A PROBABILISTIC APPROACH FOR DAMAGE DETECTION UTILIZING INCOMPLETE MODAL DATA

T. YIN[1*†] and H. P. ZHU[2]

[1] *School of Civil and Architectural Engineering, Wuhan University, Wuhan 430072, P.R. China*
[2] *School of Civil Engineering and Mechanics, Huazhong University of Science and Technology, Wuhan 430074, P.R. China*

ABSTRACT

Over the last few decades, there has been great interest in the development of a structural health monitoring (SHM) methodology based on vibration data. Following the Bayesian probabilistic approach and finite element (FE) model reduction technique, this paper proposes a probabilistic approach for structural damage detection using noisy incomplete modal parameters corresponding to natural frequencies and partial mode shapes with only limited number of sensors. Based on the measured incomplete modal data, the most probable structural model parameters together with the most probable values of the system natural frequencies and partial modes shapes with associated uncertainties can be identified by the proposed methodology. In addition, in order to construct the prior probability density function (PDF) of the incomplete system modal parameters, the approach of FE model reduction is employed to evaluate the degree of agreement that how the system modal parameters satisfy the reduced structural model. One of the significant features of the proposed method lies in that the requirement of full system mode shapes is completely avoided, and matching measured modes with corresponding calculated modes from the structural FE model is also not required, which is in contrast to many existing methods in the literature. Furthermore, an efficient iterative solution method is also presented to solve the optimization problem arisen from finding the most probable values of the structural model parameters as well as the incomplete system modal parameters. Numerical simulations conduced for a typical one-storey portal frame is used to demonstrate the proposed methodology.

Keywords: Structural health monitoring (SHM), Bayesian probabilistic approach, Probability density function (PDF), Finite element model reduction.

1. INTRODUCTION

In the majority of existing damage detection methods based on modal parameters in the literature (Carden and Fanning 2004), by assuming that the influence of damage on structural mass is

[*] Corresponding author: Email: tyin@whu.edu.cn

[†] Presenter: Email: tyin@whu.edu.cn

neglected, an objective function is usually defined in terms of the discrepancies between the experimental modal parameters and those calculated from a FE model, and it is then minimized to give the estimation of the change of stiffness parameters. However, before the formation of the above objective function, it's necessarily ensured that the measured modes are matched with the calculated ones by using the modal assurance criterion (MAC), which is not easy in real application since usually very limited number of sensors are available and only partial mode shapes could be obtained. In addition, because damage might caused changes in the order of the modes, the mode matching becomes even more difficult.

Some methods have been proposed for solving the mode matching problem in SHM by introducing the concept of system mode shapes, which is distinct from those calculated from the structural model specified by some given structural model parameters. Most of these methods employ Rayleigh quotient frequencies, which are derived from the structural model and the system mode shapes, to avoid solving the eigensystem equation directly during damage detection process, where the structural model parameters and the system mode shapes are identified simultaneously (Ching and Beck 2004). Recently, an improved method has been proposed in reference (Yuen et al. 2006) that the system natural frequencies is also included to be identified based on incomplete modal data besides of the system mode shapes.

For handling the main difficulties arisen by the limited number of sensors in real situations, mode shape expansion procedures, which expand the experimental mode shapes to fit those from the FE model, are usually adopted. The above-mentioned methods follow this thought. However, it has been revealed that the expansion process would aggregate the effects of modeling error, experimental noise, and other uncertainties to the resultant mode shapes (Zhu et al. 2003), which significantly affects the results of damage detection. In such situation, the FE model reduction method, particularly for the dynamic-reduction method (Kidder 1973), is a practical choice, since it does not introduce any error in the transformation process within a certain frequency range (Yin et al. 2009; Lam and Yin 2011). Therefore, the main objective of this paper is to extend the FE model reduction based method (Yin et al. 2009; Lam and Yin 2011) to structural damage detection using noisy incomplete modal parameters by following the Bayesian probabilistic approach and utilizing the concept of the system modal parameters (Yuen et al. 2006). One of the significant features of the proposed method lies in that the identification of full system mode shapes is completely avoided, and matching measured modes with corresponding calculated modes is never required. Furthermore, an efficient iterative solution method is also presented to solve the optimization problem arisen from finding the most probable values of the structural model parameters as well as the incomplete system modal parameters. The proposed method is demonstrated by a typical one-storey portal frame structure through numerical simulations.

