WAVE PROPAGATION ANALYSIS OF CRACKED LEVY PLATE

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ABSTRACT

Guided wave is verified to be more sensitive to small size defects than conventional various vibration characteristics (such as natural frequencies, mode shapes, and time-domain dynamic responses). Different from the conventional finite element (FE) method only approximately treats the structural mass based on static shape function, the spectral finite element (SFE) method models the mass distribution exactly by using frequency-dependent (dynamic) shape function, and it is more accurate for modeling vibration characteristics of structures in high-frequency domain, such as wave propagation phenomenon. In this paper, the SFE method is used to model the Levy plate with a part-through surface crack for investigating the wave propagation properties. It's assumed that the crack having an arbitrary length, depth and location was parallel to one side of the plate, by considering the elastic behaviour of the plate at the crack location as a line spring with a varying stiffness along the crack length. Whereas, the related works in the literature for using SFE method to model cracked plate with above damage type rarely concentrate on the effects of crack length and crack's 2D location since the dimension along one of plate axes are usually assumed to be infinite. For the Levy plate investigated in this paper, the length of plate along the free edges to which the finite-length crack parallel is finite. This paper reports the theoretical development in modeling the cracked Levy plate based on the SFE method, and the numerical simulation of the wave propagation analysis with the developed cracked spectral plate element.

Keywords: Wave propagation, Levy plate, Spectral finite element method, Part-through crack.

1. INTRODUCTION

The structural health monitoring (SHM) methodology using vibration measurements are based on the fact that the damage occurred in the structure usually causes a decrease in the structural stiffness, which produces changes in modal parameters of the structure (Carden and Fanning 2004). It is the fact that small defects usually induce negligible change in lower energy modes, while higher energy modes might only slightly perturbed in such situation. The finite element (FE) technique is most

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commonly used in vibration-based methods, and the number of FEs must be sufficiently large in order to obtain satisfactory higher energy modes as compared to that required in low-frequency range. While, the higher energy modes correspond to shorter wave length, and the FE size should be much shorter than the wave length in high frequency range. This makes the computational effort for determining higher energy modes with FE technique be extremely costly.

The spectral finite element (SFE) method (Chakraborty et al. 2008) essentially a FE method formulated in the frequency domain, can be employed to solve vibration problem in high frequency range very well. In the SFE method, the dynamic stiffness matrix is formulated from the frequency-dependent shape functions that are exact solutions of the governing differential equations by treating the mass distribution exactly. This is contrast to the conventional FE method where the mass distribution is only approximately modeled due to the use of static shape function. Thus, the SFE method can certainly provide more accurate predictions of dynamic characteristics especially in high-frequency range with few elements needed to mode large regions.

It is found from the literature that, the SFE method had already been applied to model the bar, beam (Krawczuk 2002), and plate type structures (Danial and Doyle 1995; Krawczuk et al. 2004) with and without cracks for the purpose of vibration analysis or damage detection. However, the current applications of SFE method for the cracked plates concentrate mostly on the spectral plate element with the assumption that the length of the element with the nodal spectral DOFs defined is finite, while the size of the plate in another direction parallel to the crack line is infinite although having a finite-length window. The drawback of this cracked spectral plate element is that the finite-length property of the crack can not be revealed as well as the central location of the crack in its paralleled direction.

This paper reports both the theoretical development the numerical simulations of wave propagation for a spectrally-modeled Levy-type plate with a finite-length and non-propagating surface crack lying parallel to one side of the plate. The crack assumed to have an arbitrary length, depth and location, is modeled as a line spring having a varying stiffness along the crack line.

2. THEORETICAL DEVELOPEMENTS

The dimensions of the spectral plate element is a and b in the x and y directions, and H in the thickness direction, respectively, as shown in Figure 1. The length of the crack is $2C$ and the crack depth h varies along the length of the crack. Let the (x_0, y_0) to be the coordinate of the crack center and h_0 to be the central crack depth. For convenience, it's assumed that the edges at y=0 and y=b have a simply supported boundary condition, where the other two edges are currently considered to have arbitrary boundary conditions, and the crack is assumed to be parallel to the edge x=0. Levy introduced a method employing a separable single Fourier series to solve this type of problem. By considering the two regions (1) and (2) separated by a hypothetical boundary at the crack line x=x_0, the Levy-type solutions at frequency ω_n can be expressed as

