DEFORMATION-BASED APPROACH FOR DETERMINATION OF THE EFFICIENCY OF THE CONFINEMENT IN R/C MEMBERS

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ABSTRACT

Currently, for the determination of the ultimate strength of confined concrete cores in reinforced concrete members subjected to compression, the yielding of the confining reinforcement is usually assumed. This assumption may in many cases be on the unsafe side, particularly by the use of high-strength concrete or high-strength steel for confining reinforcement. Furthermore, the question about the possible spalling of the concrete cover in a confined member under the service load is still open. In this paper, the development of a model for lateral strains of concrete in triaxial compression is presented. With the proposed model, both the actual stress in confining reinforcement and the risk of the early spalling of concrete cover can be estimated. Based on the investigations, lower and upper bounds of the ratio of confining reinforcement for the effective confinement are proposed.

Keywords: confinement, lateral strain, compressive strength, reinforcement ratio

1. INTRODUCTION

Research on the behavior of concrete under active confinement began at the early of 20th Century (Richart et al. 1928), where a linear relation between the strength of confined concrete f_{cc} and the lateral stress σ_l was introduced as:

$$\frac{f_{cc}}{f_c} = 1.0 + k \cdot \frac{\sigma_l}{f_c} \tag{1}$$

Where the coefficient k was chosen to be *4,1*.

The confinement can also be obtained in R/C members through the use of the lateral reinforcement in the form of ties or stirrups (Fig. 1). In contrast to the triaxial compression test, the confinement in R/C members is a passive phenomenon, the actual confining stress σ_{le} thus depends decisively on the shape of the cross-section, the amount of lateral reinforcement and the deformation capacity of concrete in triaxial stress states. Until now the interdependence between the lateral reinforcement and the confined concrete core is not yet clear. It is usually assumed that the confining reinforcement yields when the triaxial compressive strength f_{cc} is reached, e.g. in Model Code 2010

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(2012). This assumption is from the mechanical point of view inconsistent and cannot be confirmed by experimental investigations with low ratio of confining reinforcement or with high-strength concrete (Nagashima et al. 1992; Cusson and Paultre 1995).

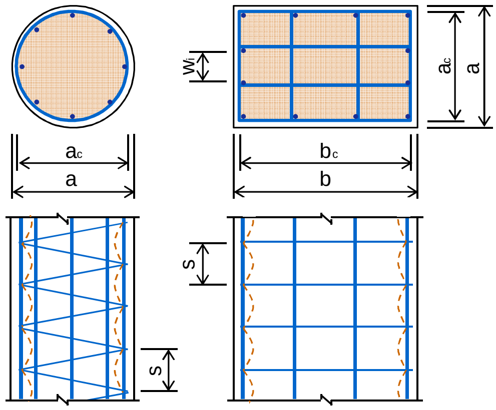

Figure 1: Stirrups und ties as confining reinforcement in R/C members

Due to the confinement the lateral strains of confined core is smaller than that of concrete cover. An early spalling of concrete cover may occur. Experimental investigations shown that the longitudinal strain at the cover spalling $\varepsilon_{c,A}$ can be lower than the peak-strain of concrete under uniaxial compression ε_{co} (Razvi 1996). Because of the fact that the maximum strength of the confined concrete core f_{cc} can be several times higher than the compressive strength of unconfined concrete in member f_{co}, the concrete cover can fail even under the service load. To assure the durability of the structures the confinement must not exceed an upper boundary. For this the determination of the strain ε_{cA} of the cover spalling and the corresponding load is of importance.

Based on the further development of an existing model for lateral strain of concrete in triaxial compression (Montoya et al. 2006), the interactions between and the material properties and the confinement considering the structural detailing of RC members are presented.

2. CONFINEMENT IN R/C MEMBERS

2.1. Lateral strain of concrete in triaxial stress states

Based on the results of the experimental study on cylinders under triaxial compression of Imran and Pantazoloulou (1996) a simple parabolic relation between the lateral strain ε_{cl} and the normalized longitudinal strain $\varepsilon_c/\varepsilon_{cc}$ was proposed by Montoya et al. (2006):

$$\varepsilon_{cl} = \left(0.0019 + 0.0242 \cdot I_e\right) \times \left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)^2 \tag{2}$$

