A CASE STUDY OF WEIGHING FOR MULTIPLE TERMS OF OBJECTIVE FUNCTION FOR THE APPROACH OF MULTIPLE OBJECTIVE OPTIMIZATION

WONGWANISHWATANA W.[1]*, BOONYACHUT S.[2]

[1]*Doctoral Candidate, Department of Civil Engineering,*

Mahanakorn University of Technology, Thailand

[2]*Asst. Prof., Department of Civil Engineering,*

Mahanakorn University of Technology, Thailand

ABSTRACT

This paper presents an example illustrating the Weighing of the terms for the Objective Function for Multiple Objective Optimization. The problem will be analysed by using the approach of Goal Programming. As there are many approaches available, two approaches are selected to demonstrate solving the problem of choosing a proper method (i.e. for bridge deck repairing and rehabilitating) by including multiple objectives into the consideration. There are Multiple Objective Optimization and Analytic Hierarchy Process (AHP), and will be demonstrated in the example. The idea shown in this paper might lead to the better decision making made by the executive officers to tackle the multiple objectives problems with the best benefit gained from the available options, or in the other word, with lease of the opportunity lost. The concept of Multiple Objective Optimization can be applied for many more practical field works if the decision makers really understand the meaning and the usage.

Keywords; goal programming, multiple objective optimization, analytic hierarchy process (AHP)

1. INTRODUCTION

Most of the decision makings made by the executive officers are based on the more experiences and senses of the expert with life-long learning judgment with respect to the main objective of the problems. The main objective for each problem might be different from each other. The pattern of judgment might not be the same as previously used. To obtain such decisions, the experts or decision makers have to structurally carry out the calculation process. It can be used as a tool to verify, describe or even educate the other co-workers. Many problems contain of many objectives along with the numbers of available alternatives in the consideration. The executive officers have to do the whole judgment without bias and without the absent of all relevant elements.

* Corresponding author and Presenter: Email: wikhanes@gmail.com

2. APPROACHES FOR THE CALCULATION

2.1 MUTIPLE OBJECTIVE OPTIMIZATION

Multiple Objective Optimization, also known as Multi-objective optimization, is an approach to solve the multiple criteria decision making. It is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. Multi-objective optimization has been applied in many fields of science, including engineering, economics and logistics where optimal decisions are taken as the sense of trade-offs between two or more important or conflicting objectives. In practical problems, the multi-objective optimization may have even more than three objectives in the calculation. The well-structured pattern of calculation and input data is needed to perform the repeatable process which may lead to the best solution in shorter period.

The study of multi-objective optimization problems may be performed in different viewpoints with different philosophies and goals to obtain the solutions. The goal or objectives may be finding and quantifying the trade-offs with the most satisfied objectives, or finding a solution that satisfies the preferences set by the decision makers.

2.2 ANALYTIC HIERACHY PROCESS

Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decisions based on mathematics and psychology. The process was developed by Thomas L. Saaty in the 1970s and it has been extensively studied and refined since then.

AHP is widely used for group decision making and decision of the situations which might have complexities in multiple objectives and many alternatives or options available. Instead of making a decision or choosing a "correct" choice based on single objective, or with the experience-based judgment to weigh for each term for multiple objectives, the AHP helps decision makers to find one that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a problem. The problem has to be well defined and split into elements. They are to be quantified or rated by comparison of their significances among them.

The decision problem has to be arranged as hierarchy which is easily comprehended the problems and able to be analyzed independently. The hierarchy pattern can relate all objectives and alternatives of the decision problem. They have to be carefully measured, quantified, or rated for the importance.

Once the hierarchy is built, the decision makers systematically evaluate its various elements by comparing one to the others with respect to the objective at a time. The decision makers may compare by using benefit of the elements, or typically use their judgments to define the relative importance. The process of evaluating for all elements in AHP needs Experts judgments, along with some information, to perform the well relative rating for all elements.

The AHP converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element in the hierarchy, allowing all elements to be compared to one another in a rational and consistent way. This capability distinguishes the AHP from other decision making techniques.

Then the numerical of weighed scores are calculated for each of the decision alternatives. These numbers represent the relative ability for alternatives to achieve the decision goal or objective function. Hence, the decision makers can pick the best prioritized alternative obtained by the highest weighed score from the calculation.