2. THEORETICAL DEVELOPEMENTS

For a class of structural models $\mathcal{M}$ discretized by FE method into N DOFs, by assuming that the mass matrix is not altered by damage, the eigen-system equation corresponding of the j-th mode to the measured (or master) set of DOF (index m) and the unmeasured (or slave) set of DOF (index s) of the FE model is given by (Yin et al. 2009; Lam and Yin 2011)

$$\left[\mathbf{K}_{\mathbf{P}}-\sum_{i=1}^{N_\theta}\theta_i\mathbf{K}_{\mathbf{P}}^i-\lambda_j\mathbf{M}_{\mathbf{P}}\right]\begin{bmatrix}\boldsymbol{\varphi}_j^m\\ \boldsymbol{\varphi}_j^s\end{bmatrix}=\mathbf{0},\ \mathbf{K}_{\mathbf{P}}=\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{sm} & \mathbf{K}_{ss}\end{bmatrix},\ \mathbf{M}_{\mathbf{P}}=\begin{bmatrix}\mathbf{M}_{mm} & \mathbf{M}_{ms}\\ \mathbf{M}_{sm} & \mathbf{M}_{ss}\end{bmatrix},\ \mathbf{K}_{\mathbf{P}}^i=\begin{bmatrix}\mathbf{K}_{mm}^i & \mathbf{K}_{ms}^i\\ \mathbf{K}_{sm}^i & \mathbf{K}_{ss}^i\end{bmatrix} \quad (1)$$

where $\lambda_j=(2\pi f_j)^2$, $\boldsymbol{\varphi}_j\in\mathbb{R}^N$, $j=1,2,\ldots,N_t$, are the j-th eigenvalue and eigenvector, respectively. N_t is the number of measured modes, and f_j is the j-th natural frequency. $\boldsymbol{\varphi}_j^m$ and $\boldsymbol{\varphi}_j^s$ are the measured and unmeasured parts of full mode shape $\boldsymbol{\varphi}_j$, with dimensions N_m and N_s, respectively, and $N_m+N_s=N$. The scaling parameters $\boldsymbol{\theta}=[\theta_1,\theta_2,\ldots,\theta_{N_\theta}]^{\mathrm{T}}\in\mathbb{R}^{N_\theta}$ allow the nominal stiffness matrix given by $\boldsymbol{\theta}=\boldsymbol{\theta}_0$ to be updated based on measured data from the real structure. N_θ is the number of unknown parameters to be identified. $\mathbf{K}^i$, $i=1,2,\ldots,N_\theta$ is the contribution of the i-th member or substructure to the global stiffness matrix.

From the second set of equation (1), the full mode shape of the j-th mode can be represented only by the corresponding measured mode shape as

$$\begin{bmatrix}\boldsymbol{\varphi}_j^m\\ \boldsymbol{\varphi}_j^s\end{bmatrix}=\mathbf{T}_j\boldsymbol{\varphi}_j^m,\quad \mathbf{T}_j=\begin{bmatrix}\mathbf{I}_{N_m}\\ \mathbf{D}_j\end{bmatrix},\ \mathbf{D}_j=-(\mathbf{K}_{ss}-\sum_{i=1}^{N_\theta}\theta_i\mathbf{K}_{ss}^i-\lambda_j\mathbf{M}_{ss})^{-1}(\mathbf{K}_{sm}-\sum_{i=1}^{N_\theta}\theta_i\mathbf{K}_{sm}^i-\lambda_j\mathbf{M}_{sm}) \quad (2)$$

where $\mathbf{T}_j$, being the function of both λ_j and $\boldsymbol{\theta}$, is the transformation matrix corresponding to the j-th mode, and $\mathbf{I}_{N_m}$ is the identity matrix with dimension $N_m\times N_m$.

