A PROSPECT GAME THEORY MODEL FOR BIDDING PRICE DECISION

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ABSTRACT

The owners of construction projects typically apply a competitive bid process to award a contract to the qualified bidder whom submits the lowest price. Bid price typically includes construction cost and profit; the profit, the latter can be presents as construction cost multiplied by mark-up. For the contractors, it is very difficult to decide an optimal mark-up to win the contract and at the same time to secures sustaining profitability as high as possible. This article proposes a bidding decision model to assist contractors to select an optimal mark-up size and set appropriate bidding prices under an extremely competition circumstances.

Keywords: Bidding Decision, Mark-up Size, Cumulative Prospect Theory, Game Theory, Fuzzy Preference Relation Fuzzy Rating.

1. INTRODUCTION

Most of constructions projects are apply a competitive bidding process and the owner award the projects to the bidder which submits the lowest bid price and meets the stated specification. Bid price typically includes construction cost and profit, the latter is the primary incentive of winning and executing contracts (Dikmen et al. 2007). Profit can present as construction cost multiplies mark-up size, then the winning contractor must be able to set a mark-up size that secures the contract while sustaining profitable (Shash and Abdul-hadi 1992) So in the bidding decision process, after deciding to submit a bid, the bidders must decide what mark-up size to use on the submitted bid (Egemen and Mohamed 2008). Due to high uncertainty, intense competition, and difficult to quantifying risks, the mark-up size deciding process is very complex, which requires simultaneous assessment of a large number of highly interrelated variables to reach a decision. For the limitations of rational and information process, the decision makers can not consider all relevant variables (Deng 1994). In practice, contractors usually make bid decisions in a highly subjective manner and based on a mixture of “guts” feelings, experiences and guesswork (Ahmad and Minkarah 1988),

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thus may actually lack a foundation in reason (Ahmad 1990). The typically emphasize factors like contractor’s experiences, intuitions, and personal preferences, which are not conducive to approach standardization (Chua and Li 2000). For an extremely competition circumstances, the companies whom participate in the competitive bid must submit a very low bid price to win the project contract. In other word, the mark-up size of all companies may very close; it makes the determination of mark-up size become more difficult. In light of such, there is a practical need for a mark-up size decision-making model to fit the construction company’s practices.

This article based on cumulative prospect theory (CPT), game theory (GT), fuzzy preference relations, and fuzzy ratting to propose a Prospect Game Theory Bidding Decision Model in extremely competition (BD-PGTMe) to assist the construction company’s PDM to determine the appropriately bid price in a extremely competitive bidding. And the test results of the case show that the method is feasible.

2. LITERATURE REVIEW

2.1. Cumulative prospect theory

Consider a prospect $X$ with outcomes $x_1 \leq \ldots \leq x_k \leq 0 \leq x_{k+1} \leq \ldots \leq x_n$ that are associated with probabilities $p_1, \ldots, p_n$. Cumulative prospect theory predicts that people will choose prospects based on the prospect value generated by:

$$V_{CPT} (X) = \sum_{j=1}^{k} \pi_i \times \lambda \times v(x_i) + \sum_{j=k+1}^{n} \pi_j \times v(x_j)$$

(1)

Where $v(x)$ is the utility function and $\lambda$ is a loss-aversion parameter. In gain condition, the value function (Kahneman and Tversky 1979; Tversky and Kahneman 1992; Tversky and Fox 1995; Gonzalez and Wu 1999) show below:

$$v(x) = x^\alpha$$

(2)

The decision weights $\pi$ is calculated by the “cumulative” probabilities $p_i$ which associated with outcomes $x_i$. In gains condition, the function of decision weights (Tversky and Kahneman 1992) is show below:

$$\pi_i = \sum_{j=1}^{n} w^+(p_j) - \sum_{j=i+1}^{n} w^+(p_j) \text{ for } 1 \leq i \leq n-1$$

(3)

The probability weighting function $w^+$ are represented the condition of gains (Tversky and Kahneman 1992; Camerer and Ho 1994; Wu and Gonzalez 1996).

$$w^+(p) = p^\gamma \left( p^\gamma + (1-p)^\gamma \right)^{-1/\gamma} \text{ for } 0 \leq p \leq 1$$

(4)
2.2. Game theory

Game theory (Von-Neumann and Morgenstern 1944) attempts to explain behaviour in strategic situations or games mathematically by recognizing that successful decision-making depends on the choices of others. The Nash equilibrium (Nash 1951) is the solutions of non-zero game.