3. ILLUSTRATING EXAMPLE

There are difficulties of choosing the best choice out of the numbers of available alternatives in multiple objectives problems. The Experts judgment for each problem is needed to relatively rate the significance of all elements in the consideration. The problem of selecting the method for repairing and rehabilitating for a Bridge Deck is selected to illustrate the AHP approach.

There are 3 main objectives in the consideration; Applicability, Recovery, and Cost Effectiveness, are set as the criteria for the decision making. The available methods for bridge repairing and rehabilitating are selected for 4 methods; assumed as method A, B, C and D. The problem is relatively structured as the figure shown below;

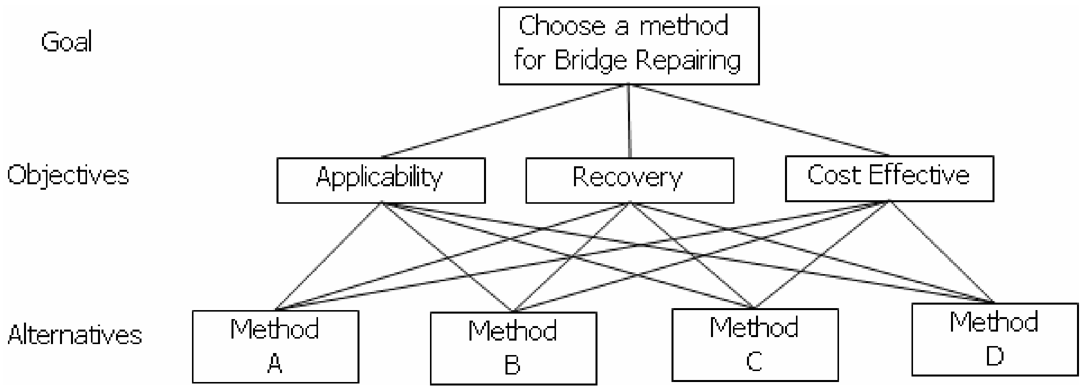


Figure 1: The relation of all elements in illustrated problem

The ***Pairwise Comparison*** is carried out to compare one-on-one with the other elements in the form of matrix. The pairwise comparison matrix of Objectives which is to relatively rate the significance of each objective compare to the others, shown in table 1;

Table 1: Pairwise Comparison among 3 Objectives

Objective	Applicability	Recovery	Cost
Applicability	1	1/2	1/2
Recovery	2	1	1/2
Cost	2	2	1
Total	5	3.5	2

The scale of rating for this example is set ranging from 1 to 3 (preferable, 5 to 10 in practice).

For table 1; in column of Applicability, the applicability is set as 1, then, let the expert relatively judge and compare with Recovery, which is relatively rated for significant as 2, and then with Cost Effectiveness as 2.

For the column of Recovery, the recovery is set as 1, then, relatively rated for Applicability as 1/2, and for Cost Effectiveness as 2.

For the last column, the Cost Effectiveness is set as 1, then, relatively rated for both Applicability and Recovery as 1/2.

The matrix is then now arranged in the triangle form with

There are many available repairing and rehabilitating methods for the bridge deck in practice. The Experts may pre-select the group of top-ranked method for the shorter calculation, assumed as method A, B, C and D.

Likewise, the matrix of pairwise comparison of each repairing method compared with the other method, are obtained, with respect to the sense of Applicability as shown in table 2, with respect to the sense of Recovery as shown in table 3, and with respect to the sense of its Cost Effectiveness as shown in table 4.

Table 2: Pairwise Comparison among 4 methods with respect to Applicability

Repair Method	A	B	C	D
A	1	2	1/2	3
B	1/2	1	1/3	2
C	2	3	1	3
D	1/3	1/2	1/3	1

Table 3: Pairwise Comparison among 4 methods with respect to Recovery

Repair Method	A	B	C	D
A	1	1/2	1	3
B	2	1	2	2
C	1	1/2	1	2
D	1/3	1/2	1/2	1

Table 4: Pairwise Comparison among 4 methods with respect to Cost Effectiveness

Repair Method	A	B	C	D
A	1	2	1	2
B	1/2	1	1/2	1/2
C	1	2	1	1/2
D	1/2	2	2	1

4. EQUATIONS AND NOTATIONS

The objective function is consisted of the 3 components, as for the applicability index, recovering effect, and cost Effectiveness. These three components are organized as the following equation.

