OPTIMAL SENSOR PLACEMENT FOR DAMAGE DETECTION OF DISTRIBUTED-PARAMETER STRUCTURES BASED ON INFORMATION ENTROPY

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ABSTRACT

The number of sensors and the corresponding locations are very important for the success of structural health monitoring (SHM) using vibration data. Most works related to the optimal sensor placement (OSP) problem in the literature are based on discrete-coordinate systems which are usually modelled by the finite element (FE) method. Although FE method can provide a convenient and practical approach to the dynamic response analysis of arbitrary structures, the solutions obtained can only approximate their actual dynamic behaviour because the motions are only represented by a limited number of displacement coordinates, where the sensors are confined to be put. For structures having typical distributed properties, it's more reasonable and accurate to be modelled as distributed-parameter system with analytical formulation. Furthermore, although much attention has been paid to the OSP problem in various fields, such as modal testing and model parameters updating, the particular purpose for structural damage detection has seldom been involved. The main purpose of this paper is to develop an effective methodology for investigating the OSP problem for distributed-parameter system based on the information entropy for the damage detection purpose. Where, the information entropy is employed as a measure to quantify the uncertainty of the identified crack parameters, and it is minimized by the generic algorithm (GA) to give the optimal sensor configurations. The proposed method is demonstrated by the numerical simulation of an Euler-Bernoulli beam model.

Keywords: Optimal sensor placement, Damage detection, Information entropy, Genetic algorithm.

1. INTRODUCTION

The quantity and quality of the measured data, i.e., the number of sensors and the corresponding locations are very important for the success of SHM utilizing measured dynamic responses. In most field tests, the sensor locations are decided based on the experience of the researchers taking into

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account a series of practical constraints. It is important to ensure that the selected sensor locations can measure the required information by following a mathematically rigorous way to determine the optimal locations for placing sensors.

The problem of OSP was first investigated by Shah and Udwadia (1978). Later, many researchers have studied the issue of optimally installing a given number of sensors on the target structure for the purposes of model and modal parameter identification (Kammer 1991; Penny et al. 1994). In 2000, Papadimitriou et al. (2000) introduced the information entropy, representing a direct measure of the uncertainties in model parameter estimates, to the OSP problem, and the optimal sensor locations is determined by minimizing the information entropy measure. Recently, the methodology proposed in reference (Papadimitriou et al. 2000) is adopted for studying the sensor placement of a transmission tower model (Chow et al. 2011).

It's found from the literature that, most relevant works for studying the OSP problem employ the discrete-coordinate systems following the FE technique. The discrete-coordinate models can provide a convenient approach to the dynamic response analysis of arbitrary and complex structures, but the solutions obtained can only approximate their actual dynamic behavior as the motions are represented by a limited number of displacement coordinates. Although the precision of the results obtained can be made as refined as desired by increasing the number of degrees of freedom (DOF) considered in the analyses, it would undoubtedly increase the computation effort as well as the number of potential DOFs for placing sensors especially with large number of iterative calculations usually required. For such reason, the sensor placement procedure is usually carried out based on a very coarse mesh model, which will induce large modeling errors. Moreover, with the coarse FE model, the sensors are confined to be placed at only a very limited number of available locations, which is "optimal" only with respect to the particular coarse model. Moreover, the majority of relevant works in the literature does not handle sensor placement problem for damage detection purpose, which is very important for real applications, since the measured responses should be as sensitive to damage as possible.

In this paper, a sensor placement technique based on the information entropy for distributed-parameter system is developed particularly for the purpose of structural damage detection. The proposed method adopts the analytical formulation to model each structural segment, and the sensor locations are identified by their coordinates that are continuous within the region of interest. Thus, the proposed method is independent of the potential number of candidate sensor locations required in other methods based on discrete-coordinate models. A numerical case studies for a beam model is utilized to demonstrate the proposed method with the binary-coded GA.

2. THEORETICAL DEVELOPEMENTS

For a parameterized class of structural models $\mathcal{M}$ with the vector of parameters $\boldsymbol{\theta} \in R^{N_\theta}$ chosen to describe the input-output behavior of a structure, the Bayesian statistical system identification methodology developed in reference (Beck and Katafygiotis 1998) is adopted to estimate the values of the parameters set $\boldsymbol{\theta}$ and their associated uncertainties using the information provided from dynamic measurement data $\mathcal{D}$, assuming that the prediction error is both spatially and temporally independent and normally distributed with zero mean and variance σ^2.

