DYNAMIC RESPONSE OF STRUCTURES LOADED BY OUTSIDE BLASTS

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ABSTRACT

The paper deals with the dynamic response of damped orthotropic reinforced concrete plates subjected to blast loading. Blast loading is a short duration load categorized as an impulsive loading. Mathematically blast loading is treated as a triangular one. The natural frequency of vibration of a plate governs its response to such loading. A rectangular reinforced concrete plate is considered for the numerical analyses with fixed boundary conditions. The natural frequencies of the system are solved from a system of two transcendental equations. The dynamic response of the structure is then determined through the superposition of natural modes. Special emphasis is focused on the mid-point displacements of the plate. The results obtained give an insight into the response of the plate under blast loading, indicating the effect of the thickness of the reinforced concrete plate and the time duration of the blast loading on its overall behavior.

Keywords: blast loading, triangular loading, concrete plates, transcendental equations, thickness, time duration.

1. INTRODUCTION

The study of blast effects on structures has received widespread attention in recent years. A bomb explosion within or immediately nearby a building can cause catastrophic damage on the building's external and internal structural frames, collapsing walls, blowing out large expanses of windows, and shutting down critical life-safety systems. Loss of life and injuries to occupants can result from many causes, including direct blast-effects, structural collapse, debris impact, fire and smoke. The indirect effects can prevent timely evacuation, thereby contributing to additional casualties. In addition, major catastrophes from gas-chemical explosions had resulted in large dynamic loads, greater than the original design loads, of many structures.

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Theoretical solutions for plates subjected to blast loading are rare. Schubak et al. (Schuback et al. 1989) and Olson (Olson 1991) studied the response of a one-way stiffened plate under intense loads, in which the stiffened plate was treated as a single symmetric beam with the plate acting as a large flange. In 2003 Yuen and Nurick (Yuen and Nurick 2003) presented the numerical results of a quadrangular plate with different stiffener configurations under blast loading. Extensive experimental studies had also been conducted to assess numerical simulation (Pan and Louca 1999, Jacob et al. 2004). Stiffened plates were among the most common considered structural elements.

In this paper the analysis of thin fixed supported damped orthotropic reinforced concrete plates subjected to blast loading is presented. The presented numerical results may serve as design guidelines of damped orthotropic plate structures under blast loading. It should be noted that explosive tests are costly and dangerous, whereby their reproducibility is not always ensured and the results of the test always show some degree of uncertainty.

2. THE GOVERNING EQUATION OF ORTHOTROPIC PLATE

In Figure 1 the coordinate system, plate dimensions and loading are shown. Common assumptions are made to simplify the mathematical model of a Kirchhoff plate. These assumptions are (1) the strain component ε_z in the perpendicular direction of the plate is sufficiently small such that it can be ignored; (2) the stress components τ_{zx}, τ_{zy} and σ_z are far less than the other stress components, so that the resulting deformation can be neglected; and (3) the displacement parallel to the horizontal direction of the plate is zero (E. J. Yoder and M. W. Witezak 1975).

Based on these assumptions and the fundamental equation of Newton's second law, the equation of motion of the undamped system can be written as:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} - \rho h \omega^2 w = 0 \tag{1}$$

where D_x is the flexural stiffness in the x direction, H is the torsional stiffness, D_y is the flexural stiffness in the y direction, ρ is the mass density, h is the thickness of the plate and ω is the natural frequency of the plate. In this study the modified Bolotin method is used to solve Eq. (1) for one set of boundary condition associated with a plate with all sides fixed, for which the boundary conditions are given by:

$$w = \frac{\partial w}{\partial x} = 0 \quad \text{at x=0 and x=a} \tag{2}$$

$$w = \frac{\partial w}{\partial y} = 0 \quad \text{at y=0 and y=b} \tag{3}$$

In the modified Bolotin method an eigen mode is initially approximated by a general solution consisting of trigonometric functions (Pevzner et al. 2000):