$$\text{Goal} \quad \text{Objective} = [A]\,[W_A] + [R]\,[W_R] + [C]\,[W_C] \qquad \text{\_\_\_\_\_\_(1)}$$

where :

A = Applicability index of selected maintenance method

R = Recovering effect

C = Cost Effectiveness

W_A, W_R, W_C = Weighed of each objective obtained from the matrix

All of the elements are shown in matrix form, and to be normalized further. The matrix of scores for each element is taken into calculation. The illustrated calculation are done in spreadsheet and shown further. The objective is to find the highest score obtained by this equation

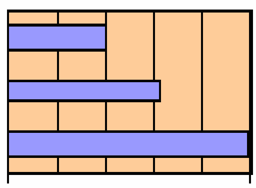
5. CALCULATION AND RESULT

All of the matrixes are normalized in column, by dividing each score by the total score of each column. This is to relatively rate the significant of each objective compared with the others.

The matrix in table 1 is normalized, and shown in table 5. Figures in first column, 0.2000 for applicability is obtained by having score 1 divided by total score in its column (1+2+2=5), figure for recovery with 0.4000, obtained by having 2 divided by 5, and with the same manner for cost effectiveness with 0.4000. The Weights of Objectives are obtained by averaging the scores in each row; i.e. for applicability, the figure 0.1976 is obtained by averaging of 0.2000, 0.1429 and 0.2500. They are all displayed along with the graph for weight, shown in table 5;

Table 5: Normalized Matrix of Objectives

Objectives	Normalized Matrix			Weight
Applicability	0.2000	0.1429	0.2500	0.1976
Recovery	0.4000	0.2857	0.2500	0.3119
Cost	0.4000	0.5714	0.5000	0.4905
Total	1.0000	1.0000	1.0000	1.0000



From table 5, it shows that the weight for cost effectiveness is 0.4905 as the highest score, 0.3119 for recovery and 0.1976 for applicability. These figures are used in goal or objective function as the weighing score for each term.

Likewise, the matrix of scores in table 2 for Applicability is normalized in the same manner and shown in table 6, the matrix in table 3 for Recovery is normalized and shown in table 7, and matrix in table 4 for Cost Effective is normalized and shown in table 8.

Table 6: Normalized Matrix of Methods with respect to Applicability

Choices	Normalized Matrix				Scores
A	0.2609	0.3077	0.2308	0.3333	0.2832
B	0.1304	0.1538	0.1538	0.2222	0.1651
C	0.5217	0.4615	0.4615	0.3333	0.4445
D	0.0870	0.0769	0.1538	0.1111	0.1072
Total	1.0000	1.0000	1.0000	1.0000	1.0000

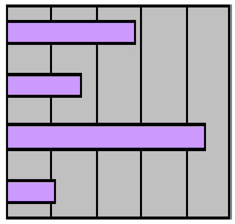


Table 7: Normalized Matrix of Methods with respect to Recovery

Choices	Normalized Matrix				Scores
A	0.2308	0.2000	0.2222	0.3750	0.2570
B	0.4615	0.4000	0.4444	0.2500	0.3890
C	0.2308	0.2000	0.2222	0.2500	0.2257
D	0.0769	0.2000	0.1111	0.1250	0.1283
Total	1.0000	1.0000	1.0000	1.0000	1.0000

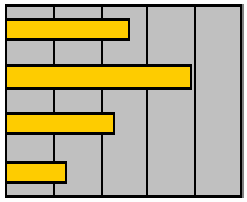
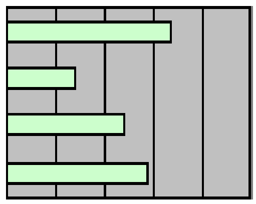


Table 8: Normalized Matrix of Methods with respect to Cost Effective

Choices	Normalized Matrix				Scores
A	0.3333	0.2857	0.2222	0.5000	0.3353
B	0.1667	0.1429	0.1111	0.1250	0.1364
C	0.3333	0.2857	0.2222	0.1250	0.2416
D	0.1667	0.2857	0.4444	0.2500	0.2867
Total	1.0000	1.0000	1.0000	1.0000	1.0000



The scores for each objective (table 6, 7 and 8) are to be calculated by equation (1) with the weight obtained from table 5 to get the final weighed scores for each repairing method, shown in Table 9.

