# EVALUATION ON PREDICTION ERROR OF CORROSION INITIATION TIME DEPENDING ON NUMBER OF CONCRETE CORES

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## ABSTRACT

In practical maintenance work, chloride ion penetration in a concrete structure has generally been evaluated with chloride ion profiles defined by a few concrete cores sampled. However, chloride ion penetration has considerable variation even in the same structure. In this paper, the variation of chloride ion penetration is evaluated by using statistical analysis. Error reduction for evaluation of chloride penetration ion prediction is discussed with the number of concrete cores sampled. It is made clear that 1) as for the variations of chloride ion penetration, surface chloride ion contents and apparent diffusion coefficients conform to the Weibull and the Gumbel distributions, respectively and 2) the prediction error of corrosion beginning time is quantified with the number of concrete core sampled.

**Keywords:** Core sampling, chloride ion penetration, reliability evaluation, variation.

## 1. INTRODUCTION

Due to various reasons, deterioration progress shows high diversity, which reveals various aspects even in one structure or in one structural member (Yokota 2006, Kato et al. 2009). It is important to well treat the variation in assessment of structures and in deterioration prediction during the life-cycle management procedure (Yokota and Hashimoto 2013). Therefore, it is important to discuss the variations in this prediction and to quantify the variations to take into account variations in the calculation parameters used for the prediction.

To estimate the corrosion beginning time of a steel bar in concrete, chloride ion content, apparent diffusion coefficient and concrete cover are needed to determine for calculating the chloride ion accumulation at the steel bar. However, as mentioned earlier, their values may have considerable variation. Since one or a few concrete cores are sampled from structural members to determine them in practical maintenance work, the range of such variations should be well understood.

In this paper, chloride ion profiles in an existing reinforced concrete slab located in the splash zone are investigated at first. Thirty concrete cores are sampled from the slab, and surface chloride ion

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contents and apparent diffusion coefficients are measured in the laboratory. Then, the probability functions of those values are discussed. The degree of conformance of those values to the normal, the lognormal, the Gumbel, and the Weibull distributions is examined. After determining the probability functions, the variation in corrosion beginning time of a steel bar is quantified and discussed by using the Mote Carlo simulation.

## 2. EXPERIMENTAL AND ANALYTICAL PROCEDURES

### 2.1. Chloride ion measurement

A part of the reinforced concrete slab is cut out from the superstructure of the open-piled wharf aged about 40 years as shown in Figure 1. The test slab measures about 400 mm wide, 1500 mm long, and 300 mm thick as shown in Figure 2. Deformed steel bars of 13 mm in diameter are embedded at the depth of about 50 mm. While mix proportions of the concrete are not clear, the maximum aggregate size is 25 mm. A total of 30 concrete cores of 33.4 mm in diameter are taken out from the non-deteriorated parts of the test slab. Phenolphthalein spray showed that carbonation of the concrete has little progress.

The concrete core is sliced with 10 mm thick from its exposed surface and is ground to be fine powder (particle size less than 105 μm). The chloride ion content in the powder sample is measured. The surface chloride ion content, $C_0$ and apparent diffusion coefficient, $D_{ap}$ are estimated by fitting Fick's 2nd law of diffusion as shown in Equation (1). By using the equation, the corrosion beginning time is estimated as the elapsed time until the concrete chloride content at the steel bar reaches 2.0 kg/m$^3$.

$$C(x,t) = C_0\left(1 - erf\frac{x}{2\sqrt{D_{ap}\cdot t}}\right) \tag{1}$$

Where $C(x,t)$ is the chloride ion content at depth $x$ and time $t$, $C_0$ is the surface chloride ion content, $D_{ap}$ is the apparent diffusion coefficient, and $erf$ is the error function.