With this model the lateral strain in triaxial stress states can be directly estimated in dependence of the confinement index $I_e = \sigma_l/f_c$. However this model doesn't consider the fact that the lateral strain decreases with the compressive strength. To improve this, further experimental data (Candappa et al. 2001; Dahl 1992) were analyzed (Tung 2009). Through a regression analysis, a relation between the lateral strain and the normalized longitudinal strain was proposed:

$$\varepsilon_{cl} = \left(0,00176 + 0,77 \cdot I_e \cdot f_c^{-0,85}\right) \times \left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)^2 \tag{3}$$

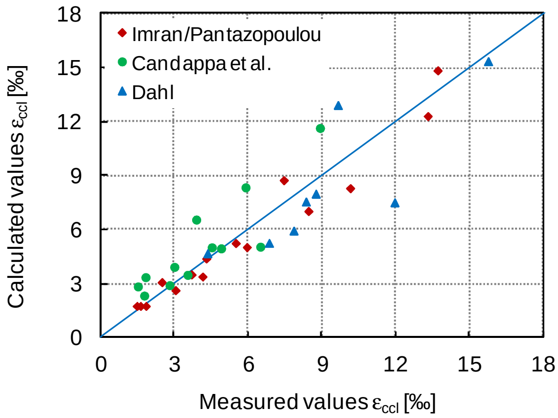

Figure 2: Comparison the lateral strain ε_{ccl} between experimental tests and test prediction

A comparison of the lateral strain ε_{ccl} at triaxial compressive strength f_{cc} calculated with Eq. (3) to the test results (Imran and Pantazoloulou 1996; Candappa et al. 2001; Dahl 1992) is given in Fig. 2. The comparison shows a good agreement between experiments and predictions, with an average value of *1,16* and a coefficient of variation of *21,5 %*.

2.2. Stress in confining reinforcement at maximum load

Considering a geometric coefficient of confinement effectiveness K_e, the confining stress in concrete core at maximum load σ_{ccl} can be calculated from the stress in confining reinforcement σ_{scc} and the geometric ratio of reinforcement $\rho_s = A_s/(a_c \times s)$ according to Eq. (4):

$$\sigma_{ccl} = K_e \cdot \rho_s \cdot \sigma_{scc} \tag{4}$$

In this contribution the widely-accepted proposal for K_e of Mander et al. (1988) is adopted. Taking as a basis of the compatibility of deformations between confined concrete core in lateral direction and confining reinforcement $(\varepsilon_{ccl} = \varepsilon_{scc})$ the strain in confining reinforcement at maximum load $(\varepsilon_c/\varepsilon_{cc} = 1)$ can be determined by substitution Eq. (4) into Eq. (3):

$$\varepsilon_{scc} = 0,00176 + 0,77 \cdot K_e \cdot \rho_s \cdot \frac{1}{f_{co}^{1,85}} \cdot \sigma_{scc} \tag{5}$$

Where $f_{co} = 0,85\, f_c$ the compressive strength of unconfined concrete in member. Assumed that the stress-strain relation of confining reinforcement is ideal elasto-plastic, the stress and strain of reinforcement at maximum load can be calculated using Eq. (5) as:

$$\varepsilon_{scc} = \frac{0,00176}{1-0,77 \cdot K_e \cdot \rho_s \cdot \frac{E_s}{f_{co}^{1,85}}}; \qquad \sigma_{scc} = E_s \cdot \varepsilon_{scc} \tag{6}$$

If the calculated strain according to Eq. (6) exceeds the yielding limit of the confining reinforcement *($\varepsilon_{scc} \geq \varepsilon_{yh}$)*, then Eq. (5) is simplified as:

$$\varepsilon_{scc} = 0,00176 + 0,77 \cdot K_e \cdot \rho_s \cdot \frac{f_{yh}}{f_{co}^{1,85}}; \qquad \sigma_{scc} = f_{yh} \tag{7}$$

2.3. Stress and strain of concrete core at maximum load

With the actual stress in confining reinforcement σ_{scc} the maximum strength of the confined concrete core f_{cc} can be estimated using Eq. (1). The coefficient k is determined to be *4,0*. Based on the relation between the crack formation and the absorbed energy (Tung 2009) the strain of concrete core at maximum loading ε_{cc} can be determined as follows:

$$\varepsilon_{cc} = \frac{\alpha_c}{\alpha_{cc}} \cdot \left(\varepsilon_{co} - \frac{f_{co}}{E_c} \right) \cdot \frac{\varepsilon_{ccl} - 0.1 \cdot \varepsilon_{co}}{0.4 \cdot \varepsilon_{co}} + \frac{f_{cc}}{E_c} \tag{8}$$

Where $\alpha_c = 0,8215 + 0,005 \cdot f_{co}$; $\alpha_{cc} = 0,665 + 0,5 \cdot I_e$ the solidity factors of the stress-strain curves of concrete under uniaxial and triaxial compression (Tung 2009).