$$w_{mn} = W(x, y)\sin\omega t \tag{4}$$

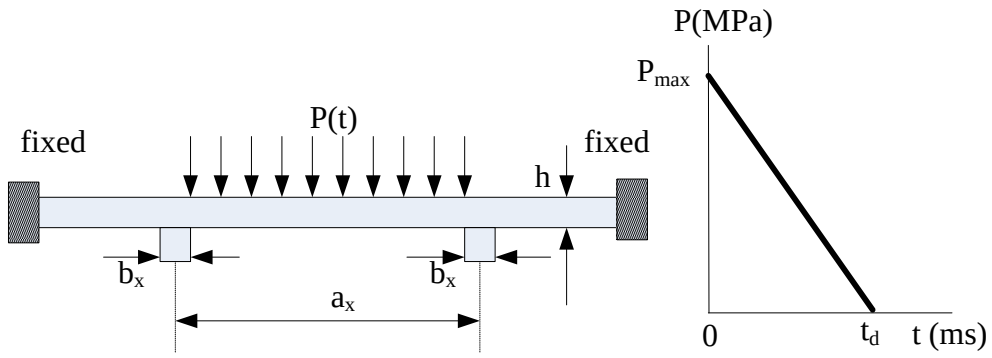


Figure 1: Orthotropic plate subjected to triangular blast load.

By substituting Eq. (4) into Eq. (1), the eigen frequency of the system is given by:

$$\omega^2 = \left(\frac{H}{\rho h}\right)\left[\frac{D_x}{H}\left(\frac{p\pi}{a}\right)^4 + 2\left(\frac{p\pi}{a}\right)^2\left(\frac{q\pi}{b}\right)^2 + \frac{D_y}{H}\left(\frac{q\pi}{b}\right)^4\right] \tag{5}$$

where p, q are real numbers such that $m \le p \le m+1$ and $n \le q \le n+1$ which are to be determined.

The next step is then to solve two auxiliary Levy-type problems.

2.1. First auxiliary Levy-type problem

The solution of Eq. (1) is assumed as

$$w_{mn} = X(x)\sin\frac{\pi q y}{b} \tag{6}$$

satisfying the boundary conditions

$$X = \frac{dX}{dx} = 0 \quad \text{at x=0 and x=a} \tag{7}$$

Substituting Eq. (6) into Eq. (1) with ω from Eq. (5) results in an ordinary differential equation for X(x):

$$X(x) = cosh(F_1 x) + \left(\frac{bp\left(c_1 - C_1\right)}{-\beta s_1 + bpS_1}\right) sinh(F_1 x) - cos(F_2 .x) - \left(\frac{-\beta c_1 + \beta C_1}{\beta s_1 - bpS_1}\right) sin(F_2 x) \tag{8}$$

where:

$$\beta = \sqrt{\frac{2Hq^2a^2}{D_x} + p^2b^2}\ ;\ \ F_1 = \frac{\beta\pi}{ab};\ \ F_2 = \frac{p\pi}{ab};\ \ C_1 = cosh\left(\frac{\pi\beta}{b}\right);\ \ c_1 = cos\left(p\pi\right);$$

$$S_1 = sinh\left(\frac{\pi\beta}{b}\right);\ \ s_1 = sin\left(p\pi\right)$$

2.2. Second auxiliary Levy-type problem

For the second auxiliary Levy-type problem the solution of Eq. (1) is assumed as:

$$w_{mn} = \sin\frac{\pi px}{a}Y(y) \tag{9}$$

satisfying the boundary conditions

$$Y = \frac{dY}{dy} = 0 \quad \text{at y=0 and y=b} \tag{10}$$

Substituting Eq. (9) into Eq. (1) with ω from Eq. (5) results in an ordinary differential equation for Y(y):

$$Y(y) = cosh(F_3y) + \left(\frac{bq\left(c_2 - C_2\right)}{-\theta s_2 + bqS_2}\right)sinh(F_3y) - cos(F_4y) - \left(\frac{-\theta c_2 + \theta C_2}{\theta s_2 - bqS_2}\right)sin(F_4y) \tag{11}$$

