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A STOCHASTIC MODEL FOR DESCRIBING TEMPORAL VARIATION OF DAMAGE ACCUMULATION IN TUNNEL LINING CONCRETE

OSAMU MARUYAMA¹**, ATSUSHI SUTOH¹, HIROAKI KANEKIYO², TAKASHI SATOH³ and HIROAKI NISHI³

¹ Department of Urban & Civil Engineering, Tokyo City University, Japan
² Department of Civil, Environmental and Applied System Engineering, Kansai University
³ Cold-Region Construction Engineering Research Group, Civil Engineering Research Institute for Cold Region, Public Works Engineering Research Institute, Japan

ABSTRACT

This paper propose a probabilistic model for describing temporal variation of damage accumulation in tunnel lining concrete under the effect of low temperature and snow in cold regions. The proposed model is based upon a stochastic differential equation driven by a Poisson white noise, whose solution process represents a damage accumulation.

Keywords: health monitoring, tunnel lining concrete, system identification, stochastic differential equation

1. INTRODUCTION

In many countries, there is a growing amount of deteriorating concrete infrastructures that not only affect the productivity of the society, but also has a great impact on human safety. The load resisting capacity of these structures may also degrade due to aging. As a structure deteriorates or approaches its design life, the existing condition may be quite different from that of the original system. The poor and uncontrolled durability with repairs and maintenance of these concrete structures may cause fatal accidents (Lemer & Moavenzadeh 1971, Mccullough 1976, Feighan et al.1987, Carnahan et al. 1987).

This paper propose a probabilistic model for describing temporal variation of damage accumulation in tunnel lining concrete under the effect of low temperature and snow in cold regions. The proposed model is based upon a stochastic differential equation driven by a Poisson white noise, whose solution process represents a damage accumulation.

Numerical demonstrations are carried out to examine the effectiveness and accuracy of the proposed model by using the inspected data obtained from existing tunnels.

* Corresponding author: Email: omaruya@tcu.ac.jp
† Presenter: Email: omaruya@tcu.ac.jp
2. INSPECTED DATA

Inspected data from existing tunnels in Hokkaido Island used for the identification of Ito stochastic differential equation. Inspection works have been carried out last several decades, such as crack width, crack length and crack expanse of over 180 tunnels in Hokkaido Island. The results categorized into as damage degree from 0.0 (no damage) to large number (critical damage). Figure.1 shows distribution of damage degree by the tunnel age. In the numerical analysis to evaluate the regional difference of degrading process, a set of data is classified into either one of six different regional categories, such as Japan Sea North (Area1), Japan Sea South (Area2), Pacific Ocean West (Area3), Pacific Ocean East (Area4), Sea of Okhotsk (Area5) and Inland (Area6) as show in Figure.2.

3. STOCHASTIC MODEL AND PARAMETER IDENTIFICATION

The present problem is the identification of the damage degree $X(t)$ of the tunnel concrete represented by the stochastic differential equation. We generally suppose that it’s mean behavior can be described by the following differential equation.

$$\frac{dX(t)}{dt} = \mu_0(t)g(X(t))$$  \hspace{1cm} (1)

where $g(x)$ = represents a shape function quantifying the growth rate; $\mu_0$ = represents a parameter describing a mean growth resistance. Further, we assume the random behavior of the damage growth is driven by a noise incorporated in the growth resistance.

$$\frac{dX(t)}{dt} = \{\mu_0(t) + W_z(t)\}g(X(t))$$ \hspace{1cm} (2)

where $W_z(t)$ = a stochastic process with mean zero representing the driving noise.

Equation (2) is then transformed into the following stochastic differential equation of Ito type.

$$dX(t) = \mu_0(t)g(X(t))dt + g(X(t-))dZ(t)$$ \hspace{1cm} (3)
In which \( Z(t) \) is an integrated process defined as

\[
Z(t) = \int_0^t W_z(s) \, ds
\]  

(4)

And \( X(t-) \) represents a left-continuous version of \( X(t) \) defined as

\[
X(t-) = \lim_{s \to t} X(s).
\]

Since actual data shows that the shape function \( g(X(t)) \) is proportional to \( X(t) \) (Maruyama & Sutoh et al. 2009), we assume that Eq. (3) is reduced to the following linear equation.

\[
dX(t) = \mu_0 X(t) dt + g(X(t-)) dZ(t)
\]  

(5)

The most widely used noise in constructing stochastic system is the well-known Gaussian white noise. That is, the stochastic process \( Z(t) \) in Eq. (3) is given as \( Z(t) = \sigma_B B(t) \) with positive constant \( \sigma_B \), where \( B(t) \) is a standardized Wiener process, which is a temporally homogeneous Gaussian process having independent increments with

\[
E\{B(t)\} = 0, \quad E\{B(t)B(s)\} = \min\{t,s\}
\]  

(6)

The growth equation is then given as

\[
dX(t) = \mu_0 X(t) dt + \sigma_B X(t-) dB(t)
\]  

(7)

which has been widely used in many application fields, such as for describing random fatigue crack growth in the structural reliability analysis (Tanaka 1999) or temporary random variation of stock price in the mathematical finance (Black & Scholes 1973).