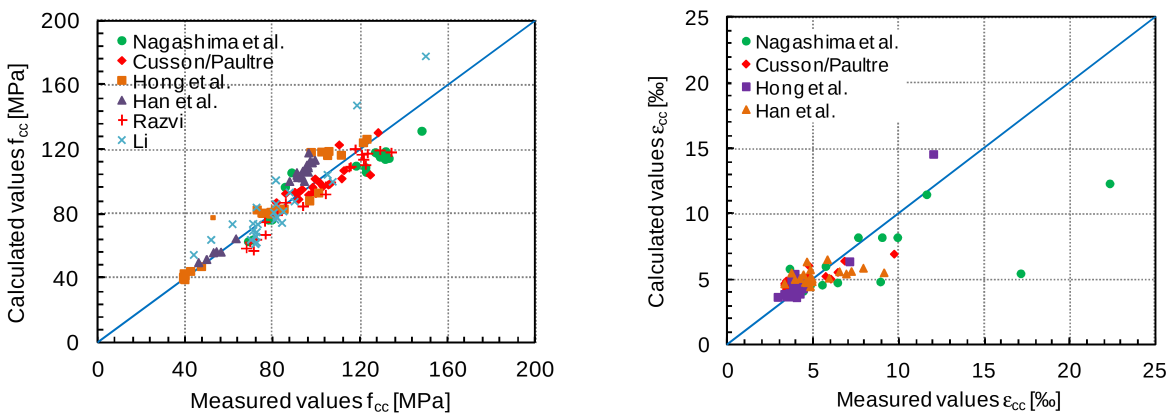

Figure 3: Comparison the triaxial strength and corresponding strain between tests and model

A comparison of the maximum strength of the confined concrete core f_{cc} and the corresponding strain ε_{cc} using Eq. (1) and (8) to the test results of 144 columns performed by different authors (Nagashima et al. 1992; Cusson and Paultre 1995; Hong et al. 2006; Han and Sihn 2003; Razvi 1996; Li 1994) is given in Fig. 3. The comparison shows a good agreement between experiments and predictions. For the prediction of the maximum strength f_{cc} an average value of *1,03* and a coefficient of variation of *10,7 %* are obtained, the prediction of the correspondent strain ε_{cc} gives an average value of *1,07* and a coefficient of variation of *20,8 %*.

3. EFFECTIVITY OF THE CONFINEMENT

3.1. Minimum reinforcement ratio

To utilize the strength capacity of reinforcement for an effective confinement the stress in the confining reinforcement should at least reach its yield strength at maximum load. That means:

$$0,00176 + 0,77 \cdot K_e \cdot \rho_s \cdot \frac{f_{yh}}{f_{co}^{1,85}} \geq \frac{f_{yh}}{E_s} \tag{9}$$

With a certain yield strength f_{yh} of the used confining reinforcement the minimum geometric ratio of reinforcement $\rho_{s,min}$ can be determined as:

$$\rho_{s,\min} = \frac{f_{yh} - 0,00176 \cdot E_s}{0,77 \cdot K_e \cdot f_{yh} \cdot \frac{E_s}{f_{co}^{1,85}}} \tag{10}$$

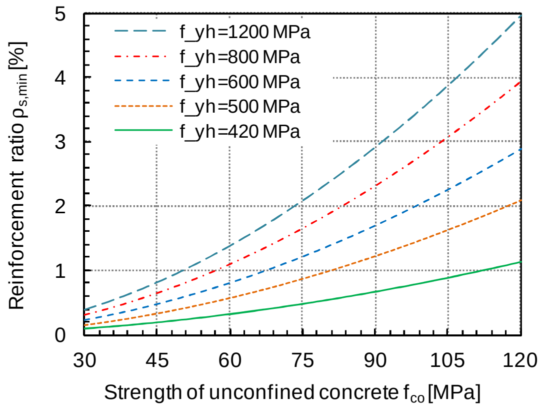

Figure 4: Minimum reinforcement ratio $\rho_{s,min}$

Fig. 4 illustrates the minimum reinforcement ratio according to Eq. (10) with $K_e = 0,65$ for different yield strengths of confining reinforcement and compressive strength. Considering the fact, that the arrangement of ties or stirrups with the reinforcement ratio more than 3 % is practically not possible, the use of high-strength steels for confining reinforcement for compressive members with HSC is not reasonable. Using yield strength to calculate the confining stresses for members with the reinforcement ratio lower than $\rho_{s,min}$ will lead to an unsafe design.