Table 9: The summary of Scores for each objective and Weight Scores as result

Objective		Applicability	Recovery	Cost	**Weighed Score**
Weight		0.1976	0.3119	0.4905	
Method	A	0.2832	0.2570	0.3353	**0.3006**
	B	0.1651	0.3890	0.1364	**0.2209**
	C	0.4445	0.2257	0.2416	**0.2767**
	D	0.1072	0.1283	0.2867	**0.2018**

Table 9 is the summary of scores for each repairing method in term of the objectives, weight of each objective for the calculation, and the resulting weighed scores. It reveals that the repairing method A has the highest weighed score of 0.3006 which is to be chosen as the first priority of repairing method. The next alternatives are method C with score of 0.2767, method B with 0.2209, and method D as the last priority with least score of 0.2018.

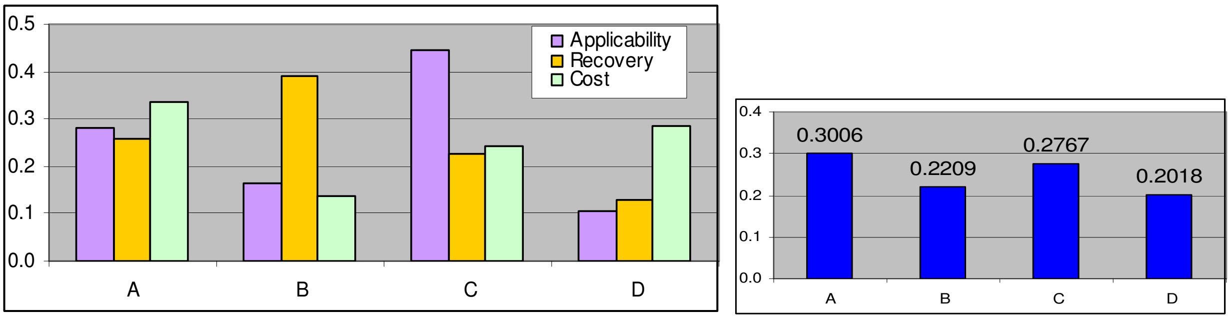


Figure 2: Individual Scores and Weighed Scores of repairing methods

For the figure 2, the left-hand graph shows all individual normalized score of all repairing methods with respected to three objectives. Method A is considerably good in all objectives, method B is good in recovery only, not applicability and cost effectiveness, method C is good in term of applicability, and method D has low scores in all objectives. The weights for objectives are now calculated with these scores by equation (1) resulting as Weighed Scores shown in right-hand graph. It shows that method A has the highest score, method C has the close-range score even though the individual normalized score for method C has the highest score for applicability, but with less weight for applicability, resulting with the less weighed score. The priority of selecting the repairing methods is A-C-B-D.

For overall comparison, the example is re-calculated with one change of the score given in table 1. This is to illustrate that the problem with different figures given by Experts somehow still gives the same result of selecting priority.

From table 1, with the score for applicability is set as 1, the score for cost effectiveness is changed from 2 to be 3 and then total score for applicability is now changed to be 6. This figure is still in the same sense showing that the cost effectiveness is more significant compared to the applicability and recovery, the score for column of cost effectiveness also changed automatically, while the other figures are remained the same, as shown in table 10.

Table 10: Pairwise Comparison among 3 Objectives with a score change

Objective	Applicability	Recovery	Cost
Applicability	1	1/2	1/3
Recovery	2	1	1/2
Cost	3	2	1
Total	6.00	3.50	1.83

The scores in table 10 lead to the change of figures in the normalized matrix for objectives shown in table 11. There are also changes for the figures shown in table 12 for the objective weights and resulting in different weighed scores for repairing methods. Anyway, the priority of selecting for a repairing method is still in the same order as A-C-B-D shown by Weighed Scores.

Table 11: Normalized Matrix of Objectives

Objectives	Normalized Matrix			Weight
Applicability	0.1667	0.1429	0.1818	0.1638
Recovery	0.3333	0.2857	0.2727	0.2973
Cost	0.5000	0.5714	0.5455	0.5390
Total	1.0000	1.0000	1.0000	1.0000

Table 12: The summary of Scores for each objective and Weight Scores as result

Objective		Applicability	Recovery	Cost	Weighed Score
Weight		0.1638	0.2973	0.5390	
Method	A	0.2832	0.2570	0.3353	0.3035
	B	0.1651	0.3890	0.1364	0.2162
	C	0.4445	0.2257	0.2416	0.2701
	D	0.1072	0.1283	0.2867	0.2102

It is noticed that the Weight and the Weighed Scores are changed and scores for method B and D are in a very close range. As if, there are many different numbers in the matrix of judgment rated by the experts, this may lead to the priority shifting.