Following the Bayes' theorem, the posterior (or updated) probability density function (PDF) of the set of structural model and prediction error parameters $(\boldsymbol{\theta},\sigma)$ for a given set of measurement data $\mathcal{D}$ is denoted by $p(\boldsymbol{\theta},\sigma\,|\,\mathcal{D},\mathcal{M})$. Employing the total probability theorem, the posterior marginal PDF for the set of structural model parameters $\boldsymbol{\theta}$, i.e., $p(\boldsymbol{\theta}\,|\,\mathcal{D},\mathcal{M})$ can be obtained by integrating with respect to σ. For a non-informative (uniform) prior PDF $\pi_\sigma(\sigma)$, the integration over σ results in

$$p(\boldsymbol{\theta}\,|\,\mathcal{D},\mathcal{M}) = c\pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})\left[J(\boldsymbol{\theta};\mathcal{D},\mathcal{M})\right]^{-N_J} \tag{1}$$

where $N_J = -(NN_O - 1)/2$. N_O is the number of measurement points, and N is the number of measured time steps at each measurement point. c is a normalizing constant such that the integration of the PDF in equation (1) over the predefined domain is equal to unity. $\pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ is the prior PDF of the parameter set $\boldsymbol{\theta}$, which allows judgment about the relative plausibility of the values of $\boldsymbol{\theta}$ to be considered. $J(\boldsymbol{\theta};\mathcal{D},\mathcal{M})$ denoting the measure-of-fit function between the measured and the model predicted time domain response is given by

$$J(\boldsymbol{\theta};\mathcal{D},\mathcal{M}) = 1/(NN_O)\sum_{n=1}^{N}\left\|\hat{\mathbf{q}}(n) - \mathbf{q}(n;\boldsymbol{\theta})\right\|^2 \tag{2}$$

where $\|\cdot\|$ is the usual Euclidian norm. $\hat{\mathbf{q}}(n)$ is the $N_O \times 1$ vector of the measured time domain response at the nth time step, $\mathbf{q}(n;\mathbf{a})$ is the $N_O \times 1$ vector of the calculated time domain response at nth time step based on the model class $\mathcal{M}$ for a specified set of parameters $\boldsymbol{\theta}$.

By utilizing $p(\boldsymbol{\theta}\,|\,\mathcal{D},\mathcal{M})$ given in equation (1), a unique scalar measure of the uncertainty in the estimate of the structural parameters $\boldsymbol{\theta}$ is provided by the information entropy as

$$H(\mathcal{D},\mathcal{M}) = \mathrm{E}_{\boldsymbol{\theta}}\left[-\ln p(\boldsymbol{\theta}\,|\,\mathcal{D},\mathcal{M})\right] = -\ln c - \mathrm{E}_{\boldsymbol{\theta}}\left[\ln \pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})\right] + N_J \mathrm{E}_{\boldsymbol{\theta}}\left[\ln J(\boldsymbol{\theta};\mathcal{D},\mathcal{M})\right] \tag{3}$$

where $\mathrm{E}_{\boldsymbol{\theta}}$ denotes mathematical expectation with respect to $\boldsymbol{\theta}$. The aim of sensor configuration design ensures that for the parameter set $\boldsymbol{\theta}$, the measured data $\mathcal{D}$ are most informative, indicating that $\boldsymbol{\theta}$ can be estimated with the least uncertainties. Thus, the sensor configuration that minimizes the information entropy is chosen as the optimal one. Unfortunately, the test data $\mathcal{D}$ are not available when design the sensor configuration, and thus the explicit dependence of information entropy on the data $\mathcal{D}$ should be dropped. This goal can be asymptotically accomplished for large number of data ($N \to \infty$) by defining the information entropy with the optimal value of the model parameters $\hat{\boldsymbol{\theta}}$ and associated prediction error $\hat{\sigma}^2$ expected for a set of measured data. Noting that an estimate of $\hat{\boldsymbol{\theta}}$ and $\hat{\sigma}^2$ can still not be achieved as the test data $\mathcal{D}$ are not available, a useful design suggested in reference (Papadimitriou et al. 2000) is to replace $\hat{\boldsymbol{\theta}}$ and $\hat{\sigma}^2$ with some nominal ones $\boldsymbol{\theta}_0$ and σ_0^2 chosen by the designer to be representative of the discrete-coordinate systems.

