# CALCULATION FORMULA FOR A MAXIMUM BENDING MOMENT AND A MAXIMUM DEFLECTION OF THE TRIANGULAR SLAB WITH CONSIDERING EFFECT OF SUPPORT CONDITION SUBJECTED TO UNIFORM LOAD

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## ABSTRACT

The purpose of this study is to propose the calculation formula for allowable stress for structural design of any shape slab. In previous paper, we proposed the calculation formula for the maximum deflection and the maximum principal bending moment of the fixed edge triangular planed slabs subjected to uniform load. In this paper, we have offered the calculation formula about the case of edges conditions change. As a process, we assume virtual slab and obtain a calculation formula solution by applying it to the calculation formula for the fixed supported all edges. And so, this paper proposes simple calculation formulas give FEM solutions.

**Keywords:** Triangular slab, Degree of fixation, Uniform load, Maximum deflection,

Maximum Principal bending moment

## 1. INTRODUCTION

In Japan, floor slabs, that are assumed to be isotropic plates, are structurally designed separately from the frames. We calculate bending moments of the plate, and then we arrange reinforced bars which allow the bending moments. For rectangular slabs, we can use the design formula of Architectural Institute of Japan. By this formula, we can calculate for the approximately bending moments on rectangular slabs. On the other hand, for non-rectangular shapes, engineering judgment would be done because the formula by which we can calculate the bending moments does not exist.

From an above-mentioned reason, the purpose of authors' study is to propose the design formula of any shape of slab. In previous paper, authors presented formulas by which we can calculate the maximum deflection and the maximum principal bending moment per unit length of the triangular planed slabs with fixed supported all edges subjected to uniform load. In this paper, we have offered the calculation formula of the triangular slab when the degree of fixation of the edges conditions decrease. When the degree of fixation of some edges decrease, values of the maximum deflection

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and the maximum bending moment except on the edges increase as against the case of support is fixed. In this paper, we obtain a calculation formula solution by applying lager slab to the calculation formula for the fixed edge slab.

Note that, the vertex angle of the triangular slabs is from 30 degree to 90 degree. Here, the distribution of displacements in the fixed edge triangular slab is shown in Fig. 1, and distribution of principal bending moments is shown in Fig. 2. The maximum values treated by the calculation formula are the deflection $\delta_{\max}$ (＋ mark in Fig. 1) and the principal bending moment on $i$-th boundary sides $M_{b2i}$ (× mark in Fig. 2) and the center part $M_{c1}$ (○ mark in Fig. 2). The targeted values of these maximum values are obtained from finite element method (FEM).

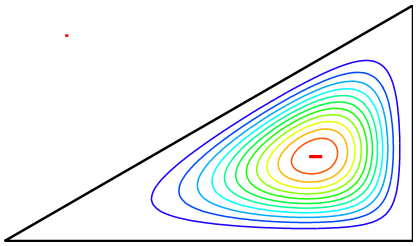

Figure 1: Contour of distribution of the displacement

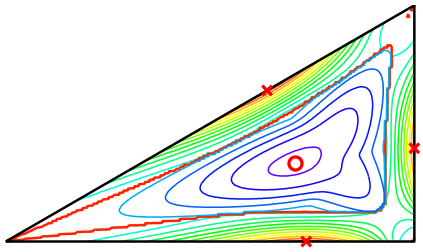

Figure 2: Contour of distribution of the principal bending moment

## 2. MAXIMUM VALUES BY THE DEGREE OF FIXATION DECREASES

### 2.1. Solution of the degree of fixation decreases

How to calculate the solution when the degree of fixation decreases is the following processes. And the process was shown in Fig. 3.

(1) An FEM solution is calculated by the fixed edge slab. The bending moment on the boundary side which lowers degree of fixation (we will call the side decrease-side) is calculated.

(2) Make the analysis model of a pin support of the decrease-side. The ratio that degree of fixation decreases is supplied to the bending moment calculated by (1), and the bending moment which negates it is supplied to the side considered as the pin support. And then, FEM solution of the slab is calculated. Note that, we will call this ratio decrease-ratio, and it will be a pin when setting the ratio as 1, will be a fixed when setting the ratio as 0.

(3) The solution of the degree of fixation decreases is superposition FEM solutions of (1) and (2).