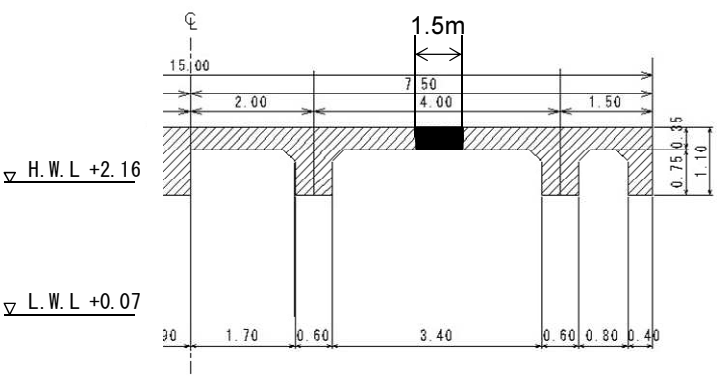


**Figure 1: Superstructure of the open-piled wharf**

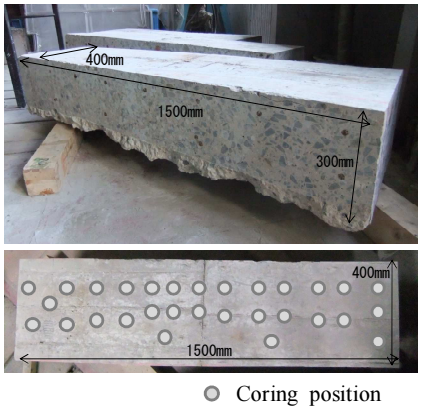


**Figure 2: Test slab**

### 2.2. Kolmogorov-Smirnov test

The Kolmogorov-Smirnov test is a technique for evaluating the degree of coincidence with a theoretical distribution form. Namely, the distribution of sampled data ($x_1$, $x_2$, ..., $x_{n-1}$, and $x_n$) can be evaluated whether they fit to the assumed probability density function $f(x)$.

Taking null hypothesis that sample $x_n$ comes from the probability density function $f(x)$ with the set of sampled data ($x_1$, ..., $x_n$), the cumulative distribution function $S_n(x)$ and the assumed cumulative distribution function $F(x)$ which is predicted by fitting in the presumed distribution are obtained. If $D_{max}$ obtained from Equations (2) and (3) is greater than the K.S. statistic, the above null hypothesis can be rejected. The significant level is set at 5% in this calculation.

$$D_i = \left| S_n(x) - F(x) \right| \tag{2}$$

$$D_{\max} = \max\left[D_1, D_n\right] \tag{3}$$

Where $D_i$ is the difference between $S_n(x)$ and $F(x)$, $D_{max}$ is the maximum value of $D_i$, $S_n(x)$ is the cumulative distribution of sample data, and $F(x)$ is the assumed cumulative distribution function.

## 3. CHLORIDE ION PENETRATION AND CORROSION INITIATION TIME

Figure 3 shows the relationship between surface chloride ion content and apparent diffusion coefficient. As shown in the figure, the chloride ion penetration varies widely even in a structural concrete member. Some parts of the test slab contain much chloride ion and others do not even in the small area (1500 mm × 400 mm) of the test beam. Such differences may be cause by local environmental conditions, non-uniformity of materials, and errors in sampling and measurement.

Figure 4 shows the frequency distribution of surface chloride ion content ($C_0$). The surface chloride ion contents vary from 4.9-20.6 kg/m$^3$ (4 times or more). Figure 5 shows the frequency distribution of apparent diffusion coefficient ($D_{ap}$). The apparent diffusion coefficients vary from 0.11-1.88 cm$^2$/year (16 times or more).