3.2. Maximum reinforcement ratio to avoid the cover spalling under service load

With the confining stress σ_{ccl} the maximum strength of the confined concrete core f_{cc} and the correspondent strain ε_{cc} can be estimated. The lateral strains ε_{cl} for confined concrete cores with two different reinforcement ratios in comparison with the lateral strain of the concrete cover are qualitatively depicted in Fig. 5. It can be seen that up to a longitudinal strain of $\varepsilon_1 \sim 0,5\ \varepsilon_{co}$ the lateral strain different between core and cover is insignificant. After that the lateral strain of cover increases disproportionately until the uniaxial compressive strength f_{co} is reached, while the lateral strain of core remains lower due to the confinement. The lateral strain different between core and cover $\Delta\varepsilon_{cl}$ depends on the reinforcement ratio.

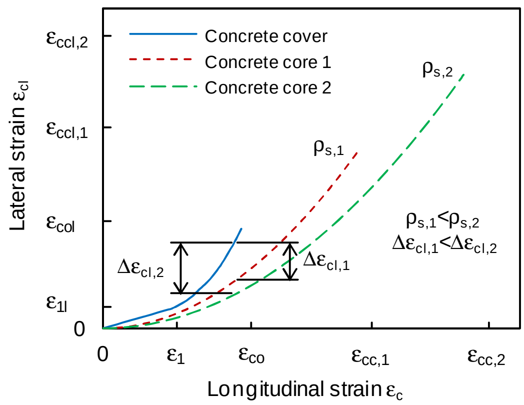

Figure 5: Schematic representation of the lateral strains for the concrete cover and core

If the strain different $\Delta\ \varepsilon_{cl}$ exceeds a tensile strain ε_{ct} of approx. *0,15 ‰* a splitting crack at the face between concrete core and cover arises, its crack width can be estimated as follows:

$$w = \tfrac{1}{2} \cdot a_c \cdot \left(\Delta\varepsilon_{cl} - \varepsilon_{ct}\right) \tag{11}$$

The load transfer at the crack can be described with a model according to MC 10 in which from a crack width $w_t = G_F/f_{ctm}$ downward the possible tensile stress is negligible. It is assumed that at this crack width the spalling of concrete cover occurs completely.

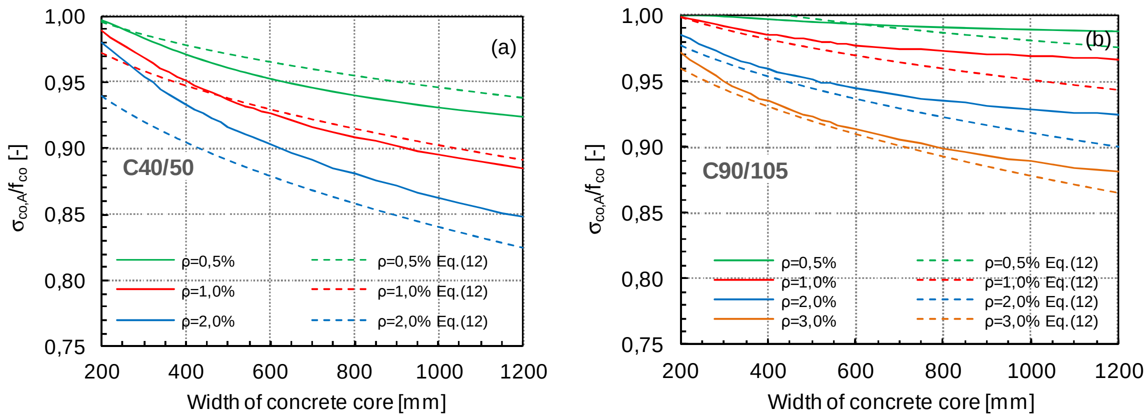

Figure 6: Relative stress of the concrete cover at spalling: (a) f_{co} = 34 MPa; (b) f_{co} = 76,5 MPa

With this assumption the strain of concrete cover at spalling can be calculated with the Eq. (3, 11). Based on the stress-strain relation according to MC 10, the stress in the concrete cover can be also determined. Fig. 6 shows the reached stress in cover at the spalling two strength classes of concrete (continuous lines). It can be seen that the stress $\sigma_{co,A}$ depends on the reinforcement ratio ρ_s, the compressive strength f_{co} and the width of concrete core. Considering these influencing parameters a relation for estimation of the stress $\sigma_{co,A}$ according to Eq. (12) is proposed, the stress $\sigma_{co,A}$ calculated with Eq. (12) is also illustrated in Fig. 6.