For the purpose of structural model updating, it's usually assumed that updated model is close to the nominal one, i.e., the updated values of the model parameters do not deviate significantly from the nominal model parameter values $\boldsymbol{\theta}_0$ and σ_0^2. The optimal sensor configuration for this purpose can be carried out by minimizing the information entropy given in equation (3). However, this is not the case for damage detection problems when using $\boldsymbol{\theta}$ to model stiffness terms in a structure, and large uncertainty in the values will arise due to the possibility of significant reduction in the structural stiffness due to severe damage. In such situation, equation (3) should be modified to account for the uncertainty in $\boldsymbol{\theta}_0$ and σ_0, which can be quantified using a prescribed PDF, i.e., $p(\boldsymbol{\theta}_0, \sigma_0 \mid \mathcal{M})$, for $\boldsymbol{\theta}_0$ and σ_0 to represent the designer's uncertainty in these parameters. The problem of optimal sensor configuration can be solved by minimizing the change of information entropy over the sensor configuration vector $\boldsymbol{\delta}$ and given by (Papadimitriou et al. 2000)

$$\Delta H = H_{\boldsymbol{\theta},\boldsymbol{\theta}_0,\sigma_0}(\boldsymbol{\delta},\mathcal{M}) - H_{\boldsymbol{\theta}_0,\sigma_0}(\mathcal{M}) = \mathrm{E}_{\boldsymbol{\theta},\boldsymbol{\theta}_0,\sigma_0}\left[-\ln p(\boldsymbol{\theta},\boldsymbol{\theta}_0,\sigma_0 \mid \boldsymbol{\delta},\mathcal{M})\right] - \mathrm{E}_{\boldsymbol{\theta}_0,\sigma_0}\left[-\ln p(\boldsymbol{\theta}_0,\sigma_0 \mid \mathcal{M})\right] \tag{4}$$

Assuming $\boldsymbol{\theta}_0$ and σ_0 being statistically independent, and after some manipulation, the change of information entropy in equation (4) takes the following simplified form

$$\Delta H = \frac{1}{2} N_\theta \left[\ln(2\pi) + 1 + \int \ln \sigma_0^2 \mathrm{d}\sigma_0\right] - \frac{1}{2}\int \ln \det \mathbf{Q}(\boldsymbol{\delta},\boldsymbol{\theta}_0) p(\boldsymbol{\theta}_0) \mathrm{d}\boldsymbol{\theta}_0 \tag{5}$$

It is obvious from equation (5) that the optimal sensor configuration are obtained by maximizing the quantity of multidimensional numerical integration over $\boldsymbol{\theta}_0$, i.e., $J_{\mathbf{Q}} = \int \ln \det \mathbf{Q}(\boldsymbol{\delta},\boldsymbol{\theta}_0) p(\boldsymbol{\theta}_0) \mathrm{d}\boldsymbol{\theta}_0$, the calculation of which can be carried out approximately but efficiently using an asymptotic expansion developed to treat this type of integral (Papadimitriou et al. 1997), valid for large number of data ($N \to \infty$). For continuous-coordinate system with the excitation locations considered, the $\mathbf{Q}$ matrix in equation (5) is proposed as

$$\mathbf{Q}(\boldsymbol{\delta},\boldsymbol{\chi},\boldsymbol{\theta}_0) = \left[\sum_{n=1}^{N}\sum_{k=1}^{N_O} \nabla_{\boldsymbol{\theta}} y(n,\boldsymbol{\delta}_k;\boldsymbol{\chi},\boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^{\mathrm{T}} y(n,\boldsymbol{\delta}_k;\boldsymbol{\chi},\boldsymbol{\theta})\right]_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \tag{6}$$

