed costs are less than the sum of the fixed costs incurred when there is a domestic firm operating in each country. Therefore, the market is not in equilibrium when there are two autarkies.

We have assumed that production costs may differ between countries. Without loss of generality, we assume that $w_1>w_2$. This coupled with condition (18) provides the following results.

Result 2
When multinational firms in Country 2 are able to survive earning zero profits, multinational firms in Country 1 always suffer losses and therefore do not enter the market.

Result 3
Only national firms based in Countries 1 and 2 and multinational firms based in Country 2 can survive in the market and at most only two of these firm types

[8] This definition of market size is essentially the same as that of Rowthorn (1992).

will exist in equilibrium.

Result 2 implies that $\pi_2^m = 0 > \pi_1^m$. While four possible types of firms may exist, Result 2 states that only three of these will enter the market. Furthermore, equations (13) and (14) imply that at most only two of the zero-profit conditions in (11) and (12) are necessary to determine the output of manufactured goods. This provides Result 3.

The number of each type of firm in the market is determined using equation (17) and the following six regions describe six possible equilibrium regimes: $(n_1, 0)$, $(n_2, 0)$, $(m_2, 0)$, (n_1, n_2), (n_1, m_2) and (n_2, m_2). These are be expressed as the number of each type of firm that exists in the market.[9)]

3. Equilibrium Analysis

Once we have determined the states of the market, as described by the types of existing firms, we can determine the values of x_{ii} and x_{ij} using equations (15) and (16). We cannot explicitly solve these. Generally speaking, however, we can determine the number of firms in the market, using equation (17). These are provided in Table 1.

Table 1：The Number Of Firms In Each Region

Region	Firm Types	Number of Firms
Region Ⅰ:	$(n_1,0)$	$n_1 = \frac{1}{x_{11}} = \frac{1}{x_{12}} = \sqrt{S_1 + w_{21}S_2}$
Region Ⅱ:	$(n_2,0)$	$n_2 = \frac{1}{x_{22}} = \frac{1}{x_{21}} = \sqrt{S_2 + w_{12}S_1}$
Region Ⅲ:	$(m_2,0)$	$m_2 = \frac{1}{x_{11}} = \frac{1}{x_{22}} = \sqrt{\frac{S_2 + w_{12}S_1}{1+\rho w_{12}}}$
Region Ⅳ:	(n_1,n_2)	$n_1 = \frac{x_{22}-x_{21}}{\triangle_1}, n_2 = \frac{x_{11}-x_{12}}{\triangle_1}$ where $\triangle_1 = x_{11}x_{22} - x_{12}x_{21} > 0$
Region Ⅴ:	(n_1,m_2)	$n_1 = \frac{x_{22}-x_{11}}{\triangle_2}, m_2 = \frac{x_{11}-x_{12}}{\triangle_2}$ where $\triangle_2 = x_{11}(x_{22}-x_{12})$
Region Ⅵ:	$(n_2,m_2,)$	$m_2 = \frac{x_{11}-x_{22}}{\triangle_3}, m_2 = \frac{x_{22}-x_{21}}{\triangle_3}$ where $\triangle_3 = x_{22}(x_{11}-x_{21})$

Continuing to assume that production costs differ between countries $(w_{12} > 1)$, it is possible to solve x_{ii} and x_{ij} implicitly using equations (15) and (16). We assume that a market equilibrium cannot occur in any region where there

[9)] If the production costs in each country are equal $(w_1 = w_2)$, then it is impossible to distinguish between the multinational firms of each country. However, if we set $m = m_1 + m_2$ in equations (16), then once again we will have again six possible equilibrium regimes: $(n_1, 0)$, $(n_2, 0)$, $(m, 0)$, (n_1, n_2), (n_1, m) and (n_2, m).

are firms earning negative profits.

Let $l_i = n_i, m_i$ denote the number of national or multinational firms in Country i.

Definition
In order for Region κ ($\kappa = I, II, \cdots, VI$) to describe an equilibrium regime, the following two conditions must be satisfied.

(1) *For firm i in Region κ, $\pi_i^{li} = 0, x_{ii} > 0, x_{ij} > 0$.*
(2) *For firm i not in Region κ, $\pi_i^{li} < 0$.*

Equations (13) and (14) imply Lemma 1.

Lemma 1
A region that contains two autarky economies cannot describe an equilibrium.

As we assume that headquarter services are freely transferable, multinational firms, under condition (18), can produce a level of output equal to that of two autarky economies, but at a lower cost.

Lemma 2
When intra-industry trade (Region IV) occurs in the equilibrium regime, $x_{ii} > x_{ij}$.

Referring to Table 1, in order for both n_1 and n_2 to be positive, $x_{11} > x_{12}$ and $x_{22} > x_{21}$. Given the symmetry of the model, each firm supplies more domestically than abroad due to the existence of transport costs. This is similar to the result obtained by Rowthorn (1992).

We know from Result 3 that at most two types of firms can exist in any possible equilibrium regime. This allows us to establish Proposition 1.

Proposition 1
Intra-industry trade and FDI cannot occur simultaneously in equilibrium. In particular, there exists a constant, ρ^0, that divides the potential equilibrium regimes.

(1) *When $\rho^0 > \rho$, Regions III, V, and VI do not describe the equilibrium regime.*

(2) *When $\rho^0 < \rho$, Region VI does not describe the equilibrium regime.*

$$\rho^0 = f(w_{12}, S_1, t).$$ [10]

See Appendix A.

[10] As shown in Appendix A, we consider the case where $S_1 + S_2 =$ constant. The effect of each variable on ρ^0 is generally ambiguous.

Result 3 already indicates that intra-industry trade and FDI cannot occur simultaneously. Proposition 1 states that a critical value for the index of barriers to trade, ρ^0, divides the regions where FDI and intra-industry trade occur. The value of ρ^0 depends on the production cost differential (w_{12}), market size (S_1), and trade costs (t).

4. Intra-Industry Trade and Multinational Firms (FDI)

If we set w_{12} and S_1 as model parameters, then Proposition 1 states that for pairs of (w_{12}, S_1) that satisfy $\rho^0 < \rho$, we have intra-industry trade and for pairs of (w_{12}, S_1) that satisfy $\rho^0 > \rho$, we have FDI. Intuitively, a firm will supply a foreign market via exports when barriers to FDI are high. We consider the intra-industry trade case first.