$$W_n^{(i)}(x,y;\omega_n)=\sum_{m=1}^{\infty}(c_{1mn}^{(i)}\mathrm{e}^{p_1x}+c_{2mn}^{(i)}\mathrm{e}^{p_2x}+c_{3mn}^{(i)}\mathrm{e}^{p_3x}+c_{4mn}^{(i)}\mathrm{e}^{p_4x})\sin(\xi_m y),\quad \begin{cases} x\in[0,x_0], & \text{for } i=1 \\ x\in[x_0,a], & \text{for } i=2 \end{cases} \tag{1}$$

where $\xi_m = m\pi/b$. $c_{1mn}^{(i)}$ to $c_{4mn}^{(i)}$ (for i=1,2) are eight unknown coefficients to be determined. Denoting $\mathbf{c}_{mn}^{(i)}=[c_{1mn}^{(i)},c_{2mn}^{(i)},c_{3mn}^{(i)},c_{4mn}^{(i)}]^{\mathrm{T}}$ for convenience, equation (1) can be further written as

$$W_n^{(i)}(x,y;\omega_n)=\sum_{m=1}^{\infty}W_{mn}^{(i)}(x;\xi_m,\omega_n)\sin(\xi_m y),\quad W_{mn}^{(i)}(x;\xi_m,\omega_n)=\mathbf{q}_{mn}\mathbf{c}_{mn}^{(i)},\quad \mathbf{q}_{mn}=[\mathrm{e}^{p_1x},\mathrm{e}^{p_2x},\mathrm{e}^{p_3x},\mathrm{e}^{p_4x}] \tag{2}$$

p_1 to p_4 are wavenumbers, which are the roots of the following characteristic equation,

$$p^4-2\xi_m^2p^2+\xi_m^4-\rho H\omega_n^2/D=0 \tag{3}$$

where ρ is the mass density, $D=EH^3/12(1-v^2)$ denotes the plate stiffness, v is the Poisson ratio, and E represents the Young's modules. The rotation angle (or slope) about the y direction in frequency domain corresponding to the two crack regions can be derived from equation (1) as $\Theta_{xn}^{(i)}(x,y;\omega_n)=\partial W_n^{(i)}(x,y;\omega_n)/\partial x$.

At the left and right edges (x=0 and x=a) of the plate element, the nodal displacements and rotations in frequency-wavenumber domain can be written by

$$\mathbf{d}_{mn}(\xi_m,\omega_n)=\begin{Bmatrix} W_{1mn}^{(1)}(\xi_m,\omega_n) \\ \Theta_{1xmn}^{(1)}(\xi_m,\omega_n) \\ W_{2mn}^{(2)}(\xi_m,\omega_n) \\ \Theta_{2xmn}^{(2)}(\xi_m,\omega_n) \end{Bmatrix}=\begin{Bmatrix} W_{mn}^{(1)}(0;\xi_m,\omega_n) \\ \Theta_{xmn}^{(1)}(0;\xi_m,\omega_n) \\ W_{mn}^{(2)}(a;\xi_m,\omega_n) \\ \Theta_{xmn}^{(2)}(a;\xi_m,\omega_n) \end{Bmatrix} \tag{4}$$

On the other hand, at the hypothetical boundary (x=x_0) induced by the crack, by utilizing the continuity conditions of slope, displacement, bending moment and shear force, respectively, one can get the following relations in frequency-wavenumber domain as,

$$\begin{aligned} &\Theta_{xmn}^{(1)}(x_0;\xi_m,\omega_n)=\Theta_{xmn}^{(2)}(x_0;\xi_m,\omega_n)+\Delta_{mn}(\xi_m,\omega_n),\quad W_{mn}^{(1)}(x_0;\xi_m,\omega_n)=W_{mn}^{(2)}(x_0;\xi_m,\omega_n), \\ &M_{xxmn}^{(1)}(x_0;\xi_m,\omega_n)=M_{xxmn}^{(2)}(x_0;\xi_m,\omega_n),\quad V_{xzmn}^{(1)}(x_0;\xi_m,\omega_n)=V_{xzmn}^{(2)}(x_0;\xi_m,\omega_n) \end{aligned} \tag{5}$$

where Δ_{mn} is the wavenumber transform of the slope discontinuity induced by the present of the crack in frequency domain Δ_n, which is given with the help of Fourier series expansion as

$$\Delta_n(y;\omega_n)=\theta(y)M_n^{(1)}(x_0,y;\omega_n)=\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty}\theta_m(\xi_{m_1})M_{mn}^{(1)}(x_0;\xi_{m_2},\omega_n)\sin(\xi_{m_1}y)\sin(\xi_{m_2}y) \tag{6}$$

where $\theta(y)$ denotes the bending flexibility at both sides of the crack (Khadem and Rezaee 2000).