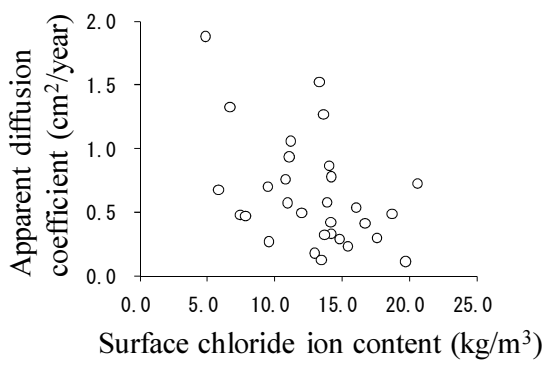


**Figure 3: Relationship between $C_0$ and $D_{ap}$**

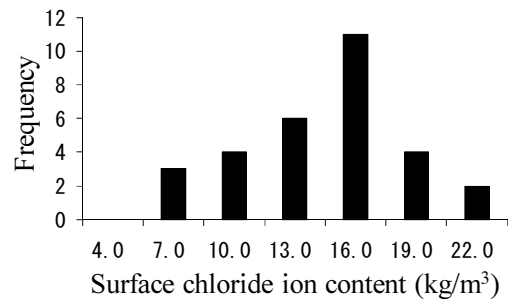


**Figure 4: Frequency distribution of $C_0$**

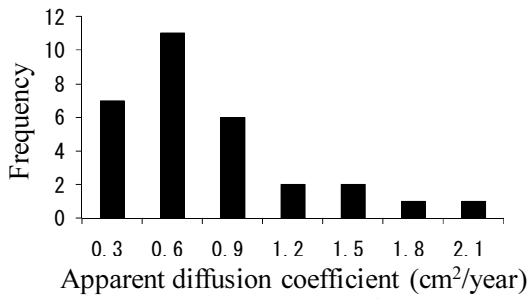


**Figure 5: Frequency distribution of $D_{ap}$**

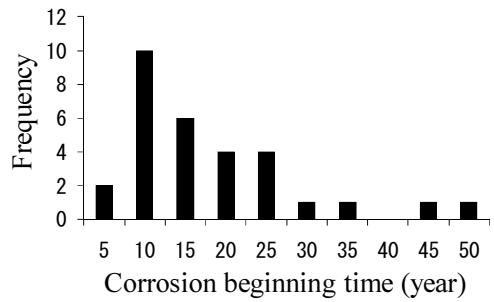


**Figure 6: Frequency distribution of $T$**

By using a set of $C_o$ and $D_{ap}$, the corrosion beginning time can be predicted. The frequency distribution of the corrosion beginning time ($T$) is shown in Figure 6. It also has the considerable variation and the most frequent range is 10-25 years, while its latest and earliest years are 47 years and 4 years, respectively. In other words, the latest corrosion beginning time is predicted to be 11 times or more than the earliest corrosion beginning time. It implies that the prediction of corrosion beginning time or even that of life time may not be easily concluded by using a small number of concrete cores sampled.

## 4. EVALUATION OF THE CHLORIDE ION PENETRATION

The degree of conformance of probability distributions of surface chloride ion content and apparent diffusion coefficient to the normal, the lognormal, the Gumbel, and the Weibull distributions is discussed. The degree of conformance is evaluated by the Kolmogorov-Smirnov test as mentioned earlier.

Figure 7 shows $D_i$ in Equation (2) of each distribution on the surface chloride ion content. Their maximum values, $D_{max}$ of all the distributions are smaller than the K.S. statistic (0.2417). When the Weibull distribution is applied, $D_{max}$ of surface chloride ion content is the smallest. Therefore, it can be said that the Weibull distribution is the most suitable distribution to describe the surface chloride ion probability distributions. Moreover, the Gumbel distribution and the normal distribution are probably applicable as well.

Figure 8 shows $D_i$ of each distribution on the apparent diffusion coefficient. Their maximum values, $D_{max}$ of all the distribution are smaller than the K.S. statistic. When the Gumbel distribution is applied, $D_{max}$ of apparent diffusion coefficient is the smallest. Therefore, it can be said that the Gumbel distribution is the most suitable distribution to describe the apparent diffusion coefficient probability distributions. Moreover, the Weibull distribution and the lognormal distribution are applicable.

The prediction error will be discussed later by using the suitable probability distributions that are found in the above analyses.