By applying the Ito formula (Ito 1942), we can drive the solution of Eq. (7) as follows.

\[
X(t) = X(0) \exp\{ (\mu_0 - 0.5\sigma_B^2) t + \sigma_B B(t) \}
\]  

(8)

Since the stochastic process \( X(t) \) given by Eq. (8) is a typical example of diffusion process having continuous path. As \( X(t) \) is the time dependent lognormal process, which characterized by following mean and variance.

\[
E[\log X(t)] = E[\log X(0)] + (\mu_0 - 0.5\sigma_B^2) t
\]  

(9)

\[
\text{Var}[\log X(t)] = \text{Var}[\log X(0)] + \sigma_B^2 t
\]  

(10)

Figure 3 shows generated ten sample processes with arbitrary number of parameters \( \mu_0 = 0.05 \), \( \sigma_B = 0.223 \) and \( X(0) = 0.0 \). We can see that each sample path shows quite frequent fluctuation process without repair works, which is not acceptable for modeling the damage accumulation process.
To remove, the fluctuation of the solution process in the diffusive model given by Eq. (7), we newly propose a mathematical model by introducing a noise of another type, i.e., a Poisson white noise (Tanaka et al. 2011). The Poisson white noise is a formal derivative of a compound Poisson process C(t), expressed as

\[ C(t) = \sum_{k=1}^{N(t)} Y_k \]  

Where \( N(t) \) = a temporally homogeneous Poisson process with an intensity \( \lambda \) and \( \{Y_k\}_{k=1,2,\ldots} \) = a family of i.i.d. (identically and indecently distributed) positive random variables, whose probability distribution function, is supposed to be given.

\[ P(Y_k \leq y) = F(y) \]  

Further, it is also assumed that \( \{Y_k\}_{k=1,2,\ldots} \) is statistically independent of the Poisson process \( N(t) \).

Setting the noise \( Z(t) \) as a compensated version of \( C(t) \), i.e.,

\[ Z(t) = C(t) - \lambda q_1 t, \quad (q_1 = E\{Y_k\}) \]  

and substituting Eq.(13) into Eq.(5), we can obtain the following Ito stochastic differential equation which describe the damage accumulation.

\[ dX(t) = \mu X(t) dt + X(t-) dC(t) \]  

\[ \mu = \mu_0(t) - \lambda q_1 \]  

The first term of Eq.(14) represents continuous growth of the damage degree according to the effect of rainfalls, salt or other usual effects. On the other hand, the second term represents large-scale growth of the damage caused by unusual effects such as large-scale earthquake, serious frost damage etc. In this study we call the former growth small scale growth and the later large scale growth.
growth. The solution of Eq.(14) can be obtained in analytical form, by applying the Ito formula (Ito 1942), as

\[ X(t) = X(0) \exp\{\mu t\} \prod_{k=1}^{N(t)} (1 + Y_k) \]

(16)

The degrading process \( X(t) \) of the proposed model has following mean and variance.

\[ E[X(t)] = X(0) \exp\{\mu_0 t\} \]

(17)

\[ \text{Var}[X(t)] = X(0)^2 \exp\{2\mu_0 t\}(\exp\{\lambda q_2 t\} - 1) \]

(18)

in which \( q_2 = E[Y_k^2] \) is a second moment of i.i.d. positive random variable \( Y_k \). In this research, we assume the exponential distribution for the \( Y_k \).

\[ f_Y(y) = (1/\nu) \exp(-y/\nu) \]

(19)

We may have,

\[ q_2 = E[Y_k^2] = 2\nu^2 \]

(20)

Comparing the mean and variance of the diffusive model and the proposed model, it leads,

\[ \sigma_0^2 = \lambda q_2 \]

(21)

The accuracy and propriety of the proposed model will be examined in the numerical example.

4. NUMERICAL EXAMPLE

Numerical examples are carried out to examine the effectiveness and accuracy of the proposed model. The regional difference of degrading process is evaluated in Figure 4 (Japan Sea North), Figure 5 (Japan Sea South), Figure 6 (Inland) and Figures 7 (Pacific Ocean West). The results indicated that a damage accumulation rapidly grow up in Japan Sea South area. In this area, the arterial community lifeline road is pass along the sea side. The intense cold, the rough wave and wind with sea salt may accelerate the deterioration.

5. CONCLUDING REMARKS

A method is developed to identify degrading process of tunnel lining concrete through inspected data of existing aged tunnels. The stochastic model of degrading process is described by the Ito stochastic differential equation. Basic examples are demonstrated based on the inspected data from existing aged tunnels.
REFERENCES