One the other hand, the wavenumber-domian transform of Δ_n can also be calculated as

$$\Delta_{mn}(\xi_m,\omega_n)=\frac{2}{b}\int_0^b \theta(y)M_n^{(1)}(x_0,y;\omega_n)\sin(\xi_m y)\mathrm{d}y \tag{7}$$

After some laborious derivations, one can get from equation (7) that

$$\Delta_{mn}(\xi_m,\omega_n)=(\frac{e_1}{e_2+e_3-e_4}+\frac{e_1}{e_2+e_3+e_4})\theta_m(\xi_{m_1})M_{mn}^{(1)}(x_0;\xi_{m_2},\omega_n) \tag{8}$$

where

$$e_1=m(-1)^m-m(-1)^{m_1+m_2},e_2=\pi m^2(-1)^{m_1+m_2},e_3=-\pi(m_1^2+m_2^2)(-1)^{m_1+m_2},e_4=2\pi m_1 m_2(-1)^{m_1+m_2} \tag{9}$$

Substituting equation (2) into equations (4) and (5), one can get that

$$\mathbf{U}_{mn}(\xi_m,\omega_n)\mathbf{c}_{mn}=\mathbf{u}_{mn}(\xi_m,\omega_n) \tag{10}$$

and

$$\mathbf{U}_{mn}(\xi_m,\omega_n)=\begin{bmatrix}\mathbf{U}_{1mn}(\xi_m,\omega_n) & \mathbf{0}_{2\times4}\\ \mathbf{0}_{2\times4} & \mathbf{U}_{2mn}(\xi_m,\omega_n)\\ \mathbf{U}_{3mn}(\xi_m,\omega_n) & \mathbf{U}_{4mn}(\xi_m,\omega_n)\end{bmatrix},\ \mathbf{c}_{mn}=\begin{Bmatrix}\mathbf{c}_{mn}^{(1)}\\ \mathbf{c}_{mn}^{(2)}\end{Bmatrix},\ \mathbf{u}_{mn}(\xi_m,\omega_n)=\begin{Bmatrix}\mathbf{d}_{mn}(\xi_m,\omega_n)\\ \Delta_{mn}(\xi_m,\omega_n)\\ \mathbf{0}_{3\times1}\end{Bmatrix}, \tag{11}$$

where $\mathbf{U}_{1mn},\mathbf{U}_{2mn}\in\mathbb{C}^{2\times4}$, and $\mathbf{U}_{3mn},\mathbf{U}_{4mn}\in\mathbb{C}^{4\times4}$.

Utilizing equation (2), the bending moment and effective shear force at the left and right edges (x=0 and x=a) of the plate element can be expressed as

$$\mathbf{F}_{mn}(\xi_m,\omega_n)\mathbf{c}_{mn}=\mathbf{f}_{mn}(\xi_m,\omega_n) \tag{12}$$

and

$$\mathbf{F}_{mn}(\xi_m,\omega_n)=\begin{bmatrix}\mathbf{F}_{1mn}(\xi_m,\omega_n) & \mathbf{0}_{2\times4}\\ \mathbf{0}_{2\times4} & \mathbf{F}_{2mn}(\xi_m,\omega_n)\end{bmatrix},\ \mathbf{f}_{mn}(\xi_m,\omega_n)=\begin{Bmatrix}M_{1mn}^{(1)}(\xi_m,\omega_n)\\ V_{1xmn}^{(1)}(\xi_m,\omega_n)\\ M_{2mn}^{(2)}(\xi_m,\omega_n)\\ V_{2xmn}^{(2)}(\xi_m,\omega_n)\end{Bmatrix}=\begin{Bmatrix}M_{mn}^{(1)}(0;\xi_m,\omega_n)\\ V_{xmn}^{(1)}(0;\xi_m,\omega_n)\\ M_{mn}^{(2)}(a;\xi_m,\omega_n)\\ V_{xmn}^{(2)}(a;\xi_m,\omega_n)\end{Bmatrix} \tag{13}$$

where $\mathbf{F}_{1mn},\mathbf{F}_{2mn}\in\mathbb{C}^{2\times4}$.