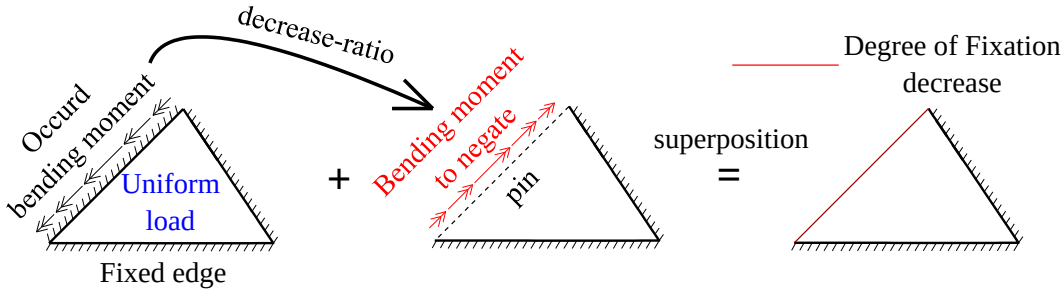


Figure 3: Process of calculate the solution when the degree of fixation decreases

Here, we change vertex angles every 5 degree and obtain 25 triangular slabs, change the decrease-ratios 0(fixed) to 1(pin) every 0.1 and obtain 10 kinds ratios and change decrease-side one to all and obtain 7 kinds combination of the sides. By all these patterns, we calculated 1750 kinds of FEM solutions. Furthermore, Their FEM solutions are calculated by midas/Gen. The element is an isotropic thin plate element and is used triangular element (DKT element) and quadrangular element (DKQ element). A divided width of the FEM model is 1/30 radius of the inscribed circle.

### 2.2. Effect on the maximum values by the degree of fixation decreases

As an example of the FEM solution, distributions of the principal bending moments are shown in Fig. 4. In the shape of these figures, vertex angles are 30, 60, 90 degrees. The decrease-side is the longest side, and the decrease-ratio is 40 % in Fig. 4(a), and is 60 % in (c). These figures were drawn in the larger absolute value in two kinds of principal bending moments. We will call the values $M_A$. As for the thick line in a figure, $M_A$ means 0.

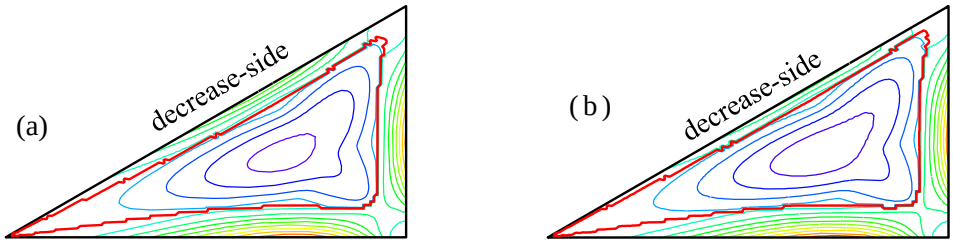


Figure 4: Contour of distribution of the $M_A$ (a) decrease-ratio is 40 %; (b) decrease-ratio is 60 %

Comparison of these figure and Fig. 2 (fixed supported all edges), by decrease-side, it proved that the line of $M_A$=0 is moving in the direction of a normal of the side. On the other hand, the shapes of the distribution of the inner part of $M_A$=0 line or the outer part resemble one of the fixed edge slab. And the trend is not concerned with the shape, the decrease-ratio or a number of the decrease-side.

Furthermore, as the example of change of the maximum value from the fixed supported all edges by lowering the degree of fixation, ratios of the maximum deflections is shown in Fig. 5. These ratios were calculated by 25 kinds of above triangle shape, 7 kinds combinations of the decrease-side and the decrease-ratios as 40 %, 60% and 80 %. The vertical axis shows the ratio of the maximum deflections, the horizontal axis shows the ratio of the length of the decrease-side to all sides. And

marker ＋ shows case of the decrease-ratios as 40 %, marker × shows as 60 % and marker ○ shows as 80 %.This figure proved that the ratio of the maximum deflection increased by the decrease-ratios and ratio of the length of the decrease-side to all sides.