Substituting equation (2) into equation (1), one can get the eigen-system equation of the reduced model with the N_m measured DOFs corresponding to the j-th mode as

$$\bar{\mathbf{K}}_m\boldsymbol{\varphi}_j^m=\lambda_j\mathbf{M}_m\boldsymbol{\varphi}_j^m,\quad \bar{\mathbf{K}}_m=(\mathbf{K}_m-\sum_{i=1}^{N_\theta}\theta_i\mathbf{K}_m^i) \quad (3)$$

where $\mathbf{K}_m$ and $\mathbf{M}_m$ are the reduced global stiffness and mass matrices of the j-th mode, respectively,

$$\mathbf{K}_m=\mathbf{T}_j^{\mathrm{T}}\mathbf{K}_{\mathbf{P}}\mathbf{T}_j,\quad \mathbf{M}_m=\mathbf{T}_j^{\mathrm{T}}\mathbf{M}_{\mathbf{P}}\mathbf{T}_j,\quad \mathbf{K}_m^i=\mathbf{T}_j^{\mathrm{T}}\mathbf{K}_{\mathbf{P}}^i\mathbf{T}_j \quad (4)$$

It's assumed herein that there are $N_t\,(N_t\ll N)$ modes of the system measured, and these modes are referred to the system modes (i.e., system eigenvalue vector $\boldsymbol{\lambda}=[\lambda_1,\lambda_2,\ldots,\lambda_{N_t}]^{\mathrm{T}}$ and mode shape vector $\boldsymbol{\varphi}^m=[(\boldsymbol{\varphi}_1^m)^{\mathrm{T}},(\boldsymbol{\varphi}_2^m)^{\mathrm{T}},\ldots,(\boldsymbol{\varphi}_{N_t}^m)^{\mathrm{T}}]^{\mathrm{T}}$) to distinguish them from the corresponding modal parameters calculated from any reduced structural model specified by $\boldsymbol{\theta}$ (see equation (3)). The measured modal parameters is represented by $[\tilde{\boldsymbol{\lambda}}^{\mathrm{T}},(\tilde{\boldsymbol{\varphi}}^m)^{\mathrm{T}}]^{\mathrm{T}}=[\boldsymbol{\lambda}^{\mathrm{T}},(\boldsymbol{\varphi}^m)^{\mathrm{T}}]^{\mathrm{T}}+\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}\in\mathbb{R}^{N_t\times(N_m+1)}$ is assumed to come from a normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$. It should be emphasized that, one of the significant merits of the proposed method is that the system partial mode shapes with only N_m measured DOFs are just the full mode shapes with respect to any given reduced structural model given in equation (3). Thus, there is no need to identify the full system mode shapes corresponding to all N DOFs of the original structural model from only N_m measured DOFs. Also, it is not required to adopt the selection matrix composed of 1's or 0's that picks the components of full mode shapes with N DOFs corresponding to the N_m observed DOFs any more.

With the measured modal parameters available, the posterior PDF for all the unknown parameters is given by employing the Bayes' theorem as (Yuen et al. 2006)

$$p\left(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta} \mid \tilde{\boldsymbol{\lambda}},\tilde{\boldsymbol{\varphi}}^m,\mathcal{M}\right)=\kappa_1 p\left(\tilde{\boldsymbol{\lambda}},\tilde{\boldsymbol{\varphi}}^m \mid \boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta},\mathcal{M}\right) p\left(\boldsymbol{\lambda},\boldsymbol{\varphi}^m \mid \boldsymbol{\theta},\mathcal{M}\right) p\left(\boldsymbol{\theta} \mid \mathcal{M}\right) \tag{5}$$

where, κ_1 is a normalizing factor. $p(\tilde{\boldsymbol{\lambda}},\tilde{\boldsymbol{\varphi}}^m \mid \boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta},\mathcal{M})$, named the likelihood function, is well represented by $\mathcal{N}([\boldsymbol{\lambda}^{\mathrm{T}},(\boldsymbol{\varphi}^m)^{\mathrm{T}}]^{\mathrm{T}};\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}})$. It represents a normal distribution with mean $[\boldsymbol{\lambda}^{\mathrm{T}},(\boldsymbol{\varphi}^m)^{\mathrm{T}}]^{\mathrm{T}}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$. The prior PDF $p(\boldsymbol{\theta} \mid \mathcal{M})$ is assumed to be $\mathcal{N}(\boldsymbol{\theta}_0;\boldsymbol{\Sigma}_{\boldsymbol{\theta}})$. Also, by assuming the normal distribution with independent and identically distributed random variables for the eigenvalue-equation errors, and the prior PDF for the system modal parameters is represented as