Intra-industry Trade $(\rho^0 < \rho)$
Consider the following sets.

$$A_1 = \{(w_{12}, S_1): \text{Region I describes the equilibrium}\}$$
$$A_2 = \{(w_{12}, S_1): \text{Region II describes the equilibrium}\}$$
$$A_3 = \{(w_{12}, S_1): \text{Region IV describes the equilibrium}\}$$

Then we have the following properties.

(1) $A_1 \cup A_2 \cup A_3$ covers the entire range (w_{12}, S_1) where $\rho^0 < \rho$ is satisfied.

(2) All the sets are disjoint with each other.

These two properties separate Regions I, II, and IV for pairs of (w_{12}, S_1) that satisfy $(\rho^0 < \rho)$. Using the zero-profit conditions we can determine the equilibrium conditions for Region I and II, and in particular the border conditions for Regions I and II and Regions II and IV. Furthermore, a comparative static analysis of these conditions provides the location of each region and the shape of each border (See Appendix B). These are shown in Figure 1.

In Figure 1, the vertical and horizontal axes measure the production costs differential (w_{12}) and relative market size (S_1/S_2) respectively. From point C, where $w_{12} = 1$ and $S_1/S_2 = 1$, a leftward movement towards point A signifies a relative increase in S_2 and a rightward movement towards point A' signifies a relative increase in S_1. Figure 1 shows that when the market of Country 1 is relatively small and Country 2 has relatively low production costs, only national firms in Country 2 will exist. Conversely, when Country 2 has a relatively small market size and high production costs, only national firms in Country 1 will exist. Within a certain range, where w_{12} and S_1/S_2 are close to one, there will be national firms in both countries and intra-industry trade will occur. The arrow drawn from Region II to point C provides an example of the convergence theory. Initially, Country 2 has an advantage in terms of rela-

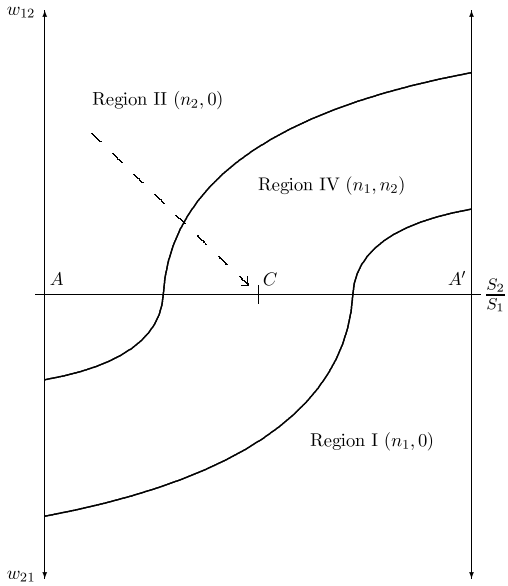

Figure 1. Intra-Industry Trade

tive production costs and market size, but as the production costs and market sizes of Country 1 and Country 2 converge this advantage is reduced and national firms appear in Country 1. Then intra-industry trade occurs (Either (1982)).

Multinational Firms (FDI) $(\rho^0 > \rho)$

Following the line of reasoning used in the intra-industry case, we consider the following sets.[11)]

$$A_1 = \{(w_{12}, S_1): \text{Region I describes the equilibrium}\}$$
$$A_2 = \{(w_{12}, S_1): \text{Region II describes the equilibrium}\}$$
$$A_3 = \{(w_{12}, S_1): \text{Region III describes the equilibrium}\}$$
$$A_4 = \{(w_{12}, S_1): \text{Region V describes the equilibrium}\}$$
$$A_5 = \{(w_{12}, S_1): \text{Region VI describes the equilibrium}\}$$

These sets have properties similar to those given in the intra-industry trade case. They cover the entire range where the pairs (w_{12}, S_1) satisfy $\rho^0 > \rho$ and are disjoint with each other. Once again, using the zero-profit conditions we can determine the equilibrium conditions for Regions I, II, and III, and the border conditions for Regions I and V, III and V, III and VI, and II and IV. A comparative static analysis of these conditions provides the location of each region and the shape of each border (see Appendix C). These are shown in Figure 2.

The basic framework of Figure 2 is the same as that of Figure 1. The ar-

[11)] These five regions are similar to the potential equilibria described by Rowthorn (1992) and Horstmann and Markusen (1992).

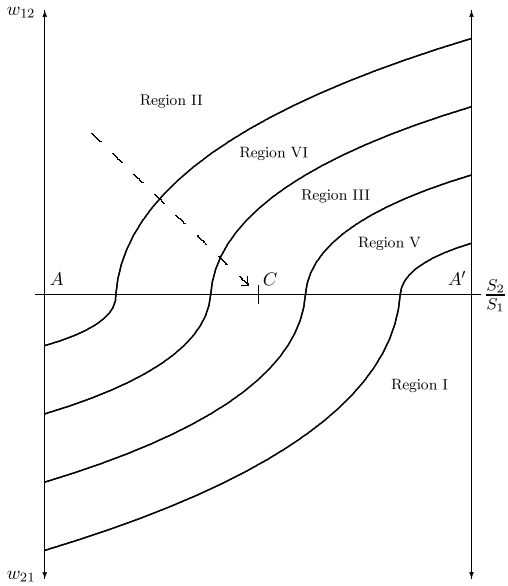

Figure 2. Active Multinational Firms

row drawn from Region II to point C shows how the equilibrium changes as production costs and market sizes converge. Initially, the larger market size and lower production costs of Country 2 lead to an equilibrium where only national firms in Country 2 exist. However, as the production cost advantage of Country 2 diminishes and the market of Country 1 expands, multinational firms based in Country 2 appear. This is described by the mixed equilibrium in Region VI, where there are both national and multinational firms based in Country 2. Further convergence in market size and production costs brings about an equilibrium described by Region III where there only multinational firms exist. In this case, the size of both markets is large enough to support the local production of multinational firms. This type of convergence is similar to that described by Markusen and Venables (1998).

This completes the analysis of all six potential equilibrium regimes and the intra-industry trade and FDI cases. The results concerning convergence are summarized in Proposition 2.

Proposition 2
As the two economies become similar, two types of convergence occur.

(1) *For $\rho < \rho^0$, the markets approach an equilibrium where only multinational firms exist.* (*Markusen and Venables (1998)*).