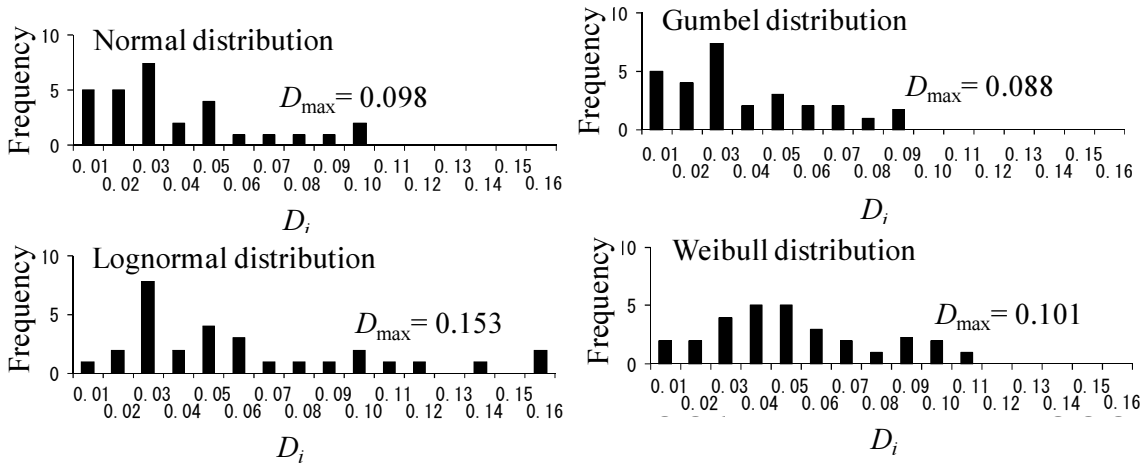


**Figure-7: $D_i$ distribution of surface chloride ion content ($C_0$)**

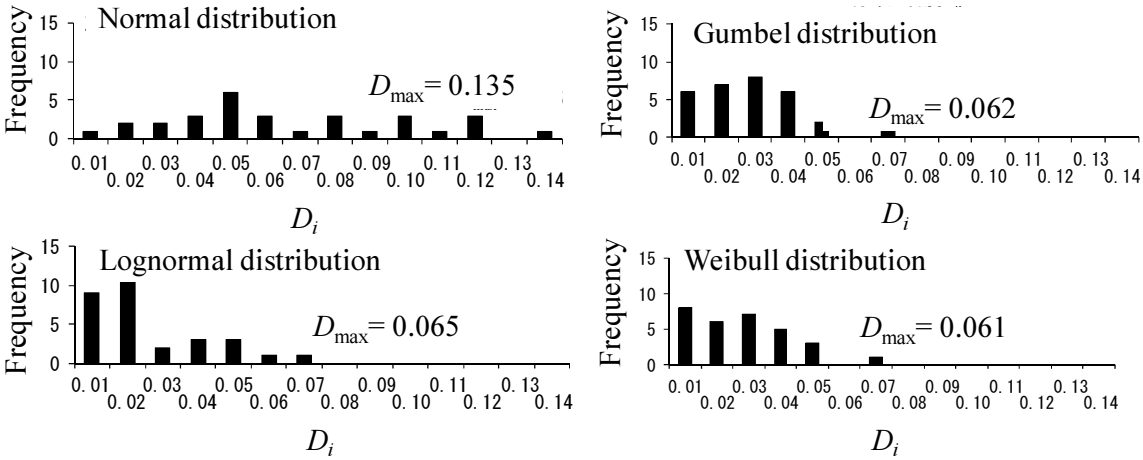


**Figure 8: $D_i$ distribution of apparent diffusion coefficient ($D_{ap}$)**

## 5. QUANTIFICATION OF PREDICTION ERROR

### 5.1. Method of calculation

The mean value errors of the surface chloride ion content and the apparent diffusion coefficient with the number of core are discussed by using the Monte Carlo simulation. Equation (4) is used to calculate the mean value error. During the calculation, uniform random numbers are generated by using the Mersenne Twiser (Matsumoto et al. 1998).

$$E_j = \frac{1}{10000} \sum_{i=1}^{10000} \left| \mu_{30} - \mu_{i,j} \right| \tag{4}$$

Where $E_j$ is the mean value error, $\mu_{30}$ is the mean value of 30 cores, and $\mu_{i,j}$ is the mean value of $i$-th data set under the total of $j$ cores.