2.3. Fuzzy preference relationships

Many important decision models have been developed which focus mainly on: 1) multiplicative preference relations (MPR) and 2) FPR (Herrera-Viedma et al. 2004). In MPR, an expert assigns a value which reflects the degree of preference to each pair of alternatives. For a set of alternatives $X$ is represented by matrix $A = [a_{ij}] \subseteq X \times X$, $a_{ij} = [1/9, 9]$ and $a_{ij} a_{ji} = 1$ for $i, j \in \{1, ..., n\}$. Saaty suggested measuring $a_{ij}$ using a ratio scale 1-9 (Saaty 1980; Saaty 1994). When $a_{ij} = 9$ denotes that $x_i$ is preferred absolutely to $x_j$, and $a_{ij} = 1$ represents no difference in preference between $x_i$ and $x_j$. A FPR on a set of alternatives $X$ is represented by a matrix $B$. Matrix $B$ is a fuzzy set on product set $X \times X$ that is characterized by membership function $\mu_B : X \times X \rightarrow [0, 1]$. Therefore, in $B = [b_{ij}]$ and $b_{ij} = \mu_B(x_i, x_j)$ for $i, j \in \{1, ..., n\}$, where $\mu_B$ is a membership function, and $b_{ij}$ is the preference ratio of the alternative $x_i$ over $x_j$. While $b_{ij} = 0.5$ denotes that $x_i$ and $x_j$ are indifferent, and $b_{ij} = 1$ represents that $x_i$ is preferred absolutely to $x_j$. Matrix $A$ can be transferred into matrix $B$ by using transform equation $b_{ij} = (1 + \log_9 a_{ij})/2$. The relative weights $w_i$ for all alternatives $i$ can be obtained by using $w_i = \frac{\sum_j b_{ij}}{\sum_i \sum_j b_{ij}}$.

Figure 1: Flowchart of BD-PGTMe.
3. PROSPECT GAME THEORY MODEL FOR BIDDING PRICE DECISION

The flowchart of BD-PGTMe is shown in Figure 1. To illustrate the feasibility of the model, a test case will be used to illustrate A company how to apply the model to determine an appropriate bid price.

3.1. Phase I – Information Collection

The aim of this phase is to identify the companies that may participate in the bidding, the type of bidding strategy, the submitted bid price for each bidding strategy, and the implementation probability of each bidding strategy.

3.1.1. Data Collection

When receiving an invitation for competitive bidding for a construction project, the experts of A company assess that there are two companies (B company and C company) that will participate in the competitive bidding and submit bids.

3.1.2. Determine Bidding Strategies and Profits

The cost estimated by each construction company may be very similar, and the variations in competitor's bids are mainly due to their selected markup size (Moselhi et al. 1993). In the test case, the construction cost calculated by A company is $10,000×10^4 NTD, and A company's PDM decides on the bidding strategy and the size of markup. Table 1 shows the expected profit for each bidding strategy.

3.1.3. Assign Criterion’s Implement Probability of Bidding Strategy

The FPR is used to estimate the relative weights of bidding strategies for A company’s PDM. The linguistic terms used in FPR are {AH, VH, SH, WH, EQ, WL, SL, VL, AL} with {9, 7, 5, 3, 1, 1/3, 1/5, 1/7, 1/9} to compare to corresponding neighboring factors. Via the computational process (Cheng et al. 2011), relative weights can be calculated from estimating results, Table 1 shows estimated results.