where $\boldsymbol{\delta} \in R^{N_O N_D}$ is the sensor configuration vector and given by $\boldsymbol{\delta} = \{\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \ldots, \boldsymbol{\delta}_{N_O}\}^{\mathrm{T}}$, including the coordinates of all measurement points. Similarly, $\boldsymbol{\chi} = \{\boldsymbol{\chi}_1, \boldsymbol{\chi}_2, \ldots, \boldsymbol{\chi}_{N_I}\}^{\mathrm{T}} \in R^{N_I N_D}$ denotes the actuator configuration vector and includes the coordinates of all excitation points, and N_I is the number of excitation points. $\boldsymbol{\delta}_k \in R^{N_D}$ ($k = 1,2,\ldots,N_O$), $\boldsymbol{\chi}_l \in R^{N_D}$ ($l = 1,2,\ldots,N_I$) denote the coordinate for the kth measurement and the lth excitation points, respectively. $N_D = 1,2,3$ is the dimension of the target structural model. $\nabla_{\boldsymbol{\theta}}$ denotes the usual gradient vector with respect to the parameter vector $\boldsymbol{\theta}$. $y(n,\boldsymbol{\delta}_k;\boldsymbol{\chi},\boldsymbol{\theta})$ is the kth element of $\mathbf{q}(n;\boldsymbol{\theta})$. Noting that for the case of ambient vibration, the vector $\boldsymbol{\chi}$ indicating the excitation locations should be dropped. Unlike the majority of existing methods, the problem of OSP in the proposed method runs into a continuous optimization procedure, which can be efficiently solved by GA, a simple yet very powerful routine for obtaining global minimums.

3. NUMERICAL SIMULATIONS

To demonstrate the proposed method for distributed-parameter systems, an Euler-Bernoulli beam simply-supported at one end and clamped at the other end with a single open and non-propagating crack is employed (see Figure 1). The beam is uniform in both material and geometry properties. The length L, width b, and height h of the beam are 1.25 m, 0.0094 m, and 0.0158 m, respectively. The Young's modulus E and mass density ρ are 5.8×10^{10} N/m^2 and 2780 Kg/m^3, respectively. The crack is characterized by the relative location $\zeta_C = L_C / L$ and relative depth $d_C = h_C / h$. It's assumed that the structure is excited by a single impulsive load applied in the transverse direction, and the dynamic response of the beam is calculated by the modal superposition method involving the first four lower vibration modes. A uniform 1% modal damping ratio is assumed for all modes, and the sampling frequency is 5000 Hz with a duration of 0.2 s.

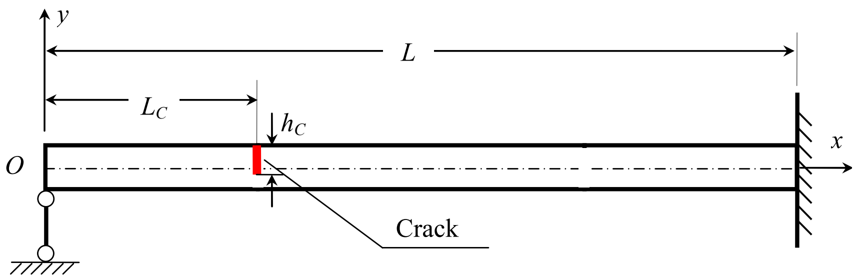

Figure 1: Euler-Bernoulli beam with a transverse crack.

It should be noted that larger crack depth would certainly induce more significant influence on the dynamic response, and including crack-depth parameter in optimization process would always make the identified most probable value touch the pre-defined upper boundary. But this is not the case for the crack-location parameter. Thus, in the present study, only crack-location parameter are considered for identification by keeping the crack depth to be constant, and the non-dimensional scalar parameter θ with unity nominal value is used to scale the relative crack location ζ_C. Binary-coded GA is employed, and the number of bits for coding each variable is denoted by N_{bits}. It divides the 1-D searching domain into $2^{N_{\text{bits}}} - 1$ regions, or equivalently, provides the solution with a resolution of $1/(2^{N_{\text{bits}}} - 1)$. Without loss of generality, two typical values of N_{bits} (i.e., N_{bits}=4 and 8) with the number of sensors ranging from 1 to 4 are employed in present case study.