$$p\left(\boldsymbol{\lambda},\boldsymbol{\varphi}^m \mid \boldsymbol{\theta},\mathcal{M}\right)=\kappa_2 \exp\left(-\frac{1}{2}\sigma_{\boldsymbol{\psi}}^{-2}\sum_{j=1}^{N_t}\left\|\boldsymbol{\psi}_j\right\|^2\right),\quad \boldsymbol{\psi}_j=(\bar{\mathbf{K}}_m-\lambda_j\mathbf{M}_m)\boldsymbol{\varphi}_j^m\in\mathbb{R}^{N_{mt}},\ \ j=1,2,\ldots,N_t \tag{6}$$

where $\|\bullet\|$ denotes the Euclidean norm, and the prescribed variance $\sigma_{\boldsymbol{\psi}}^2$ controls the size of equation errors with respect to the reduced structural model.

Therefore, the posterior PDF in equation (5) can be further assumed to take the following form as

$$p\left(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta} \mid \tilde{\boldsymbol{\lambda}},\tilde{\boldsymbol{\varphi}}^m,\mathcal{M}\right)=\kappa_3 \exp\left[-\frac{1}{2}J\left(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta}\right)\right]\simeq\mathcal{N}\left(\left[\hat{\boldsymbol{\lambda}}^{\mathrm{T}},\left(\hat{\boldsymbol{\varphi}}^m\right)^{\mathrm{T}},\hat{\boldsymbol{\theta}}^{\mathrm{T}}\right]^{\mathrm{T}};\boldsymbol{\Sigma}\right) \tag{7}$$

where κ_3 is a normalizing constant, and $J(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta})$ is a measure-of-fit function as defined by

$$J\left(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta}\right)\propto\sigma_{\boldsymbol{\psi}}^{-2}\sum_{j=1}^{N_t}\left\|\boldsymbol{\psi}_j\right\|^2+\begin{bmatrix}\tilde{\boldsymbol{\lambda}}-\boldsymbol{\lambda}\\ \tilde{\boldsymbol{\varphi}}^m-\boldsymbol{\varphi}^m\end{bmatrix}^{\mathrm{T}}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}^{-1}\begin{bmatrix}\tilde{\boldsymbol{\lambda}}-\boldsymbol{\lambda}\\ \tilde{\boldsymbol{\varphi}}^m-\boldsymbol{\varphi}^m\end{bmatrix}+\left(\boldsymbol{\theta}-\boldsymbol{\theta}_0\right)^{\mathrm{T}}\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1}\left(\boldsymbol{\theta}-\boldsymbol{\theta}_0\right) \tag{8}$$

The covariance matrix $\boldsymbol{\Sigma}$ in equation (7) can be calculated by the inverse of the Hessian of the function $-\ln p(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta} \mid \tilde{\boldsymbol{\lambda}},\tilde{\boldsymbol{\varphi}}^m,\mathcal{M})$ evaluated at the optimal parameters. The most probable values of all the unknown parameters can be estimated by maximizing the posterior PDF in equation (7), which is equivalent to minimize the measure-of-fit function $J(\boldsymbol{\lambda},\boldsymbol{\varphi}^m,\boldsymbol{\theta})$ in equation (8). However, the objective function J is highly nonlinear and very challenging by directly solving it in high-dimensional parameter space through the conventional optimization routines.