(2) *For $\rho > \rho^0$, the markets approach an equilibrium where only intra-industry trade occurs.* (*Ethier (1982)*).

5. Trade Policy Analysis

Before analyzing the effects of several trade policies, we examine how ρ^0 is affected by parametric changes.

Corollary 1
Under plausible conditions, we have the following:

$$\frac{\partial \rho^0}{\partial w_{12}} < 0, \frac{\partial \rho^0}{\partial S_1} < 0, \frac{\partial \rho^0}{\partial t} < 0.$$

See Appendix D.

Corollary 1 provides some intuitive explanations. As the production cost differential increases (in this case the production cost in Country 2 becomes relatively cheaper), firms in Country 2 will choose to export rather than becoming multinational firms. On the other hand, as the market of Country 1 expands, the cost of investing in a foreign plant becomes less expensive relative to potentially incurred trade costs and national firms expand to become multinational firms. In the following we investigate how the economies are affected by various trade policies. We consider: (1) a production cost subsidy, (2) a market growth policy, (3) a restrictive trade policy and (4) a FDI incentive policy. Our chief concern is the affect of these trade policies on intra-industry trade and FDI.

To facilitate the following analysis, we define both an index of intra-industry trade, I_t, and an index of FDI, I_m. First, suppose production locations are described by Region IV, where intra-industry trade occurs. Then the index of intra-industry trade, I_t, can be defined as

$$I_t = 1 - T, \text{ Where}$$

$$T = \frac{|n_1 P_2 x_{12} - n_2 P_1 X_{21}|}{n_1 P_2 X_{12} + n_2 P_1 X_{21}}.$$

As $n_1 P_2 X_{12}$ are exports from Country 1 and $n_2 P_1 X_{21}$ are imports from Country 2, I_t is crucially related to Country 1's import-export ratio, Q. Referring to Table 1,

$$Q = \frac{n_2 P_1 X_{21}}{n_1 P_2 X_{12}} = \frac{(x_{11} - x_{12}) x_{12} S_1 w_{12}}{(x_{22} - x_{21}) x_{12} S_2} = Q(w_{12}, S_1, t).$$

Next, suppose the economies are located in Region V, where we have national firms in Country 1 and multinational firms in Country 2. Then, I_m can be defined as

$$I_m = \frac{m_2 x_{21}^m}{n_1 x_{11}^n + m_2 x_{21}^m} = \frac{m_2}{n_1 + m_2} = \frac{x_{11} - x_{12}}{(x_{22} - x_{11}) + (x_{11} - x_{12})}.$$

5.1 Production Cost Subsidy

Suppose Country 1 proposes to increase employment by promoting the expansion of the manufacturing sector through the subsidization of production costs.[12] The effect on intra-industry trade will depend on the initial trade balance and Q. From the definition of I_t, if Country 1 has a trade surplus, then the effect of the production cost subsidy on I_t and Q will be the same.

Lemma 3
For Region IV

(1) *If Country* 1 *has a trade surplus,* $\frac{\partial I_t}{\partial w_{12}} > 0.$

(2) If *Country* 1 *has a trade deficit,* $\frac{\partial I_t}{\partial w_{12}} < 0.$

For Region V, $\frac{\partial I_m}{\partial w_{12}} > 0.$

See Appendix F.

Lemma 3 states that the production cost subsidy of Country 1 has opposing effects on I_t that depend on whether the manufacturing sector in Country 1 has a trade surplus or a trade deficit. When it enjoys a trade surplus, I_t reduces to the import ratio of total trade. As the production cost subsidy reduces w_{12}, the relative share of imports in total trade will be decreased. On the other hand, when Country 1 suffers from trade deficits in the manufacturing sector, the index of intra-industry trade, I_t, reduces to the ratio of exports from country 1 to total trade. The production cost subsidy increases the competitiveness of Country 1 in the world market and raises the ratio of exports.

Next, suppose the economies are in Region V. Then, as a result of the production cost subsidy, national firms in Country 1 become more competitive as fixed costs in country 1 are also reduced. Therefore, the number of n_1 firms increases and the number of m_2 firms decreases.

Finally, we briefly touch upon the effect of a production cost subsidy on trade patterns. Suppose that initially $\rho^0 > \rho$. In this case the subsidy lowers w_{12} and therefore ρ^0. If this policy is continued, the markets may change from an equilibrium regime with intra-industry trade to one with FDI.

As shown in Appendix B, the production cost subsidy moves both of the boundaries in Figure 1 upward. Therefore, Region I expands and Region II contracts; an anticipated result.

Similarly, referring to Appendix C, we find that the boundaries in Figure 2 also shift upward as a result of the production cost subsidy.

[12] The production cost subsidy is similar to the support provided to the Japanese automobile industry by MITI. Between 1951 and 1959, 369 million yen in commissions and subsidies were provided to the automobile industry. Further support was provided through low interest loans, a special depreciation system and exemption from import duties on machinery and equipment (Imai (1980)).

5.2 Market Growth Policy

In this section, we consider a case where macroeconomic policy is used to induce market growth in Country 1. Here, we simply assume there is a contraction in the market size of Country 2. This infers the relative expansion of S_1 resulting in two opposing effects.[13] The first is a *switch-over effect* where there is an increase in imports and a relative decrease in exports. The second is an *expansion effect* where the number of market entrants increases.

Lemma 4

Suppose the expansion effect dominates the switch-over effect in Region IV.

(1) *If Country* 1 *has a trade surplus,* $\frac{\partial I_t}{\partial S_1} > 0$.

(2) If *Country* 1 *has a trade* $\frac{\partial I_t}{\partial S_1} < 0$.

For Region V, $\frac{\partial I_m}{\partial S_1} < 0$.

See Appendix F.

Lemma 4 states that a market growth policy, undertaken in Country 1, will increase the number of firms in Country 1 and reduce the number of firms in Country 2. In addition the switch-over effect, from exports to local production, is larger in Country 1. Therefore the switch-over effect tends to reduce I_t while the expansion of the market increases I_t. These combined effects are stated in Lemma 4.

Referring to Appendixes B and C, we find that a market growth policy shifts all borders in both Figures 1 and 2. In both cases, Region I expands and Region II contracts.