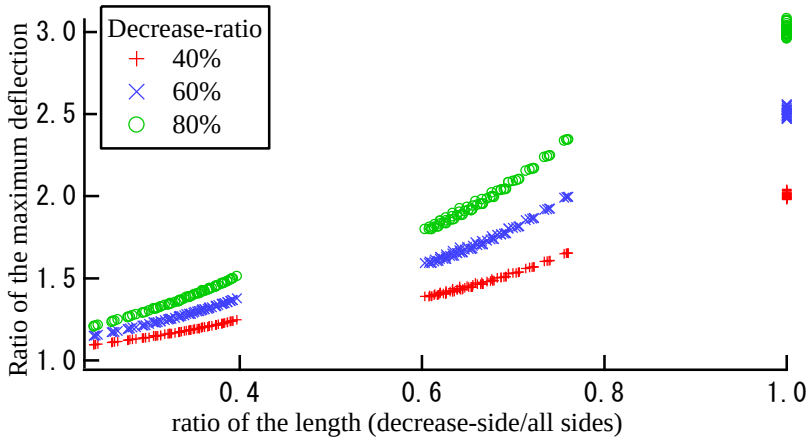


Figure 5: Ratios of the maximum deflections
(case of the degree of fixation decrease to case of the fixed supported all edges)

## 3. ASSUMPTION OF THE CALCULATION FORMULA

As mentioned above, the distribution of the bending moment when the degree of fixation decreases resembled one of the fixed supported all edges. From an above-mentioned reason, we considered that we can calculate the maximum values by supposing the fixed edge slab which large to the direction of the normal of the decrease-side. Such slab was shown in Fig. 6 with the broken line, and we will call it virtual-slab.

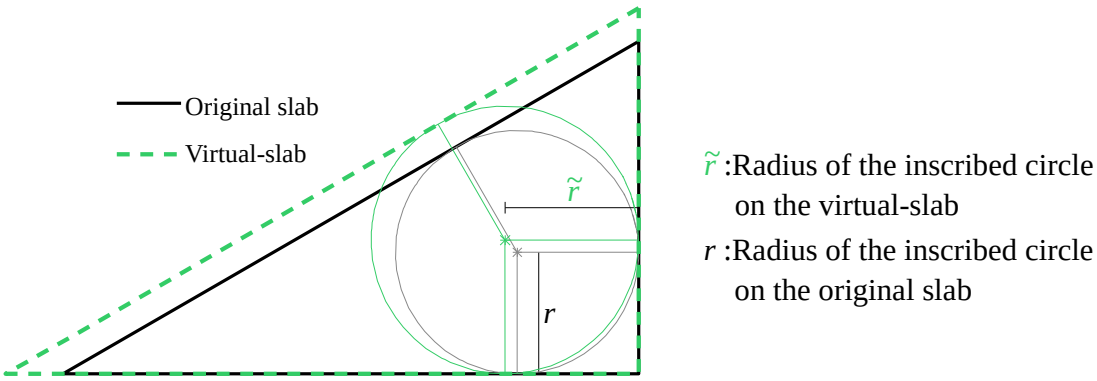


Figure 6: virtual-slab and its radius of the inscribed circle $\tilde{r}$

And, the calculation formula of the fixed edge triangular slabs is formulated in authors' previous paper. Here, these formulas are shown in Eqs. (1) to (3).

$$\delta_{\max} = \alpha_t \frac{wr^4}{Et^3}, \alpha_t = 0.258(1+\frac{0.0847}{\theta_1})(1+0.15\theta_4^2) \tag{1}$$

$$M_{b2i} = \beta_{ti} w r^2 , \beta_{ti} = 0.198(\frac{L_i}{r})^{0.175} \tag{2}$$

$$M_{c1} = \gamma_{t1} w r^2 , \gamma_{t1} = 0.137(1-0.272\theta_1)(1+0.08\theta_4^{1.5}) \tag{3}$$

Here, $\delta_{\max}$ is the maximum deflection, $w$ is the uniform load, $r$ is the radius of the inscribed circle, $E$ is the young's module, $t$ is the thickness of the slab, $\theta_1$ is the minimum vertex angle, $\theta_4$ is calculated by $(\theta_3\text{-}\theta_2)/2$, $M_{b2i}$ is the maximum principal bending moments in $i$-th boundary sides, $L_i$ is the length of $i$-th boundary sides, $M_{c1}$ is the maximum principal bending moment in the center part.