3.1.4. Forecast Competitor’s Profit and Implement Probability of Each Bidding Strategy

The fuzzy rating method is used to forecast the profit of B company’s and C company’s on each bidding strategy. Figure 2 shows the fuzzy number membership function of linguistic variables, which is obtained by fuzzy statistic analysis method (Cheng and Ko 2003).
The FPR is used to forecast the B company’s and C company’s implement probability on each bidding strategies. Table 1 shows the forecasted results.

Table 1: All company’s profits and implement probabilities

<table>
<thead>
<tr>
<th>Bidding strategy</th>
<th>A company</th>
<th>B company</th>
<th>C company</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profit (&lt;10^6NTD)</td>
<td>Bid price (&lt;10^6NTD)</td>
<td>Implement probability</td>
</tr>
<tr>
<td>S1</td>
<td>2000</td>
<td>102000</td>
<td>0.3446</td>
</tr>
<tr>
<td>S2</td>
<td>3000</td>
<td>103000</td>
<td>0.3785</td>
</tr>
<tr>
<td>S3</td>
<td>4000</td>
<td>104000</td>
<td>0.2769</td>
</tr>
</tbody>
</table>

3.2. Phase II – Obtain PDM’s preference

The goals of this phase are to elicit the value function and probability weighting function of the “criterion” (A company’s PDM), and forecast the foregoing functions of competitors.

3.2.1. Elicit value function and probability weighting function of criterion

This article combine certainty equivalent method (Tversky and Kahneman 1992; Gonzalez and Wu 1999) and bisection method (Abdellaoui 2000, Bleichrodt and PintoL 2000) to elicit the value function and probability weighting function of A company’s PDM. Figure 4 shows the elicited value function and probability weighting function of A company’s PDM.

3.2.2. Forecast competitors PDM’s value function and probability weighting function

In Figure 3 shows the concept of forecasting competitor’s value functions and probability weighting function. Fuzzy rating methods are used to evaluating the difference rates of the competitor on the emphasis on money and the risk attitudes, which are in compared with the criterion. The area under the curve of value function represents the attitude of emphasis on money; difference in the area can be regarded as the difference in emphasis on money. The point of PRN on probability weighting
function represents the risk attitude in neutral; the rate of moving distances can be regarded as the difference in risk attitude. Figure 4 shows the forecasted result.

![Value Function and Probability Weighting Function](image)

**Figure 4: All PDM’s value function and probability weighting function**

3.3. Phase III – Deciding bid price

Bidder takes the presumed strategies of competitor bidders into consideration before formulating a bid strategy and setting a bid price. This article adopts non-cooperative games to describe the analysis process and uses the prospect value of each bidding strategy to represent the payoff in game.

3.3.1. Calculate joint probability for bidding strategy combination

In competitive bidding, the bidder can adopt different bidding strategy and form bidding strategy combination. The probability of the combination achieving can be represented by the joint probability of PDMs adopted strategy. For example, the probability of companies A, B and C all adopt bidding strategy S1, the probability for this situation occurred is 0.33%.

3.3.2. Calculate PDM’s prospect value of bidding strategy

The prospect value $V_{CPT}$ for a bidding strategy can obtain by prospect value equation. The $x$ is the expected profit of adopted bidding strategy, while $p$ is the joint probability of bidding strategy combinations. For example, the PDM of A company, B company, and C company are all select bidding strategy S1, the corresponding prospect values are $69.1 \times 10^4$, $60.3 \times 10^4$, and $71.5 \times 10^4$. Table 2 shows the normal form of bidding game.

3.3.3. Forecast competitors bidding strategies and bid prices

This article use the static non-cooperative game to forecast the PDMs adopted bidding strategies and bid prices. The best-response analysis method (Myerson 1991) is used to find the Nash equilibrium of the game. In Table 2, the payoff set in bold face type is the Nash equilibrium. From the results of the game, this article forecast that A companies, B companies, and C companies trend
to adopt bidding strategies S3, S2, and S2, respectively.