Using equations (10), (11), and (12), one can relate the nodal displacement $\mathbf{d}_{mn}$ to the nodal force $\mathbf{f}_{mn}$ through the following,

$$\mathbf{K}_{mn}(\xi_m,\omega_n)\mathbf{d}_{mn}(\xi_m,\omega_n)=\mathbf{f}_{mn}(\xi_m,\omega_n)-\mathbf{g}_{mn}(\xi_m,\omega_n)\Delta_{mn}(\xi_m,\omega_n) \tag{14}$$

where $\mathbf{K}_{mn}\in\mathbb{C}^{4\times4}$, being the first four columns of the matrix $\mathbf{F}_{mn}\mathbf{U}_{mn}^{-1}$, is the frequency-wavenumber domain element stiffness matrix for the cracked Levy plate. $\mathbf{g}_{mn}\in\mathbb{C}^{4\times1}$ denotes the 5th column of the matrix $\mathbf{F}_{mn}\mathbf{U}_{mn}^{-1}$, and $\mathbf{g}_{mn}\Delta_{mn}$ can be considered as the body force induced by the present of the crack.

3. NUMERICAL CASE STUDIES

Figure 1 shows the employed Levy plate with transverse open and non-propagation surface crack lying parallel to the y axis. The plate is simply supported at edges y=0 and y=b, and has arbitrary boundary conditions at the other two edges. The dimensions and material properties of the plate are: length in x direction a=1.5 m, length in y direction b=4 m, thickness H=0.003 m, Young's modules E=7.2 × 10^{10} Pa, mass density ρ=2750 kg/m^3, and Poisson ratio ν=0.33.

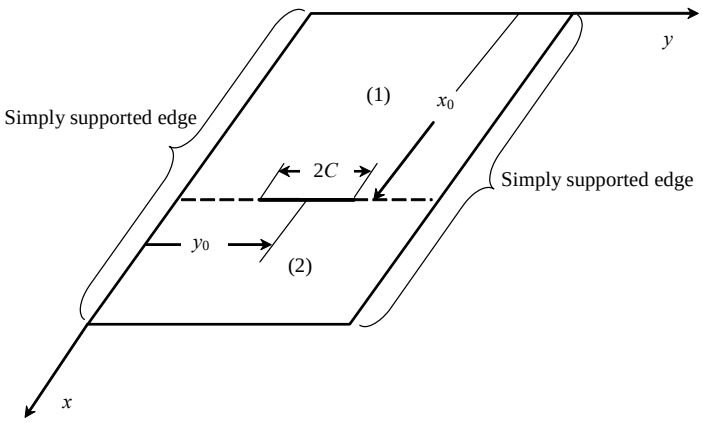
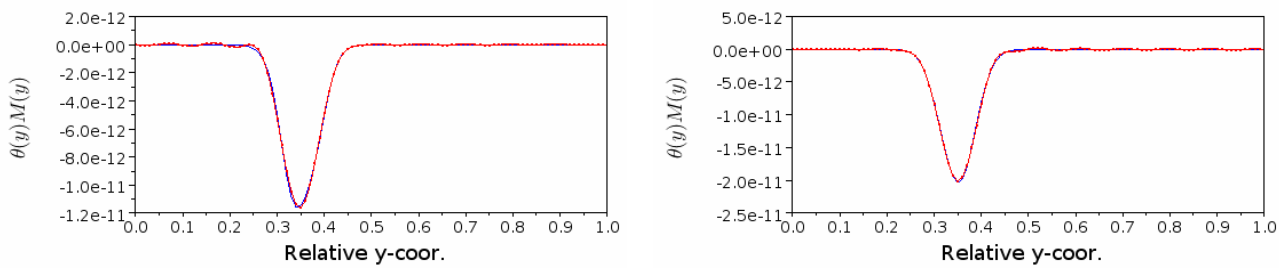


Figure 1: Levy plate with a part-through finite-length crack.

The crack parameters are given as: crack half-length C=b/10, crack central depth h_0=0.05H, and the coordinate of crack center (x_0, y_0)=(0.5a, 0.35b). The plate in Figure 1 is modeled using three SFEs, i.e., one SFE and two throw-off elements to remove the effect from the plate boundary. The plate is excited by an distributed impact at the left edge. The plate is excited by an transverse distribution force with uniform amplitude applied to the left edge, consisting of a 100 kHz narrow-band ten-cycle sinusoidal tone burst modulated by a Hanning window. The guided wave response is measured along the line at x=0.3a, and the number of Fourier terms throughout this paper is chosen to be 21.