A maximum difference in $\delta_{\max}$ between the solution by Eq. (1) and FEM solution is around 1%, in $M_{b2i}$ is around 4% and in $M_{c1}$ is around 2%

As indicated in these formulas, a dominant variable of the maximum values are the radius of the inscribed circle. Therefore, we suppose the calculation formula of the radius of the inscribed circle on the virtual-slab. And we will call this radius $\tilde{r}$. And so, when the degree of fixation in the some edge decreases, the decrease-ratio and the ratio of the length of the decrease-side to all sides influence the solution. Therefore, these two ratios are used for the calculation formula of $\tilde{r}$.

A targeted value is defined for to formulate the calculation formula of $\tilde{r}$. The target value of $\tilde{r}$ is a solved value which gave the FEM solution ($\delta_{\max}$, $M_{b2i}$, $M_{c1}$) and the variable used for the FEM analysis ($w$, $E$, $t$, $\theta_1$, $\theta_4$, $L_i$) to Eqs. (1) to (3). We will call the target value $r_F$. Here, obtained $r_F$ are shown in Fig. 7. The vertical axis shows the ratio of $r_F$ to the original radius $r$, the horizontal axis shows the value which multiplied the decrease-ratio by the ratio of the length of the decrease-side to all sides. And marker × shows case of the $\delta_{\max}$, marker + shows $M_{b2i}$, marker ○ shows $M_{c1}$.

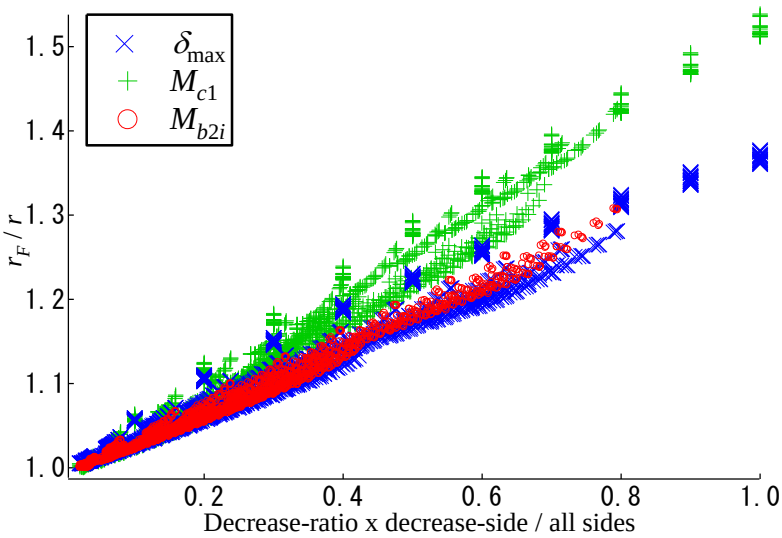


Figure 7: Target values $r_F$ for $\tilde{r}$

From the distribution of Fig. 7, we consider that $r_F$ is can calculate using variable used horizontal axis. Where, Slope differs for each maximum value, and some nonlinearity is observed in each distribution. Therefore, we will formulate for the calculation formula of $\tilde{r}$ for each maximum

value, and variable used horizontal axis changes and uses a trend. $\tilde{r}$ was assumed such as Eqs. (4) and (5).

$$\tilde{r}_k = r(1 + I_k^r \zeta) \tag{4}$$

$$\zeta = \frac{1}{L_1^n + L_2^n + L_3^n} \sum_i (B_r L_{bi})^n \tag{5}$$

Here, $\tilde{r}_k$ ($\delta_{\max}$:$d$, $M_{b2i}$:$b$, $M_{c1}$:$c$) is the radius of the inscribed circle on the virtual-slab in each maximum value, $r$ is the original radius of the inscribed circle, $L_1$ to $L_3$ are the lengths of the sides, $B_r$ is decrease-ratio (fixed:0, pin:1), $L_{bi}$ is the length decrease-side, $I_k^r$ and $n$ are undecided constants.

These undecided constants are found out in order that each maximum value by the Eqs. (4) and (5) and the $r_F$ may be congruous.