### Table 2: Normal form of bidding game

<table>
<thead>
<tr>
<th></th>
<th>A company</th>
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<tbody>
<tr>
<td></td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
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<tr>
<td>A company</td>
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<tr>
<td>B company</td>
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<td>C company</td>
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<td></td>
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<tr>
<td>VCPT $(\times 10^4)$</td>
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<td></td>
</tr>
<tr>
<td>S1</td>
<td>69.1</td>
<td>72.8</td>
<td>49.5</td>
<td>105.9</td>
<td>111.5</td>
<td>75.9</td>
<td>109.7</td>
<td>115.6</td>
<td>78.4</td>
</tr>
<tr>
<td>S2</td>
<td>60.3</td>
<td>63.9</td>
<td>41.6</td>
<td>64.9</td>
<td>68.8</td>
<td>44.8</td>
<td>50.8</td>
<td>53.8</td>
<td>35.0</td>
</tr>
<tr>
<td>S3</td>
<td>71.5</td>
<td>106.8</td>
<td>94.9</td>
<td>76.2</td>
<td>113.8</td>
<td>101.2</td>
<td>61.6</td>
<td>92.0</td>
<td>81.5</td>
</tr>
<tr>
<td>S1</td>
<td>90.7</td>
<td>95.4</td>
<td>65.2</td>
<td>138.8</td>
<td>146.1</td>
<td>100.0</td>
<td>144.2</td>
<td>151.8</td>
<td>103.5</td>
</tr>
<tr>
<td>S2</td>
<td>117.4</td>
<td>124.3</td>
<td>81.3</td>
<td>126.3</td>
<td>133.7</td>
<td>87.5</td>
<td>99.0</td>
<td>104.9</td>
<td>68.4</td>
</tr>
<tr>
<td>S3</td>
<td>92.9</td>
<td>138.7</td>
<td>123.8</td>
<td>98.9</td>
<td>147.6</td>
<td>131.9</td>
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<td>119.7</td>
<td>106.6</td>
</tr>
<tr>
<td>S1</td>
<td>63.8</td>
<td>67.2</td>
<td>45.7</td>
<td>97.8</td>
<td>103.0</td>
<td>70.1</td>
<td>101.3</td>
<td>106.7</td>
<td>72.3</td>
</tr>
<tr>
<td>S2</td>
<td>101.8</td>
<td>107.8</td>
<td>70.2</td>
<td>109.6</td>
<td>116.1</td>
<td>75.6</td>
<td>85.7</td>
<td>90.8</td>
<td>59.0</td>
</tr>
<tr>
<td>S3</td>
<td>66.2</td>
<td>98.9</td>
<td>87.8</td>
<td>70.6</td>
<td>105.4</td>
<td>93.6</td>
<td>57.0</td>
<td>85.2</td>
<td>75.4</td>
</tr>
</tbody>
</table>

3.3.4. Comparison and Decision Making

From the forecasted result and refer to Table 1, shows the C company’s bid price $103033\times10^4$ NTD is the lowest. Therefore, the A company wants to win the project, it should submit a bid price lower than C company's bid price.

4. CONCLUSIONS

This article develops a Prospect Game Theory Bidding Decision Model in extremely competition (BD-PGTMe) to help construction companies to determine appropriate bid prices. Article contributions include:

1. Provide a systematic decision model to determine an appropriate bid price.

2. Use FPR to simplify the process of forecasting implementation probability for bidding strategies and overcome traditional reliance on evaluator experience and guesswork.

3. Adopt fuzzy rating to forecast value functions, probability weighting functions, and bid price of competitor’s PDM, which may reduce inherent uncertainty in evaluator’s ratings and also eliminate the predicament of being unable to obtain value functions and probability weighting functions directly from competitor’s PDM.

4. Use CPT to calculate the preference values can reflect the PDMs’ risk preference on each bidding strategy.

5. Adopt GT to simulate the conflict of competitive bid process, allow the PDM to set an optimal bid price able to secure both the contract award and as high a profit as possible.
REFERENCES