Finally, we briefly touch upon the effect a market growth policy has on ρ^0. Initially, suppose that $\rho^0 > \rho$. In this case, we would expect FDI to prevail. If Country 1 maintains a market growth policy, ρ^0 may continue to decrease until the economies move from an equilibrium regime with FDI to a regime with intra-industry trade.

5.3 Restrictive Trade Policy

Suppose both countries raise tariff rates. Then substitution of local production for exports may occur in each country.[14] The effect of such a policy on I_t is generally ambiguous. However, as stated in Corollary 1, continuing this policy raises ρ^0 and may induce a tariff-jumping effect that influences a firm's decisions on the location of production. In this case the equilibrium will change

[13] As we have fixed the size of the total market to a constant $S_0 = S_1 + S_2$, growth in S_1 implies contraction in S_2.

[14] In May of 1981 a VER was introduced on Japanese automobile exports to the United States. Japanese automobile manufacturers responded by investing in plants in the United States.

suddenly from a regime with intra-industry trade to one with FDI.

For Region V, we have Lemma 5.

Lemma 5

In Region V, $\frac{\partial I_m}{\partial t} > 0$. *Furthermore, raising trade costs expand Region III in Figure* 2.

See Appendix F.

Because only firms in Country 1 are exporting, the tariff is imposed by Country 2. This reduces the number of firms in Country 1 and increases the number of firms in Country 2. This tariff-jumping effect works around both the border with Region V and that with Region VI. From Appendix C, $\frac{\partial G^{IV}}{\partial t} < 0$ and $\frac{\partial G^{IV}}{\partial t} > 0$. Therefore, the rise in t expands Region III through shifts in both the upper and lower borders.

5.4 FDI Incentive Policy

Finally, we examine the case where Country 1 implements a policy that creates incentives for FDI. In reality, many countries offer land at reduced prices and/or preferential tax rates to potential multinational firms who are interested in building plants.[15] Since these policies reduce the value of ρ, the FDI incentive policy has two implications. First, suppose the equilibrium is described by Region II. Then a FDI incentive policy implemented by Country 1 reduces ρ and pushes the economies into Region VI rather than into Region IV.

Second, if the equilibrium is described by Region V, the FDI promoting policy will increase the index of FDI, I_m for Country 1.

Lemma 6

Supposing the equilibrium is described by Region II, Country 1 may increase the number of multinational firms by using a FDI incentive policy.

6. Equal Production Costs

When production costs are equalized, trade patterns depend on trade costs (t) and market size (S_1). Since equal production costs are realized through factor price equalization, our argument can be described with a parallelogram similar to that of Grossman and Helpmann (1992)).

[15] When Japanese automobile manufacturers invested in the United States during the 1980's state and local governments offered various incentive packages to attract investment in their areas. These incentive packages generally included tax abatements, low-interest loans and land as well as investment in infrastructure such as roads and water access. The largest package, valued at $ 147 million in direct state investment was offered to Toyota when it built its first U.S assembly plant in Kentucky (Karan (2001)).

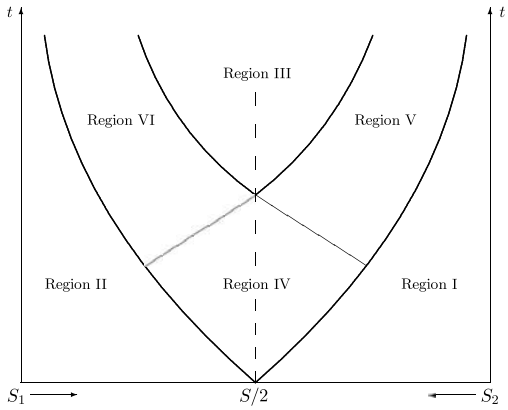

Figure 3. Equal Production Costs Case

The following analysis is similar to that presented in the previous section. If we set $w_{12} = 1$ in equations (16), the zero-profit condition will be the same for multinational firms based in either country. In other words, multinational firms will be indifferent when deciding which country to locate their headquarters in. As a result, in equation (17) the number of multinational firms $m_i = m_1 + m_2$. Now, there exists a $\rho^0 = \rho^0(1, S_1, t)$ that divides activity between intra-industry trade and FDI. As t approaches unity, x_{12} and x_{21} respectively approach x_{22} and x_{11}, and national firms always dominate multinational firms.[16] Therefore, there are pairs of t and S_1 which satisfy $\rho^0 = \rho$ for a given ρ. We call this locus the *trade-convertibles line.* It has a positive slope and is located between Region IV and Region V. We have already determined that the boundaries between Region IV and Region II, RegionII and Region VI, Region VI and Region III, all have positive slopes. An argument similar to that presented in Appendix B and C can be used to define the remaining regions in Figure 3.

Figure 3 shows the importance of market size in determining trade patterns. Because domestic production has a relative cost advantage over exports, firms decide to locate their headquarters in the larger market. Furthermore, Figure 3 clearly shows the existence of the tariff-jumping effect originally described in Rowthorn (1992) and Horstmann and Markusen (1992).

Proposition 3
Propositions 1 and 2 still hold in the equal production cost case, where $\rho^0 = \rho^0(1, S, t)$.

Proposition 3 states that the trade patterns and the effects of trade policies will not be altered in the case of factor price equalization.

[16] If we ignore fixed costs, national firms in either country can supply their products at the same cost.

7. Concluding Remarks

In this paper we have shown, using a partial equilibrium framework, that trade patterns can be explained by production cost differentials, market size, trade costs and a barriers to investment index. In particular we find that intra-industry trade and FDI occur as the production costs and market sizes of two countries converge. For a given level of trade costs intra-industry trade occurs in equilibrium when barriers to investment are large and FDI occurs in equilibrium when they are small. These results maintain for both the case where production costs differ and the case where they are equal. We have also examined the effects of several trade policies the results of which are provided in Section 5.

We have assumed, as have most previous authors, that the same technology is used to produce both commodities and headquarter services. In reality, however, it is possible to model different technologies for each. For example, labor is used mainly to produce commodities, while both labor and capital are required to produce headquarter services such as R&D. In such a case, we can determine which types firms will survive in each country and derive trade patterns using the framework derived in the paper.