where:

$$\theta = \sqrt{\frac{2Hp^2b^2}{D_y} + q^2a^2}\ ;\ \ F_3 = \frac{\theta\pi}{ab};\ \ F_4 = \frac{q\pi}{ab};\ \ C_2 = cosh\left(\frac{\pi\theta}{a}\right);\ \ c_2 = cos\left(q\pi\right)$$

$$S_2 = sinh\left(\frac{\pi\theta}{a}\right);\ \ s_2 = sin\left(q\pi\right)$$

The unkwown quantities *p* and *q* are calculated by solving the system of transcendental equations as follows (Alisjahbana, S.W. and Wangsadinata, W. 2012):

$$\frac{\pi}{a^2b^2}\left(2bp\beta - 2bp\beta cos[p\pi]cosh\left[\frac{\pi\beta}{b}\right] - (b^2p^2 - \beta^2)sin[p\pi]sinh\left[\frac{\pi\beta}{b}\right]\right) \tag{12}$$

$$\frac{\pi}{a^2b^2}\left(2aq\theta - 2aq\theta cos[q\pi]cosh\left[\frac{\pi\theta}{a}\right] - (a^2q^2 - \theta^2)sin[q\pi]sinh\left[\frac{\pi\theta}{a}\right]\right) \tag{13}$$

The mode shapes are determined as the product of Eq. (8) and Eq. (11).

3. DYNAMIC ANALYSIS

The natural frequencies of vibration are computed from the equation of displacements:

$$w(x,y,t)=\sum_{m=1}^{m}\sum_{n=1}^{n}\left[e^{-\gamma\omega_{mn}t}\left(a_{mn}\cos\left(\sqrt{1-\gamma^2}\omega_{mn}\right)t+b_{mn}\sin\left(\sqrt{1-\gamma^2}\omega_{mn}\right)t\right)\right]+$$

$$\sum_{m=1}^{m}\sum_{n=1}^{n}\left[X(x)Y(y)\frac{\int_0^a X(x)dx\int_0^b Y(y)dy}{\rho hQ_{mn}\left(\sqrt{1-\gamma^2}\omega_{mn}\right)}\int_0^t\left[p(x,y,\tau)e^{-\gamma\omega_{mn}(t-\tau)}\sin\left(\sqrt{1-\gamma^2}\omega_{mn}\right)(t-\tau)\right]d\tau\right] \quad (14)$$

where a_{mn}, b_{mn} are constants that are determined from the initial conditions, γ is the damping ratio, Q_{mn} is the orthogonally factor; and $p(x,y,\tau)$ is the dynamic load.

The orthogonally factor Q_{mn} in Eq. (14) can be expressed as:

$$Q_{mn}=\int_0^a\int_0^b X_{mn}{}^2(x)Y_{mn}{}^2(y)dxdy \quad (15)$$

The dynamic load $p(x,y,t)$ for blast loading is idealized by a linearly decaying pressure time history (Kadid 2008):

$$P(t)=P_{\max}\left(1-\frac{t}{t_d}\right) \quad (16)$$

By using the convolution integral in Eq. (14) the idealized blast loading can be solved analitically.

4. RESULTS AND DISCUSSIONS

The adopted material properties for the orthotropic plate are a=4.5 m, b=5.5 m, E= 210 GPa, υ= 0.2, a_x=2.75 m, b_x= 0.2 m, P_{max} = 1.3 MPa (Kadid 2008). In order to study the effect of time duration, three time intervals t_d have been used in this study as follows: 2 ms, 10 ms and 20 ms. On the other hand to study the effect of plate thickness, three models have been used in this work as follows: model 1, h=16 cm; model 2, h=18 cm and model 3, h= 20 cm. Modified Bolotin Method (MBM) had been applied to obtain the natural frequencies of the orthotropic plate. The first 4 modes in the x direction (n=1,2,..4) and the first 5 modes in the y direction (m=1,2,...,5) of the orthotropic plate are shown in Table 1 for different values of plate thickness.