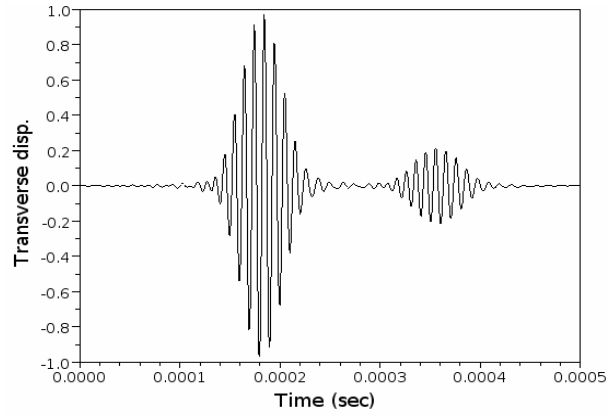


(a) (b)

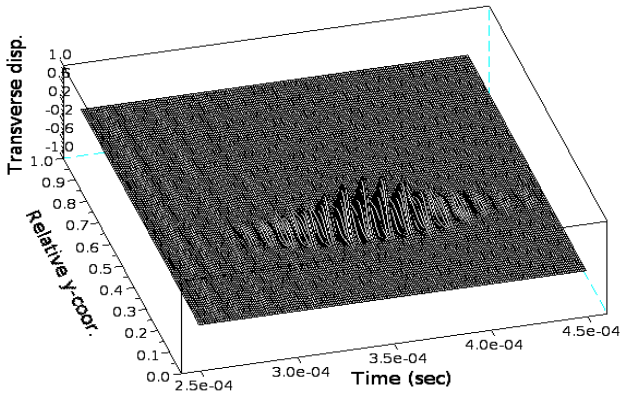
Figure 2: Variance of $\theta(y)M_n(x_0, y; \omega_n)$ along the crack line at ω_n=3135.32 Hz (exact value: blue solid line; predicted value: red dotted line): (a) real part, and (b) imagery part.

In order the verify the proposed SFE for modeling the crack in the Levy plate, the frequency-domain representation of the slope discontinuity Δ_n obtained by using equations (8) and (9), are compared to that being directly calculated from the definition, i.e., the product of $\theta(y)$

with M_n. Herein, by reasonably assuming that M_n is not greatly perturbed by the crack, we use moment of the intact plate (without crack) instead of that in cracked statues, which is unknown beforehand. It is clearly found from Figure 2 that the predicted Δ_n by the proposed method is very close to the exact values by definition, indicating that the proposed modeling method is feasible.



(a)



(b)

Figure 3: Scattering due to transverse crack: (a) response measured from the point located at (0.3*a*, 0.3*b*), and (b) response measured from the line *x*=0.3*a*.

Figure 3 shows the time histories of the transverse displacement at the measurement point located at (0.3*a*, 0.3*b*) during the time interval (0, 5×10^{-4} s) as well as those along the measurement line within the time interval (2.5×10^{-4} s, 4.5×10^{-4} s). It is clear from Figure 3(a) that the first peak arrived at around 1.8×10^{-4} s is the input pulse received at the measurement point, and there is an extra reflection from the crack front which arrives at around 3.6×10^{-4} s due to the present of the crack. In addition, in order to show the scatting of the finite-length crack, the measured response along the line x=0.3a during the time interval (2.5×10^{-4} s, 4.5×10^{-4} s) is shown in Figure 3(b). It is seen that the finite-length crack affects only finite-length area around the crack region.

4. CONCLUSIONS

In this paper, the SFE method is used to model the Levy plate with a transverse open and non-propagation surface crack for investigating the wave propagation phenomenon, where the crack having an arbitrary length, depth and location was parallel to one side of the plate. The SFE for the cracked Levy plate is firstly derived, and the numerical simulation of the wave propagation analysis with the developed cracked spectral plate element is perform. Different from the majority of relevant works in the literature, the plate in this paper is finite in both direction, and the effect of crack length and crack position can be well investigated. The obtained results from numerical case studies clearly show that the proposed SFE model for modeling cracked Levy plate is feasible and effective.

5. ACKNOWLEDGMENTS

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