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Appendix A Proof of Proposition 1

Suppose Region Ⅳ is an equilibrium region. Then equations (14) and equations (15) together determine x_{11}, x_{22}, x_{12}, and x_{21} as functions of w_{12}, S_1, S_2, and t. However, using the constraint for world resource endowments, $S_1+S_2=$constant, we can express the equilibrium values of x_{ii} and x_{ij} explicitly. Attaching the superscript "Ⅳ," these values are

$$x_{ii}^{IV}=x_{ii}^{IV}(w_{12},S_1,t) \text{ and } x_{ij}^{IV}=x_{ij}^{IV}(w_{ij},S_1,t).$$

Next, substituting x_{ii}^{IV} and x_{ij}^{IV} into equation (14), we find that the necessary equilibrium condition for Region Ⅳ is

$$(x_{22}^{IV})^2S_2+w_{12}(x_{21}^{IV})^2S_1<1+\rho w_{12} \tag{A. 1}$$

Using equation (9) and subtracting equation (13) for $i=2$ from equation (A. 1), we derive the following inequality.

$$S_1\left\{(x_{11}^{IV})^2-(x_{21}^{IV})^2\right\}<\rho.$$

Therefore,

$$\rho^0=S_1\left\{(x_{11}^{IV}(\cdot))^2-(x_{21}^{IV}(\cdot))^2\right\}=\rho^0(w_{12},S_1,t).$$

If $\rho^0<\rho$, multinational firms do not satisfy the zero profit condition. On the other hand, if $\rho^0>\rho$, then intra-industry trade will not occur in the equilibrium region.

Appendix B Illustrations of Figure 1

Intra-Industry Case $(\rho^0<\rho)$

First, we consider the case where Region I is the equilibrium region. Since $n_2=0$ in equation (17), we have $n_1=\frac{1}{x_{11}}=\frac{1}{x_{12}}$. Substituting these into equation (15), we obtain

$$x_{11}=x_{12}=\frac{1}{\sqrt{S_1+w_{21}S_2}}$$

Therefore, $\frac{\partial x_{11}}{\partial x_{12}}>0$, $\frac{\partial x_{11}}{\partial S_1}<0$ and the necessary equilibrium condition for Region I becomes

$$(x_{22})^2S_2+w_{12}(x_{21})^2S_1-1<0$$

Note that from equation (11),

$$x_{21}=1-tw_{21}+tw_{21}x_{11} \text{ and}$$
$$x_{22}=(x_{12}+tw_{12}-1)/tw_{12}.$$

Define

$$G^I=(x_{22})^2S_2+w_{12}(x_{21})^2S_1-1=G^I(w_{12},S_1,t)$$

Then

$$\frac{\partial x_{21}}{\partial w_{12}}=\frac{t(1-x_{11})}{(w_{12})^2}+\frac{t}{w_{12}}\frac{\partial x_{11}}{\partial w_{12}}>0.$$
$$\frac{\partial x_{22}}{\partial w_{12}}=\frac{1}{t(w_{12})^2}\left\{\frac{\partial x_{12}}{\partial w_{12}}+1-x_{12}\right\}>0$$

Therefore, $\frac{\partial G^I}{\partial w_{12}}>0$ and $\frac{\partial G^I}{\partial S_1}=w_{12}(x_{21})^2-(x_{22})^2+2\left\{x_{22}\frac{\partial x_{22}}{\partial S_1}S_1+w_{12}x_{21}\frac{\partial x_{21}}{\partial S_1}S_2\right\}$. Note that $\frac{\partial x_{22}}{\partial S_1}<0$

and $\frac{\partial x_{21}}{\partial S_1} < 0$. Furthermore, in order for n_1 to take positive values, $x_{22} > x_{21}$. As a result, for relatively small values of $w_{12} > 1$, it is likely that $x_{22} > w_{12}x_{21}$. Therefore, $\frac{\partial G^I}{\partial S_1} < 0$ and the boundary between Region I and Region Ⅳ takes a positive slope as shown in Figure 1.

Finally, we compute the effect of a change in t on G^I.

$$\frac{\partial G^I}{\partial t} = \frac{2(1-x_{11})}{t^2 w_{12}}\{x_{22}S_2 - x_{21}t^2 w_{12}S_1\}$$

Now, we consider the case where Region Ⅱ is the equilibrium region. Given that $n_1 = 0$,

$$x_{22} = x_{21} = \frac{1}{n_2}.$$

Substituting these into equation (15) we obtain,

$$x_{22} = x_{21} = \frac{1}{\sqrt{S_2 + w_{12}S_1}}$$

Therefore, the necessary equilibrium condition for Region Ⅱ becomes

$$G^{II} = (x_{11})^2 S_1 + w_{21}(x_{12})^2 S_2 - 1 < 0.$$

Using equations (14),

$$\frac{\partial x_{11}}{\partial w_{12}} = -\frac{1-x_{21}}{t} + \frac{w_{12}}{t}\frac{\partial x_{21}}{\partial w_{12}} < 0$$

$$\frac{\partial x_{12}}{\partial w_{12}} = -t(\ (1-x_{22}) + tw_{12}\frac{\partial x_{22}}{\partial w_{12}} < 0$$

Therefore, $\frac{\partial G^{II}}{\partial w_{12}} < 0$ and $\frac{\partial G^{II}}{\partial S_1} = (x_{11})^2 - w_{21}(x_{12})^2 + 2\left\{x_{11}S_1\frac{\partial x_{11}}{\partial S_1} + w_{21}x_{12}\frac{\partial x_{11}}{\partial S_1}S_2\right\}$. Using the relations stated in equations (14), we obtain the following.

$$\frac{\partial G^{II}}{\partial S_1} = \left[(x_{11})^2 - w_{21}(x_{12})^2\right] - \left\{\frac{x_{11}S_1}{tw_{21}} + tx_{12}S_2\right\}(S_2 + w_{12}S_1)^{-\frac{3}{2}}(w_{12}-1).$$

The first bracketed term is positive, since $n_2 > 0$ implies $x_{11} > x_{12}$ and the second bracketed term is negative. Therefore, the sign of $\frac{\partial G^{II}}{\partial S^1}$ is generally ambiguous. However, if we assume that w_{12} takes values close to unity, then $\frac{\partial G^{II}}{\partial S_1} > 0$. This implies that the boundary line between Region Ⅱ and Region Ⅳ is positively sloped as drawn in Figure 1.