## 4. PROPOSAL OF THE CALCULATION FORMULA

The result calculated from the FEM solution is shown. Note that, we calculated for $I_k^r$ by the triple figures significant figure, and calculated for $n$ by a single figure. And we made these formulas as they are bigger than in all $r_F$.

$$\tilde{r}_d = r(1 + 0.380\zeta) \tag{6}$$

$$\tilde{r}_c = r(1 + 0.562\zeta) \tag{7}$$

$$\tilde{r}_b = r(1 + 0.408\zeta) \tag{8}$$

$$\zeta = \frac{1}{L_1^{0.7} + L_2^{0.7} + L_3^{0.7}} \sum_i (B_r L_{bi})^{0.7} \tag{9}$$

Each maximum value is required by calculating $\tilde{r}_k$ by Eqs. (6) to (8), by substituting it for $r$ of Eqs. (1) to (3).

Here, these $\tilde{r}_k$ and $r_F$ are shown in Fig. 8. Fig. 8 (a) shows the case of $\delta_{\max}$ ($\tilde{r}_d$ calculated by Eq. (6)), (b) shows the case of $M_{b2i}$ ($\tilde{r}_b$ calculated by Eq. (7)), (b) shows the case of $M_{c1}$ ($\tilde{r}_c$ calculated by Eq. (8)). The vertical axis shows the ratio of $r_F$ or $\tilde{r}_k$ to the original radius $r$, the horizontal axis shows $\zeta$ calculated by Eq. (9). And marker × shows $r_F$ case of the $\delta_{\max}$, marker ＋ shows $M_{b2i}$, marker ○ shows $M_{c1}$, the solid lines of each figure are the calculated values of Eq. (6), Eq. (7) or Eq. (8). These figures indicate that the calculation formula can simply express $r_F$.