Table 1: Fundamental frequencies and number of mode

m	n	ω (rad/s) h=16 cm	ω (rad/s) h=18 cm	ω (rad/s) h=20 cm	ω (rad/s) h=16 cm	ω (rad/s) h=18 cm	ω (rad/s) h=20 cm
		without stiffener			1 stiffener		
1	1	763.342	832.778	904.76	829.531	893.74	961.469
	2	1255.87	1390.59	1528.14	1308.92	1437.34	1570.04
	3	2084.51	2325.11	2568.6	2129.28	2363.4	2602.03
	4	3218.53	3602.15	3988.65	3258.88	3635.98	4017.7
	5	4663.88	5227.79	5794.66	4704.42	5261.51	5823.47
2	1	1840.36	2006.38	2168.48	2046.41	2189.08	2340.52
	2	2291.52	2502.54	2721.5	2477.16	2671.85	2877.63
	3	3058.21	3350.39	3684.64	3230.79	3519.97	3822.53

	4	4146.58	4593.35	5049.93	4307.59	4732.86	5173.07
	5	5544.57	6167.82	6801.3	5696.77	6297.33	6913.84
3	1	3493.83	3781.65	4081.89	3880.92	4145.02	4425.38
	2	3917.76	4256.82	4610.15	4297.46	4608.28	4938.39
	3	4647.39	5076.68	5522.48	5015.2	5410.67	5829.28
	4	5692.1	6252.19	6830.7	6046.07	6567.21	7115.1
	5	7050.52	7781.3	8532.02	7391.2	8078.97	8796.54
4	1	5685.27	6150.13	6635.1	6327.03	6753.78	7206.67
	2	6102.93	6617.08	7153.41	6738.69	7209.71	7710.22
	3	6813.91	7413.73	8038.0	7439.43	7989.16	8573.11
	4	7829.14	8553.44	9305.81	8441.29	9107.9	9814.01
	5	9153.44	10041.9	10961.2	9750.78	10574.5	11442.7

Table 1 shows the natural frequencies of the orthotropic plates for different concrete plate thickness. It is shown that by increasing the plate thickness, the natural frequency of the system increases.