Finally, we examine the effect of a rise in t on the G^{II} function.

$$\frac{\partial G^{II}}{\partial t} = 2(1-x_{22})\{x_{11}w_{12}S_1 - x_{12}t^2 S_2\}$$

Appendix C Illustrations of Figure 2

C. 1 Region Ⅵ as an Equilibrium Regime

First we consider the case where Region Ⅵ, which is bordered by Region Ⅱ and Region Ⅲ, is the equilibrium regime.

Boundary with Region Ⅱ

When Region Ⅱ is the equilibrium regime $n_1 = m_2 = 0$. Using equations (15) and (16) we can obtain solutions for x_{22} and x_{21}.

$$x_{22} = x_{21} = \frac{1}{n_2} = \frac{1}{\sqrt{S_2 + w_{12}S_1}}$$

Therefore, $\frac{\partial x_{22}}{\partial w_{12}} < 0$ and $\frac{\partial x_{22}}{\partial S_1} < 0$.[17] As a result the necessary equilibrium condition for Region Ⅱ is

[17] In fact, when $x_{22} = x_{21}$, $m_2 = 0$

$$G^{III}(w_{12},S_1,t,\rho)=(x_{22})^2S_2+w_{12}(x_{11})^2S_1-(1+\rho)<0.$$

From equation (14), we have the following.

$$\frac{\partial x_{11}}{\partial w_{12}}=-\frac{(1-x_{11})}{t}+\frac{w_{12}}{t}\frac{\partial x_{21}}{\partial w_{12}}<0.$$

Then

$$\frac{\partial G^{III}}{\partial w_{12}}=2\left\{x_{22}S_2\frac{\partial x_{22}}{\partial w_{12}}+w_{12}x_{11}S_1\frac{\partial x_{11}}{\partial w_{12}}\right\}+(x_{11})^2S_1.$$

Because $x_{11}<1$, we assume that $\frac{\partial G^{III}}{\partial w_{12}}<0$.

Now consider the effect of a change in S_1 on G^{III}.

$$\frac{\partial G^{III}}{\partial S_1}=w_{12}(x_{11})^2-(x_{22})^2+2\left\{x_{22}S_2\frac{\partial x_{22}}{\partial S_1}+w_{12}x_{11}S_1\frac{\partial x_{11}}{\partial S_1}\right\}$$

Because $n_2>0$, $x_{11}>x_{22}$. The value of the bracketed term on the right-hand-side of the above equation is zero when $w_{12}=1$. Therefore, we assume $\frac{\partial G^{III}}{\partial S_1}>0$, which implies that the boundary, $G^{III}=1+\rho$, has a positive slope and that the area above is Region Ⅱ.

Finally, we consider the effect of a rise in t on G^{III}. Using equation (13), it is easy to obtain

$$\frac{\partial G^{III}}{\partial t}=\frac{1-x_{11}}{t^2w_{21}}>0.$$

Boundary with Region Ⅲ

Next we consider the case where Region Ⅲ is the equilibrium. We can obtain the equilibrium values for x_{22} and x_{11} from equation (13).

$$x_{11}=x_{22}=\frac{1}{m_2}=\sqrt{\frac{1+\rho}{S_2+w_{12}S_1}},$$

where

$$\frac{\partial x_{11}}{\partial w_{12}}<0,\ \frac{\partial x_{11}}{\partial S_1}<1\quad\text{and}\quad\frac{\partial x_{11}}{\partial \rho}>0.$$

The necessary equilibrium condition for Region Ⅲ is

$$G^{IV}(w_{12},S_1,t,\rho)=G^{IV}=(x_{22})^2S_2+w_{12}(x_{21})^2S_1-1<0.$$

Given that $x_{22}^{III}=\sqrt{1+\rho x_{22}^{II}}$, we have the following sign conditions.

$$\frac{\partial G^{IV}}{\partial w_{12}}<0\quad\text{and}\quad\frac{\partial G^{IV}}{\partial S_1}>0.$$

Furthermore,

$$\frac{\partial G^{IV}}{\partial t}=-(x_{21})(1-x_{11})S_1<0,\quad\text{and}\quad\frac{\partial G^{IV}}{\partial \rho}>0.$$

As the values of functions G^{III} and G^{IV} both move in the same direction with respect to changes in w_{12} and S_1, Region Ⅲ in Figure 2 is located in the same position as Region Ⅳ in Figure 1 and $x_{22}^{III}>x_{22}^{II}$.

C. 2 Region Ⅴ as an Equilibrium

Next we consider the case where Region Ⅴ, which is bordered by Region Ⅲ and Region I, is the equilibrium.

Boundary with Region Ⅲ

We have already obtained the equilibrium solutions values of x_{11} and x_{22} for Region Ⅲ. These are

$$x_{11} = x_{22} = \sqrt{\frac{1+\rho}{S_2 + w_{12}S_1}}$$

We have the following sign conditions.

$$\frac{\partial x_{11}}{\partial w_{12}} < 0, \frac{\partial x_{11}}{\partial S_1} < 0, \quad \text{and} \quad \frac{\partial x_{11}}{\partial \rho} > 0.$$

This necessary equilibrium condition for Region Ⅲ is

$$G^V(w_{12}, S_1, t, \rho) = (x_{11})^2 S_1 + w_{21}(x_{12})^2 S_2 - 1 < 0.$$

Given that $\frac{\partial x_{12}}{\partial w_{12}} = -t(1-x_{22}) + tw_{12}\frac{\partial x_{22}}{\partial w_{12}} < 0$, the effect of a change in w_{12} on G^V is as follows.

$$\frac{\partial G^V}{\partial w_{12}} = 2\left\{x_{11}S_1\frac{\partial x_{11}}{\partial w_{12}} + w_{12}x_{12}S_2\frac{\partial x_{12}}{\partial w_{12}}\right\} - (w_{21})^2(x_{12})^2 S_2 < 0,$$

Similarly,

$$\frac{\partial G^V}{\partial S_1} = \left[(x_{11})^2 - w_{21}(x_{12})^2\right] + 2\left\{x_{11}S_1\frac{\partial x_{11}}{\partial S_1} + w_{21}x_{12}S_2\frac{\partial x_{12}}{\partial S_2}\right\}$$