$$\frac{\sigma_{co,A}}{f_{co}} = 1.05 - 0.5 \cdot \left(\frac{\rho_s \cdot K_e \cdot f_{yh}}{f_{co}} \right)^{0.5} \cdot a_c^{0.4} \leq 1 \qquad a_c \text{ in m} \tag{12}$$

It is assumed that the stress in concrete core at the spalling $\sigma_{cc,A}$ equals approximately to the stress in cover $\sigma_{co,A}$. With the total cross-sectional area of the member, A_{tot}, and the area of concrete core bounded by centerline of ties or stirrups, A_{cc}, the load at spalling can be calculated as:

$$P_{c,A} = \sigma_{co,A} \cdot (A_{tot} - A_{cc}) + \sigma_{cc,A} \cdot A_{cc} \approx \sigma_{co,A} \cdot A_{tot} \tag{13}$$

To ensure the durability the load at spalling must be larger than the service load calculated from the maximum load $P_{cc} = f_{cc} \cdot A_{cc}$. That is:

$$P_{c,A} \geq P_{cc} / (\gamma_E \cdot \gamma_C) = f_{cc} \cdot A_{cc} / \gamma \tag{14}$$

Where γ_E, γ_C are the partial safety factors for actions and the material concrete.

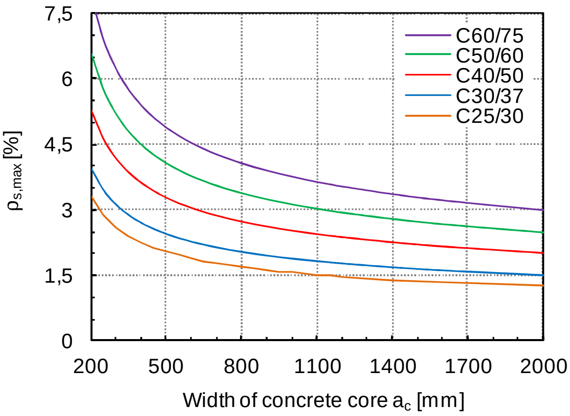

Figure 7: Maximum reinforcement ratio $\rho_{s,max}$

Inserting Eq. (1), (12), (13) in Eq. (14) the maximum reinforcement ratio $\rho_{s,max}$ can be drawn.

$$\rho_{s,\max} = \frac{1}{64K_e} \cdot \frac{f_{co}}{f_{yh}} \cdot \left[\sqrt{\left(0.5 \cdot a_c^{0.4} \cdot \frac{A_{tot}}{A_{cc}} \cdot \gamma\right)^2 + 16.8 \frac{A_{tot}}{A_{cc}} \cdot \gamma - 16} - 0.5 \cdot a_c^{0.4} \cdot \frac{A_{tot}}{A_{cc}} \cdot \gamma \right]^2 \tag{15}$$

The maximum reinforcement ratio $\rho_{s,max}$ for a member with $c_{nom} = 25\ mm$, $f_{yh} = 500\ MPa$ and $K_e = 0,6$ calculated with Eq. (15) is illustrated in Fig. 7. In the calculation a total safety factor $\gamma = \gamma_E \cdot \gamma_C = 2,1$ is used. It is clearly that maximum reinforcement ratio increases with the compressive strength and decreases with the width of the concrete core. In general, a lateral reinforcement ratio of more than 3% can hardly be arranged, so the risk of spalling of the concrete cover under service loads for members made of high-strength concrete is avoided.

4. CONCLUSION

In contrast to the triaxial compression test, the confinement in R/C members depends strongly on the deformation capacity of the concrete in lateral direction, the reinforcement ratio and the material law of confining reinforcement. Based on the compatibility of deformations between confined concrete core in lateral direction and confining reinforcement the actual stress in confining reinforcement can be determined.

Considering that the confining reinforcement should at the latest yield when the triaxial compressive strength is reached for an effective confinement, the minimum ratio of the confining reinforcement is introduced. It is shown that high-strength steels are less suitable for confining reinforcement.

By comparing the transverse strain in cover and core concrete, the phenomenon of spalling of the concrete cover was cleared. In view of the durability of the R/C members an upper limit for the ratio of confining reinforcement was determined depending on the concrete strength and the cross-sectional width.

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