4.1. Effect of plate thickness

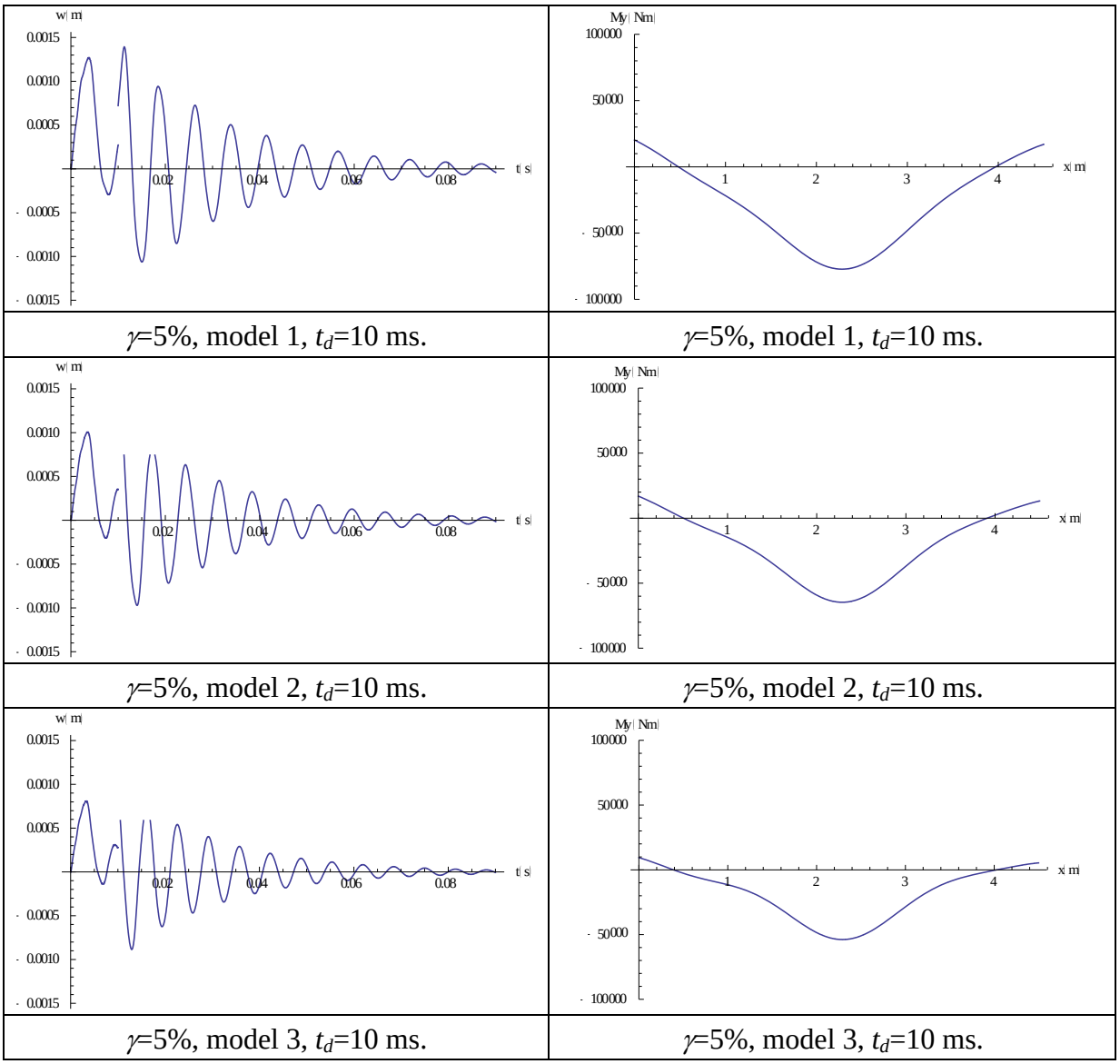
By increasing the thickness of the concrete plate, the mid-point displacement will decrease significantly; the mid-point displacement for model 1, t_d=10 ms, γ=5% is 0.00126834 m, the mid-point displacement for model 2, t_d= 10 ms, γ=5% is 0.00100618 m, while the mid-point displacement for model 3, t_d= 10 ms, γ=5% is 0.000808496 m. Therefore the plate thickness plays a very important role in reducing the dynamic deflection of the system. Increasing the thickness of the concrete plate has also resulted in a decrease of the distribution of the internal moment in the y direction (M_y) along the x axes for all values of thickness considered in this study, as shown in Figure 2. Therefore, the thickness of the concrete plate also plays an important role in determining the level of response of the orthotropic plate.

4.2. Effect of time duration

For model 1 increasing the time duration by a factor of 5 from t_d=2 ms to t_d=10 ms has resulted in an increase in the mid-point displacement by a factor of 2.33. For model 2 increasing the time duration by a factor of 5 from t_d=2 ms to t_d=10 ms has resulted in an increase in the mid-point displacement by a factor of 2.29, as shown in Table 2.

4.3. Effect of plate stiffener

As shown in Table 2, for model 1 with t_d=10 ms adding 1 stiffener parallel to the x axes, has resulted in a decrease in the mid-point displacement of the plate by 17.68% compare to the mid-point displacement of the plate without stiffener. For model 1 with t_d=10 ms adding 2 stiffeners parallel to the axes, has resulted in a decrease in the mid-point displacement of the plate significantly by 40.57% compare to the mid-point displacement of the plate without stiffener. The internal forces of the plate subjected to the blast loading for a plate without stiffener and a plate with stiffener can be seen in Figure 3.