For m_2 to be positive, $x_{11} > x_{12}$. Therefore, the first bracketed term on the right-hand-side of the above equation is positive. The second bracketed term is zero for values of w_{12} that are close to unity. Therefore, we assume $\frac{\partial G^V}{\partial S_1} > 0$. This implies that the boundary between Region Ⅲ and Region V has a positive slope and that the area above the is Region Ⅲ. The effects of changes in t and ρ on G^V are as follows.

$$\frac{\partial G^V}{\partial t} = -2x_{12}S_2(1-x_{22}) < 0 \quad \text{and}$$

$$\frac{\partial G^V}{\partial \rho} = 2\left\{x_{11}S_1\frac{\partial x_{11}}{\partial \rho} + w_{21}x_{12}S_2\frac{\partial x_{12}}{\partial \rho}\right\} > 0.$$

Boundary with Region I

When the equilibrium region is Region I, $m_2 = n_2 = 0$. The gives the equilibrium solutions

$$x_{11} = x_{12} = \frac{1}{n_1} = \frac{1}{\sqrt{S_1 + w_{21}S_2}}$$

Note that

$$\frac{\partial x_{11}}{\partial w_{12}} > 0 \quad \text{and} \quad \frac{\partial x_{11}}{\partial S_1} < 0.$$

The necessary equilibrium condition for Region I is

$$G^{VI}(w_{12}, S_1, t, \rho) = (x_{22})^2 S_2 + w_{12}(x_{21})^2 S_1 - (1+\rho).$$

Note that

$$\frac{\partial G^V}{\partial w_{12}} = (x_{12})^2 S_1 + 2\left\{x_{12}S_2\frac{\partial x_{22}}{\partial w_{21}} + w_{11}x_{21}\frac{\partial x_{21}}{\partial w_{12}}\right\}.$$

It is likely that $\frac{\partial X_{22}}{\partial w_{12}}$ is negative which enables us to assume that $\frac{\partial G^{VI}}{\partial w_{12}} > 0$. Similarly,

$$\frac{\partial G^{VI}}{\partial S_1} = w_{12}(x_{21})^2 - (x_{22})^2 + 2\left\{x_{22}S_2\frac{\partial x_{22}}{\partial S_1} + w_{12}x_{21}S_1\frac{\partial x_{21}}{\partial S_1}\right\}.$$

For areas of Region V close to the boundary, $n_1 > 0$. This infers that $x_{22} > x_{21}$. Therefore, we assume that w_{21} is close to unity and that $\frac{\partial G^{VI}}{\partial S_1} < 0$. This sign condition implies that the Region I is

below Region V as shown in Figure 2. Once again, we can obtain the following.

$$\frac{\partial G^{VI}}{\partial t} = 2x_{22}S_2\frac{\partial x_{22}}{\partial t} = 2x_{22}S_2\frac{1-x_{22}}{t^2} > 0$$

This completes the upper half of the phase diagram given in Figure 2. The lower part is drawn in a similar fashion denoting the vertical axis as $w_{21}(=1/w_{12})$.

Appendix D Proof of Corollary 1

Referring to Appendix A,

$$\rho^0 = S_1\left\{(x_{11}^{IV})^2 - (x_{21}^{IV})^2\right\}.$$

In the equation above, superscript "Ⅳ" shows the equilibrium values in Region Ⅳ. To simplify the notation we omit this superscript.

First, consider the equilibrium state of Region Ⅳ.

$$H^1 = (x_{11})^2S_1 + w_{21}(x_{12})^2S_2 - 1 = H^I(x_{11},x_{22};w_{12},S_1,t)$$
$$H^2 = (x_{22})^2S_2 + w_{21}(x_{21})^2S_1 - 1 = H^2(x_{11},x_{22};w_{12},S_1,t)$$

where $x_{ij} = 1 - tw_{ij} + tw_{ij}x_{ji}$. Then we derive the following

$$\frac{\partial H^1}{\partial x_{11}} = 2x_{11}S_1, \frac{\partial H^1}{\partial x_{22}} = 2tx_{12}S_2, \frac{\partial H^2}{\partial x_{11}} = 2tx_{21}S_1, \frac{\partial H^2}{\partial x_{22}} = 2x_{22}S_2$$
$$\frac{\partial H^1}{\partial w_{12}} = -(w_{21})^2(x_{12})^2S_2 - 2w_{21}x_{12}S_2t(1-x_{22}) < 0,$$
$$\frac{\partial H^2}{\partial w_{12}} = (x_{21})^2S_1 + tw_{21}x_{21}(1-x_{11})S_1 > 0,$$
$$\frac{\partial H^1}{\partial S_1} = (x_{11})^2 - w_{21}(x_{12})^2 > 0, \frac{\partial H^2}{\partial S_1} = -(x_{22})^2 - w_{12}(x_{21})^2 < 0$$
$$\frac{\partial H^1}{\partial t} = -2x_{12}(1-x_{22})S_2, \frac{\partial H^2}{\partial t} = -2x_{21}(1-x_{11})S_1.$$

Now

$$\triangle = \begin{vmatrix} \frac{\partial H^1}{\partial x_{11}} & \frac{\partial H^1}{\partial x_{22}} \\ \frac{\partial H^2}{\partial x_{11}} & \frac{\partial H^2}{\partial x_{22}} \end{vmatrix} = 4S_1S_2(x_{11}x_{22} - t^2x_{12}x_{21}).$$

We assume that

$$X_{ii} - tX_{ij} > 0 \ \ (i \neq j = 1,2)$$

Condition A. 1 implies that for each firm production for the domestic market is greater than gross production for the foreign market. This guarantees that $\triangle > 0$. Since $n_1 > 0$ and $n_2 > 0$ in Region Ⅳ, $x_{11} > x_{12}$ and $x_{22} > x_{21}$ (see Table 1). Therefore, we assume that $\frac{\partial H^2}{\partial S_1} < 0$ and immediately obtain the following:

$$\frac{\partial x_{11}}{\partial w_{12}} > 0, \frac{\partial x_{22}}{\partial w_{12}} < 0, \frac{\partial x_{11}}{\partial S_1} < 0, \frac{\partial x_{22}}{\partial S_1} > 0.$$

Simple calculations shows that the following to hold.

$$\frac{\partial x_{11}}{\partial t} = \frac{4x_{12}S_2}{\triangle}\{X_{22} - tX_{21}\}$$
$$\frac{\partial x_{22}}{\partial t} = \frac{4x_{21}S_1}{\triangle}\{X_{11} - tX_{12}\}.$$