Random numbers of the normal distribution and the lognormal distribution are generated by using Equations (5), (6) and (7) with uniform random numbers. Random numbers of the Weibull

distribution and the Gumbel distribution are generated by using Equations (8) and (9) with uniform random numbers.

$$Z = \sqrt{-2\ln U_1} \cdot \sin(2\pi U_2) \tag{5}$$

$$x_N = \sigma \cdot Z + \mu \tag{6}$$

$$x_{ln} = \exp(\delta \cdot Z + \xi) \tag{7}$$

$$x_w = \beta \left( \ln \frac{1}{1-U_3} \right)^{1/\alpha} \tag{8}$$

$$x_g = \alpha - \beta \ln \ln \frac{1}{U_3} \tag{9}$$

Where $Z$ is the normal random number, $U_1$, $U_2$ and $U_3$ are uniform random numbers, $\mu$ is the average value, $\sigma$ is the standard deviation, $\delta$ is the log-average value, $\xi$ s the log-standard deviation, $\alpha$ and $\beta$ are the distributional parameters, and $x_n$, $x_{ln}$, $x_w$ and $x_g$ are generated random numbers on each distribution.

The corrosion beginning time is estimated by Equation (1) and the mean values of the surface chloride ion content and the apparent diffusion coefficient obtained with 30 cores. Accordingly, the time predicted with 30 cores, $T_{30}$ is a true value in the range of the data obtained in this study. In addition, the error of corrosion beginning time is evaluated by $C_0 \pm E^c$ and $D_{ap} \pm E^d$. That is, the upper limit of corrosion beginning time is calculated in case of $C_0 - E^c$ and $D_{ap} - E^d$, while its lower limit is calculated in case of $C_0 + E^c$ and $D_{ap} + E^d$. In this paper, the results in the case of 1, 2, 4 and 6 cores are discussed.

### 5.2. Error reduction with number of cores sampled

Figure 9 shows the mean errors of the surface chloride ion content with the number of concrete cores sampled. They decrease with an increase in the core sampling numbers. When the Gumbel and the Weibull distributions are applied, the errors are almost equal. The mean errors of 2 cores are 2.0-2.5 kg/m$^3$ (20% of the mean value on 30 cores), and are 1.5-1.8 kg/m$^3$ (15% of the mean value on 30 cores) in case of 4 cores. When 6 cores can be taken, the mean error is reduced to 1.2 kg/m$^3$, which is about 10% of the mean value of 30 cores.

The mean errors of the apparent diffusion coefficient with the number of concrete cores sampled are shown in Figure 10. They decrease with an increase in the number of cores sampled. When the Weibull and the normal distributions are applied, the errors are almost equal. The mean errors of 2 cores are 0.20-0.26 cm$^2$/year (40% of the mean value on 30 cores), and are 0.15-0.19 cm$^2$/year (30% of the mean value on 30 cores) in case of 4 cores. When 6 cores are taken, the mean errors are 0.13-0.16 cm$^2$/year (20% of the mean value on 30 cores).