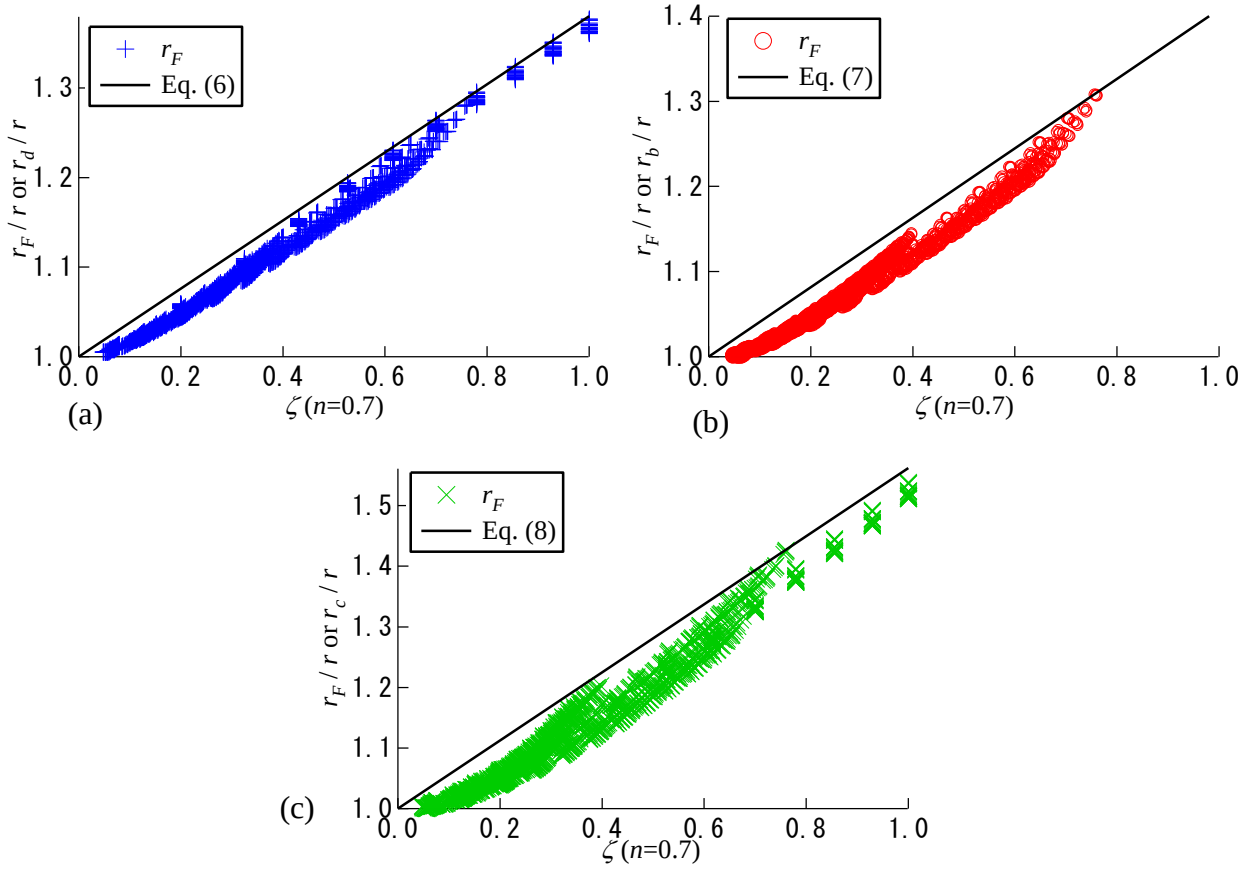


Figure 8: $r_F$ and $\tilde{r}_k$ (a): Case of $\delta_{\max}$ (b): Case of $M_{b2i}$ (c): Case of $M_{c1}$

## 5. ACCURACY OF THE SOLUTION BY THE CALCULATION FORMULA

A cumulative frequency distribution of the difference of the FEM solution and the solution of this calculation formula is shown in Fig. 9. The vertical axis shows the cumulative frequency, the horizontal axis shows the difference of the FEM solution and the solution of this calculation formula. Both of the axes are percentage notations. Note that, in $M_{b2i}$, the frequency is calculated for each boundary sides. And the side number 1 is the longest side, and 3 is shortest side.

A maximum difference in $M_{b2i}$ is around 10%. As for the difference, 80 percent or more is distributed between 5 to 10%. A maximum difference in $\delta_{\max}$ is around 15%. As for the difference, 80 percent or more is distributed between 5 to 15%. A maximum difference in $M_{c1}$ is around 18%. As for the difference, 80 percent or more is distributed between 5 to 15%.

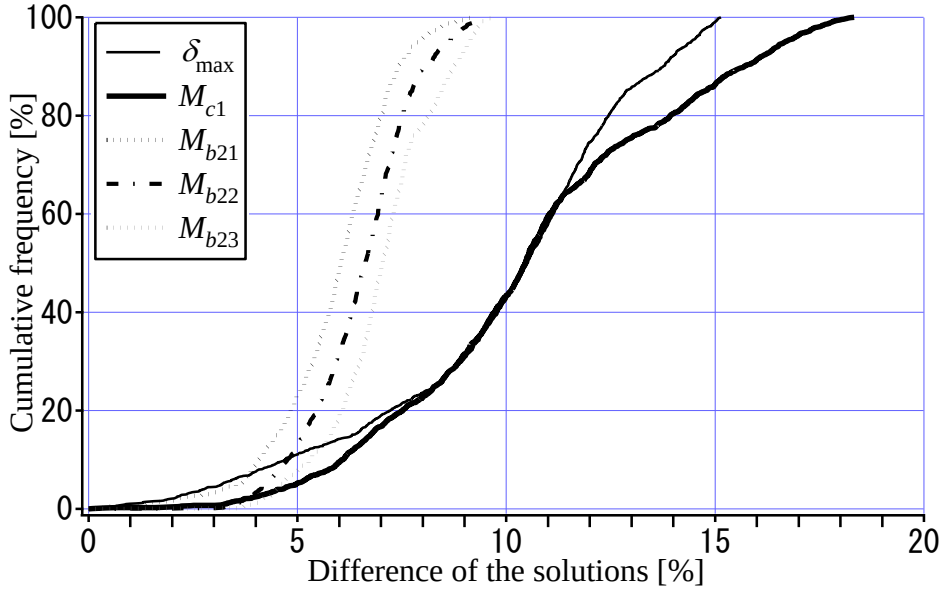


Figure 9: Cumulative frequency distribution
of the differences of the FEM solutions and solutions of this calculation formula

## 6. CONCLUSION

In this paper, we proposed the calculation formula of the triangular slab when the degree of fixation of the edges conditions decrease. When degree of fixation of some boundary side decreases, we considered that we can calculate the maximum values by supposing the fixed edge slab which large to the direction of the normal of the decrease-side. We call this fixed edge slab virtual-slab. Then, we proposed the calculation formula of the radius of the inscribed circle on virtual-slab. The maximum values can be calculated that the radius is applied to calculation formula of the fixed edge triangular slab. It is useful that we can use the formula of fixed edge slabs together. And the difference of the calculation formula solution and the FEM solution are less than 20%.

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