Table 1 shows the optimal sensor locations obtained by the proposed method with various N_{bits} and number of sensors N_O, where the excitation locations χ_1 get the values of 0.3 and 0.5, respectively. It's clear from the 4-th column of this table that the objective function value J_Q increases (or equivalently, the change of information entropy in equation (5) decreases) with the increase of the number of sensors, implying that involving more sensors could reduce the uncertainties of the identified crack parameters. Although not shown in this paper, it is also found from the numerical simulations that involving more number of modes in modal superposition process can also reduce the associated uncertainties.

Table 1: Optimal sensor locations obtained by the proposed methodology

Excitation Location χ_1	N_{bits}	N_O	Objective Function J_Q	Optimal Sensor Locations			
				δ_1	δ_2	δ_3	δ_4
0.3	4	1	**0.8545**	0.4000			
		2	**0.8563**	0.4000	0.7333		
		3	**0.8951**	0.4000	0.6667	0.7333	
		4	**0.9052**	0.3333	0.4000	0.6667	0.7333
	8	1	**0.9757**	0.7020			
		2	**1.0920**	0.3922	0.7020		
		3	**1.1626**	0.3882	0.3922	0.7020	
		4	**1.2187**	0.3882	0.3922	0.3961	0.7020
0.5	4	1	**0.6377**	0.5333			
		2	**0.7219**	0.5333	0.8000		
		3	**0.7733**	0.2667	0.5333	0.8000	
		4	**0.7998**	0.2667	0.5333	0.8000	0.8667
	8	1	**0.6476**	0.5294			
		2	**0.7428**	0.5294	0.5373		
		3	**0.7987**	0.5294	0.5373	0.5412	
		4	**0.8400**	0.5255	0.5294	0.5333	0.5373

In order to study the effect of excitation locations on the OSP results by the proposed method, the impulse excitation at relative locations 0.3 and 0.5 are considered, and summarized in Table 1. It is clear from the table that the optimal sensor locations depend very much on the excitation location, and the sensors should be installed as close to the excitation location as possible, which is similar to the results in reference (Chow et al. 2011).

Another important observation found from Table 1 is that the optimal locations of some sensors are very close to each other for larger number of bits, and the location differences between each pair of close sensors are calculated. Taking the results with excitation location χ_1=0.3 as an example, it's clear that the distances between the closely placed sensors for N_{bits}=8 are all equal to 0.003922. Noting that this distance value is just the resolution for the binary-coded GA with N_{bits}=8. Moreover, it can be further anticipated that by increasing N_{bits}, the distance between these sensors would continue to decrease to the value of resolution corresponding to the new N_{bits}, until the limit state being reached, i.e., these sensors were finally put at the same location (if allowed). This implies that for the OSP problem with respect to distributed-parameter system for the purpose of damage detection, minimizing the change of information entropy in equation (5) alone leads to the situation that some of the sensor locations were overlapped.

Although not provided here, by comparing the dynamic response measured from optimal sensor locations, one can find that the responses from each channel are much more similar to each other for N_{bits}=8 than for N_{bits}=4, especially for the case with excitation location at 0.5. This is expected referring to Table 1, i.e., the optimal sensor locations are more different for N_{bits}=4 as compared with those for N_{bits}=8, and the closer the sensors are, the more similar the response would be.

4. CONCLUSIONS

In this paper, a statistical methodology based on Bayesian theorem and information entropy is developed for the OSP problem on distributed-parameter system for the purpose of structural damage detection. This is performed by minimizing the change of information entropy measure (or uncertainties) associated with the identified crack modeling parameters with the sensor configurations as the minimization variables. A simple distributed-parameter system, i.e., a Euler-Bernoulli beam with a single crack is taken as the numerical example to demonstrate the proposed methodology. It's concluded from the numerical simulation results that, for the OSP problem with respect to the distributed-parameter system with the purpose of structural damage detection, minimizing the change of information entropy alone would lead to the situation that part of sensors tend to be placed in an overlapping manner, and the collected dynamic response from these sensors would be extremely similar to each other. However, this phenomenon is not practical since the sensors could not be placed so close to each other in practice and the duplicated data would be measured. The improved approach for avoiding this situation is under investigation, and will be presented in more detail in the coming papers.

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