Under condition (A. 1),

$$\frac{\partial x_{11}}{\partial t} > 0 \quad \text{and} \quad \frac{\partial x_{22}}{\partial t} > 0.$$

The effects of changes in w_{12}, S_1, and t on ρ^0 can now be obtained by the following:

$$\frac{\partial \rho^0}{\partial w_{12}} \equiv -2S_1\left\{(x_{21}tw_{21}-x_{11})\frac{\partial x_{11}}{\partial w_{12}}+t(w_{21})^2x_{21}(1-x_{11})\right\},$$
$$\frac{\partial \rho^0}{\partial S_1} = \frac{\rho^0}{S_1}-(tw_{21}x_{21}-x_{11})\frac{\partial x_{11}}{\partial S_1},$$
$$\frac{\partial \rho^0}{\partial t} = w_{21}(1-x_{11})-[tw_{21}x_{21}-x_{11}]\frac{\partial x_{11}}{\partial t}$$

Under intra-industry trade, we assume $x_{11} > x_{21}$ and $x_{22} > x_{12}$, both of which imply that $tw_{12} > 1$ and $tw_{21} > 0$ (see equations(14)). Then we consider the situation where $x_{11}-tw_{21}x_{21} \approx 0$. This derives Corollary 1.

Appendix E Equilibrium Conditions in Region V

The equilibrium conditions for Region V can be stated as

$$H^3 = (x_{11})^2S_1+w_{21}(x_{21})^2S_2-1 = 0,$$

$$H^4 = (x_{22})^2S_2+w_{12}(x_{11})^2S_1-1-\rho w_{12} = 0$$

From this we can compute the following

$$\frac{\partial H^3}{\partial x_{11}} = 2x_{11}S_1, \frac{\partial H^3}{\partial x_{22}} = 2tx_{12}S_2, \frac{\partial H^4}{\partial x_{11}} = 2w_{12}x_{11}S_1, \frac{\partial H^4}{\partial x_{22}} = 2x_{22}S_2,$$
$$\frac{\partial H^3}{\partial w_{12}} = -(w_{12})^2(x_{12})^2S_2-2tw_{21}x_{12}S_2(1-x_{22}) < 0,$$
$$\frac{\partial H^4}{\partial w_{12}} = (x_{11})^2S_1-\rho, \frac{\partial H^3}{\partial S_1} = (x_{11})^2-w_{21}(x_{12})^2 > 0,$$
$$\frac{\partial H^4}{\partial S_1} = w_{12}(x_{11})^2-(x_{22})^2, \frac{\partial H^3}{\partial x_{11}} = -2x_{12}S_2(1-x_{22}) < 0,$$
$$\frac{\partial H^4}{\partial t} = 0, \frac{\partial H^3}{\partial \rho} = 0, \frac{\partial H^4}{\partial \rho} = -w_{12} < 0,$$

In a situation where mark-up revenues exceed fixed costs ρ, $\frac{\partial H^4}{\partial S_1} > 0$ and $n_1 > 0$, $x_{22} > x_{11}$ and it is possible to assume that $\frac{\partial H^4}{\partial S_1} < 0$. Given this assumption the following conditions can be derived.

$$\frac{\partial x_{11}}{\partial w_{12}} > 0, \frac{\partial x_{22}}{\partial w_{12}} < 0, \frac{\partial x_{11}}{\partial S_1} < 0, \frac{\partial x_{22}}{\partial S_1} > 0,$$
$$\frac{\partial x_{11}}{\partial t} > 0, \frac{\partial x_{22}}{\partial t} < 0, \frac{\partial x_{11}}{\partial \rho} < 0, \frac{\partial x_{22}}{\partial \rho} > 0,$$

Appendix F Trade Policy Effects

Employment Promoting Policy

Suppose Region Ⅳ describes the market equilibriums of Countries 1 and 2. If Country 1 has a trade surplus, then

$$I_t = 2\frac{n_2P_1X_{21}}{n_1P_2X_{12}+n_2P_1X_{21}} = \frac{Q}{1+Q}$$

and

$$\frac{\partial I_t}{\partial \alpha} = \frac{Q}{(1+Q)} = \frac{\partial Q}{\partial \alpha} \quad \alpha = \alpha(w_{12}, S_1, t)$$

On the other hand, if Country 1 has a trade deficit, then $I_t = \frac{1}{1+Q}$ and

$$\frac{\partial I_t}{\partial \alpha} = -\frac{1}{(1+Q)^2}\frac{\partial Q}{\partial \alpha}, \quad \alpha = \alpha(w_t, S_1, t)$$

Referring to Appendix C, $\frac{\partial Q}{\partial w_{12}} > 0$. In Region V $I_m = \frac{T_1}{T_1 + T_2}$, where $T_1 = x_{11} - x_{12}$ and $T_2 = x_{22} - x_{11}$. Therefore,

$$\frac{\partial I_m}{\partial \beta} = \frac{1}{(T_1 + T_2)^2} \left\{ T_2 \frac{\partial T_1}{\partial \beta} - T_1 \frac{\partial T_2}{\partial \beta} \right\},$$

where $\beta = w_{12}, S_1, t$. Referring to Appendix D,

$$\frac{\partial T_1}{\partial w_{12}} > 0 \quad \text{and} \quad \frac{\partial T_2}{\partial w_{12}} < 0.$$

Therefore, $\frac{\partial I_m}{\partial w_{12}} > 0$.

Market Growth Policy

First, we note that $Q = Q_1 \frac{S_1}{S_2}$, where $Q_1 = \frac{(x_{11} - x_{12}) x_{21} w_{12}}{(x_{22} - x_{21}) x_{12}}$. Then,

$$\frac{dlogQ}{dlogS_1} = \frac{\overline{S}}{S_2} + \frac{dlogQ_1}{dlogS_1}.$$

The first term on the right-hand-side of the above equation shows the expansion effect and the second term, the switching effect, which is negative.

As for the effect on I_m, note that

$$\frac{\partial T_1}{\partial S_1} < 0 \quad \text{and} \quad \frac{\partial T_2}{\partial S_1} > 0.$$

Trade Cost Policy

Referring to Appendix C, $\frac{\partial T_1}{\partial t} > 0$ and $\frac{\partial T_2}{\partial t} < 0$. Therefore, $\frac{\partial I_m}{\partial t} > 0$.