6. SUMMARY OF CALCULATIONS

The example shown above illustrates the use of AHP approach applying for the multi-objective problem of choosing a repairing method for a bridge deck. The structural hierarchy in figure 1 is then now added with the result obtained from calculation, shown in figure 3 below. The problem is then now summarized in the hierarchy and lead to the decision making with ease. The figure 3 (a) is the result for the first example and figure 3 (b) is for the example with a score change. They both give the result with the same priority of repairing method selection as A-C-B-D.

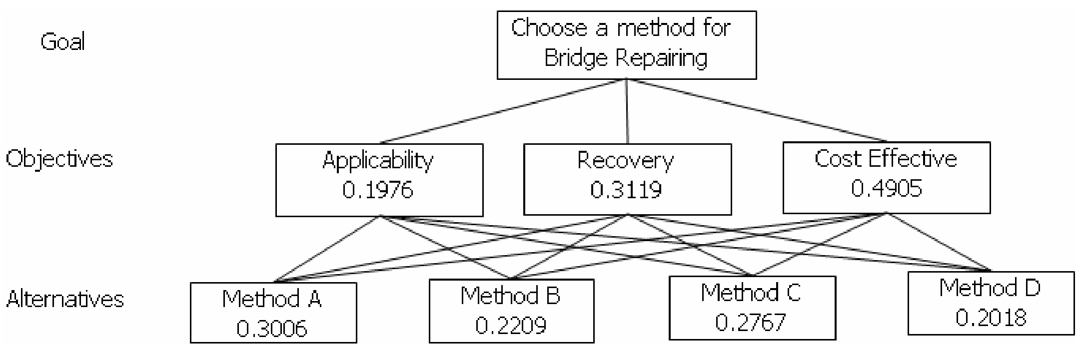


Figure 3 (a): Structural Hierarchy for problem with result (method A-C-B-D)

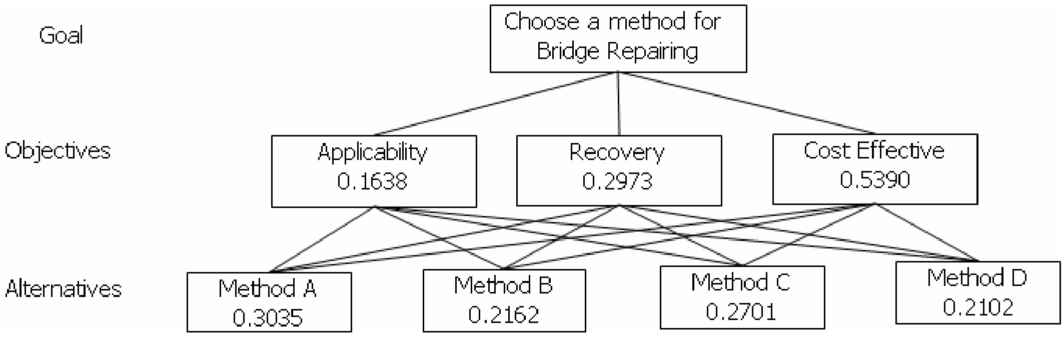


Figure 3 (b): Structural hierarchy for problem with result (method A-C-B-D) - change

7. CONCLUSIONS

The Expert Judgment is very helpful for guiding the overall concept in dealing with multiple objectives decision making. The guiding weights given for multiple objectives (terms) in the calculation, most of the time, are based on the sense of life-long experience. This has to be structurally defined to represent the meaning of their values. The AHP approach is applied in the example is to illustrate the calculation showing the process to achieve the weighing and results. It is to relatively rate among all the elements for each matrix. The experts or decision makers, who perform the evaluations, have to really focus on the judgment and relative rating for all elements, objectives and alternatives, the higher score means the more significant. This will lead to the better solution obtained from the calculation.

The scale of rating shown in this example is set from 1 to 3 which are very close to each other. The wider range of scale (preferable 5 to 10) might give the finer solutions.

The results obtained from calculations in many problems may have the close range of figures. This may be the case that it needs additional factors (objectives) put into the equation to give the better solutions. The Expert Judgment is now expressed and can be re-adjusted with figures instead of giving only Sense from experiences.

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