γ=5%, model 1, t_d=10 ms. | γ=5%, model 1, t_d=10 ms.

γ=5%, model 2, t_d=10 ms. | γ=5%, model 2, t_d=10 ms.

γ=5%, model 3, t_d=10 ms. | γ=5%, model 3, t_d=10 ms.

Figure 2. Dynamic deflection time history at mid-point and moment-y (M_y) distribution along x axes subjected to blast load.

Table 2: The maximum dynamic deflection of a damped orthotropic plate (γ=5%) subjected to a blast loading as a function of t_d.

h (cm)	t_d= 10 ms w max (m)	t_d=20 ms w max (m)	t_d=10 ms w max (m)	t_d= 20 ms w max (m)	t_d=10 ms w max (m)	t_d= 20 ms w max (m)
	without stiffener		1 stiffener		2 stiffeners	
16	0.00154078	0.00177352	0.00126834	0.00144454	0.000915634	0.00102986
18	0.00119563	0.00135706	0.00100618	0.00113385	0.000749621	0.000834418
20	0.000941155	0.00105514	0.000808496	0.000902103	0.000617292	0.000681519

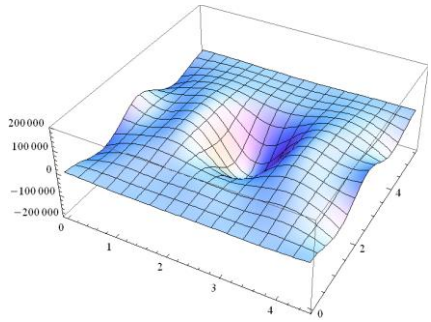
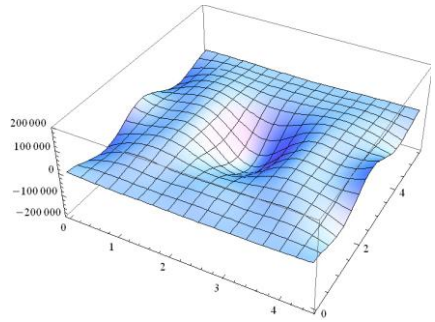
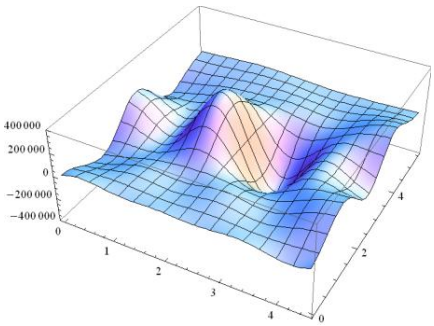
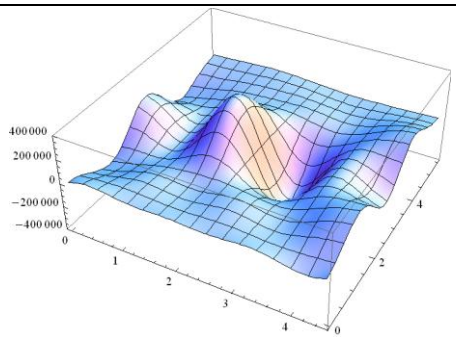
	
M_x distribution –plate without stiffener	M_x distribution –plate with 1 stiffener
	
Q_x distribution –plate without stiffener	Q_x distribution – plate with 1 stiffener

Figure 3. M_x and Q_x distribution for model 2 without stiffener and model 2 with 1 stiffener subjected to blast loading. Parameter: γ= 5%, t_d= 10 ms.