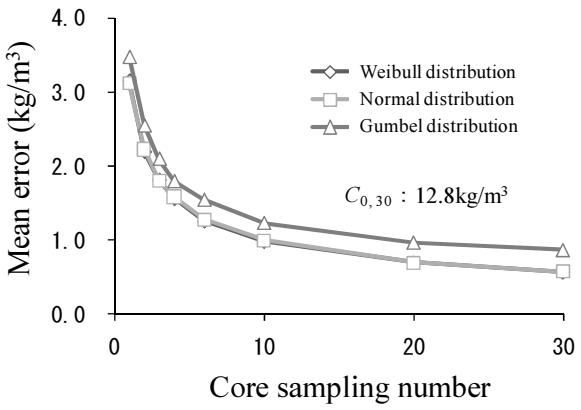


**Figure 9: Mean error of $C_0$**

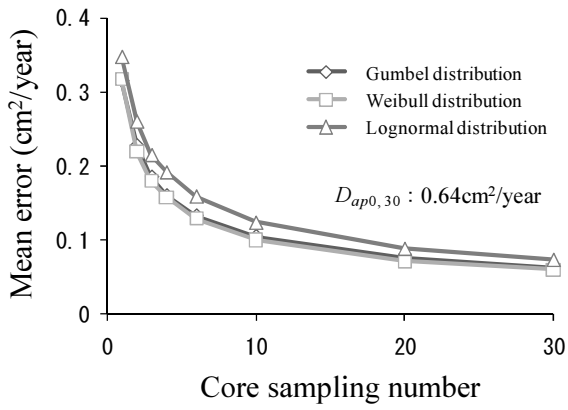


**Figure 10: Mean error of $D_{ap}$**

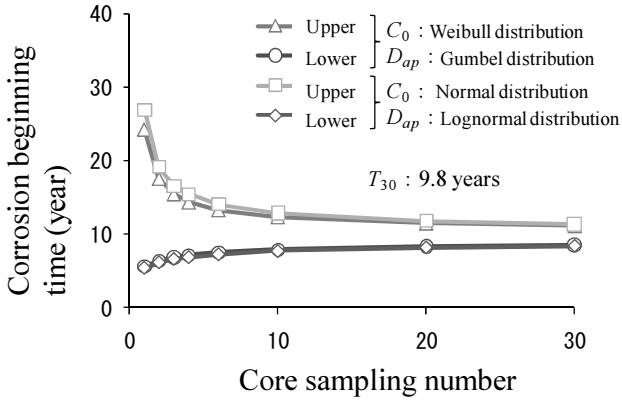


**Figure 11: Expected errors in corrosion beginning time with numbers of cores**

Figure 11 shows the prediction error of the corrosion beginning time. Its prediction errors decrease with an increase in the number of cores sampled. In particular, in case 2 cores, the prediction error becomes about 50% of $T_{30}$. In case of 4 cores, the prediction error is about 40% of $T_{30}$, while it is about 30% of $T_{30}$ in case of 6 cores. Compared to the case that the Weibull distribution and the Gumbel distribution are applied for $C_0$ and $D_{ap}$ respectively, which shows the smallest error, the difference of prediction errors is considerably small when other sets of distribution functions are applied (10% difference at the maximum). Therefore, the surface chloride ion content and the apparent diffusion coefficient should be assumed to follow the normal distribution and the lognormal distribution, respectively.

In a structural member, considerable variation may exist in chloride ion profiles of concrete. During practical investigation of existing structures, the chloride ion profile of concrete has generally been estimated based on one or a few cores sampled. However, it seems that such a few numbers of concrete cores may not be representative.

## 6. CONCLUSIONS

The following conclusions are drawn in this paper:

1) The chloride ion penetration shows considerable variation in a structural concrete member. Based on widely varied surface chloride ion content and apparent diffusion coefficient measured in a reinforced concrete slab located in a sea splash zone, the latest calculated corrosion beginning time is about 11 times or more than the earliest one.

2) Applying the Weibull distribution and the Gumbel distribution for $C_0$ and $D_{ap}$ predictions respectively, the smallest error is obtained for the prediction of corrosion beginning time. For easier application, however, they can be modeled with the normal distribution and the lognormal distribution respectively to obtain a reasonable prediction.

3) The relationship between the number of concrete cores sampled and the mean value error of surface chloride ion content and apparent diffusion coefficient is made clear. Furthermore, based on those results, the prediction error of corrosion beginning time can be identified with the number of concrete core sampled.

## 7. ACKNOWLEDGMENTS

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