One of the main purpose of this paper is to develop an effective iterative solution strategy for solving the above minimization problem, motivated by reference (Yuen et al. 2006). In the proposed solution method, the first-order partial derivatives of J with respect to the stiffness scaling parameters $\boldsymbol{\theta}$ and system eigenvalues $\boldsymbol{\lambda}$ and mode shapes $\boldsymbol{\varphi}^m$ are firstly calculated. Then, by setting these first-order partial derivatives equal to zero leads to two implicit nonlinear optimization problems with respect to $\boldsymbol{\theta}$ and $\boldsymbol{\lambda}$, which can be solved efficiently by a simple iterative method developed previously by the authors (Yin et al. 2009; Lam and Yin 2011). While, the system mode shapes can be explicitly calculated. After that, the proposed solution method sequentially updates $\boldsymbol{\theta}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\varphi}^m$ in an iterative manner by using successively the previous optimization results until certain prescribed convergence criterion is met. Finally, the uncertainties of each parameters can be quantified from covariance matrix $\boldsymbol{\Sigma}$ in equation (7).

3. NUMERICAL CASE STUDIES

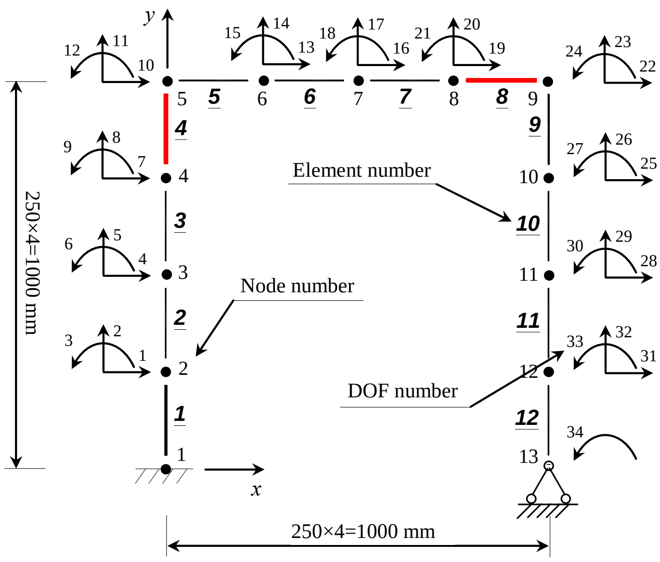
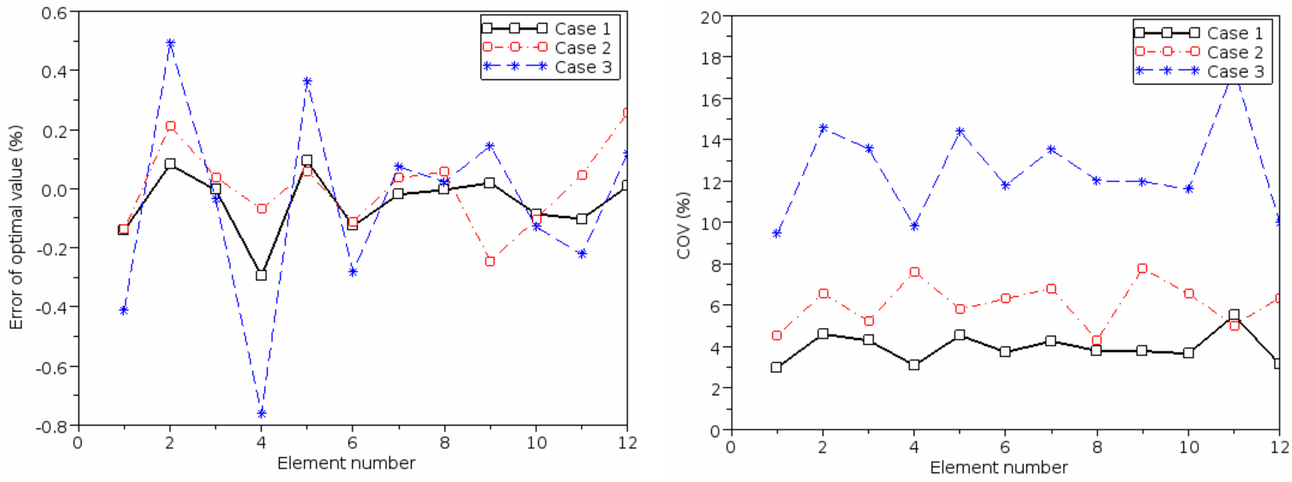


Figure 1: FE model of one-storey portal frame.