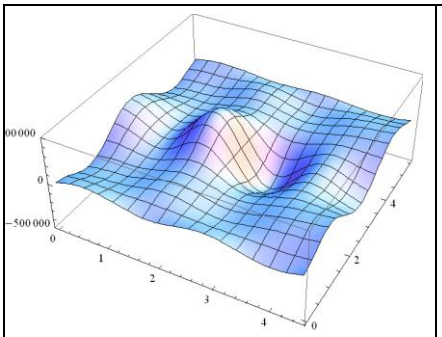
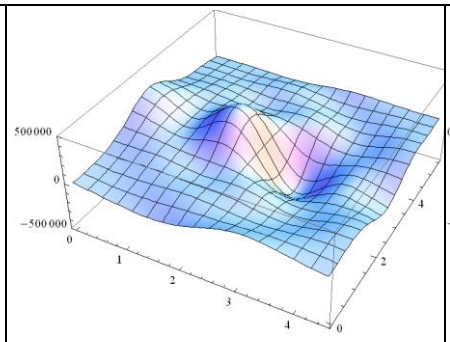
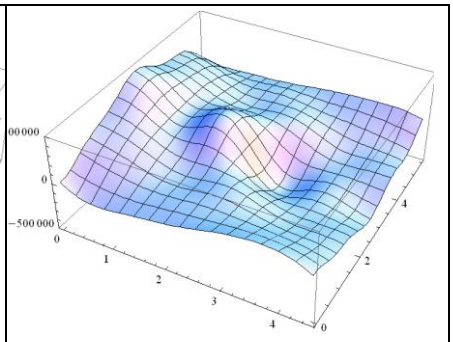
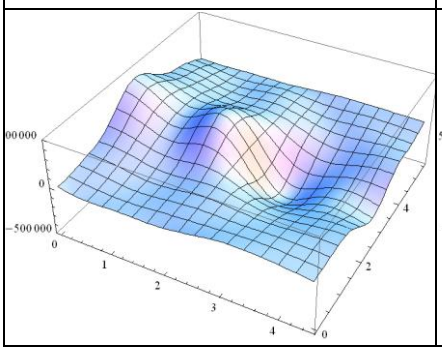
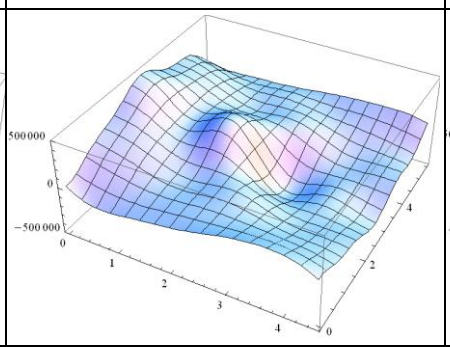
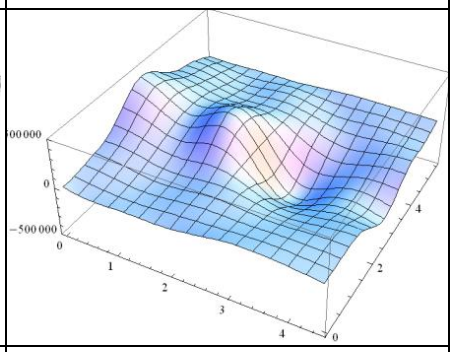
		
t=0.001s	t=0.002s	t=0.003s
		
t=0.004s	t=0.005s	t=0.006s

Figure 4. Q_x distribution along x axes and y axes for model 2 subjected to blast loading. Parameter: γ=5%, t_d=10 ms.

6 CONCLUSIONS

From the dynamic analyses of the damped orthotropic reinforced concrete plate subjected to blast loading the following conclusions can be drawn:

1. The effect of the stiffeners configuration is very important, since it affects drastically the overall behaviour of the plate especially if the number of stiffener is more than one.
2. The effect of the plate thickness is not as dominant in reducing the overall behaviour of orthotropic reinforced concrete plate as increasing the number of stiffener in the reinforced concrete plate
3. The time duration of the blast loading is one of the most important parameters since it has an influence on the dynamic deflection of the plate.

While this paper deals mainly with computational results, Kim and Nurick (Kim and Nurick 2000) reported on the experimental results on the significance of the thickness of the plate when subjected to localised blast loading. Both approaches provide satisfactory correlation and create better understanding of blast loading and the significance of the plate thickness.

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