A typical one-storey portal frame is employed in the numerical verification. The sectional and material properties of the frame are: Young's modulus $E = 2.0 \times 10^{10}$ N/m^2, cross-sectional area $A = 3.03 \times 10^{-4}$ m^2 (for the two columns) and 2.43×10^{-4} m^2 (for the beam), moment of inertia $I = 9.09 \times 10^{-10}$ m^4 (for the two columns) and 7.29×10^{-10} m^4 (for the beam), and material density $\rho = 7.67 \times 10^3$ kg/m^3. The 2D FE model of the one-storey frame is shown in Figure 1. Note that the rotation DOF of the right column-base point is released to mitigate the effects of symmetry.

The measured DOF numbers are 1, 4, 7, 11, 14, 20, 23, 25, 28, and 31, referring to Figure 1, and the first six modes (including both the natural frequencies and partial mode shapes) are included in the damage detection process. The measurement noise is simulated by adding a sample of zero-mean Gaussian white noise with 1% standard deviation to the calculated squared natural frequencies and mode shapes from the FE model, respectively. Damage is simulated on elements 4 and 8 of the structure by a 10% and 20% reduction in stiffness, respectively. It is noted that the changes in the modal parameters due to the damage are very small in such situation, and the maximum reduction in natural frequencies among the first six modes does not exceed 2% through the computer simulation. To investigate and compare the influence of some important factors on the damage detection results and associated uncertainties, three cases are considered.



(a) (b)

Figure 2: Comparisons of the results obtained by the proposed method for all cases: (a) percentage error between the identified optimal values, and (b) COV values.

Case 1 is a nominal case with above-mentioned default parameters to serve as a comparison with the other cases. Case 2 is different from Case 1 that only the first four modes are used for the damage detection so as to investigate the proposed method with relatively less modal information. By noting that the mode shapes identified from the vibration response in practice are usually less accurate than that of the natural frequencies, Case 3 is designed to investigate the feasibility of the proposed method in the presence of relatively high levels of noise by increasing the standard deviation of the Gaussian noise from 1% to 3% for the partial mode shapes. For convenience, the coefficient of variation (COV) (%) is adopted to investigate the uncertainties associated with the damage detection results. The accuracy of the identified optimal values is evaluated by examining the percentage discrepancy between the identified and actual values.

The comparison of the obtained results for all cases by the proposed method are shown in Figure 2. Specifically, Figure 2(a) compares the accuracy of the identified optimal values of stiffness scaling parameters in all cases, and Figure 2(b) shows the uncertainties associated with these identified optimal values. It is clearly seen from Figure 2(a) that the influence of the measurement noise on the accuracy of the identified optimal values investigated (see Case 3) is more significant, while the influence of another factor, i.e., fewer modes, is not very obvious as shown in Case 2.

By comparing the magnitudes of the COV values for all three cases as shown in Figure 2(b), the above phenomenon about the greater effect of measurement noise is reinforced. In this paper, the convergence of the proposed solution algorithm for the optimal values of stiffness scaling parameters is also studied by taking Case 1 as an example (noting that the pre-defined convergence criterion is set to be 1.0×10^{-6} in this paper). Although not shown in this paper, results indicate that the iteration history of the optimal stiffness scaling parameters that the convergence rate is very fast, and it takes only 5 iteration steps for Case 1 to achieve the final most probable solution. Similar phenomena are observed from the convergence studies of the other two cases.

4. CONCLUSIONS

In this paper, a probabilistic methodology based on FE model reduction technique for structural damage detection using noisy incomplete modal parameters with only limited number of sensors is developed. One of the significant features of the proposed methodology lies in that the requirement of full system mode shapes and mode matching are never required. Numerical simulation results for a typical one-storey portal frame with a set of cases show that the proposed method can give the most probable structural model parameters as well as the most probable values of the system natural frequencies and partial modes shapes with associated uncertainties. Note that the importance of the numerical case studies is to show that the proposed method provides a convenient and reliable means of estimating uncertainties associated with the damage detection results, but not to identify the most significant factor affecting the uncertainties which varies from case to case. Moreover, the convergence rate of the proposed method is very fast for the present numerical example.

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