THE ENDOCHRONIC MODEL FOR THE MECHANICAL BEHAVIORS OF CONCRETE UNDER CYCLIC LOADING

Hsin-Han Wu[1†], Jun-Kai Lu[1*], Meng-Hao Tsai[1]

[1] *Department of Civil Engineering, National Pingtung University of Science and Technology, Pingtung, Taiwan, ROC*

ABSTRACT

Present paper discusses the model of the mechanical behaviors of concrete under cyclic loading. The constitutive laws for mechanical behaviors of concrete using endochronic theory were derived in a previous study. Mechanical responses of concrete material may be simplified in the model and decomposed into a hydrostatic response and a deviatoric response. Also, the proposed model has been used to describe the mechanical and thermal behaviors of reinforced concrete members and plain concrete under different loading and thermal situations. In this paper, the mechanical behaviors of concrete under cyclic loading are discussed based on the previous study. A modified endochronic model is derived. In present model, the shear modulus has been modified. Also, hardening function and the internal variable's evolution equations of the deviatoric response have been redefined. The modified model is then used to describe the mechanical behaviors of concrete under cyclic loading condition. The theoretical results have been compared with experimental data in this study. It is seen that the results of present model are satisfied.

Keywords: endochronic model, constitutive laws, concrete, cyclic loading.

1. INTRODUCTION

Some of researchers investigated the mechanical behaviors of concrete by experiment (Sinha et al. 1964; Bahn and Hsu 1998). The others worked on the description of the relations of the stress-strain by the statistical regression or mathematical model. Lu and Wu (Lu and Wu 1998) derived the constitutive equations for concrete by using the endochronic model. Several researchers (Lu and Wu 2001; Lu and Chen 2002; Lu and Yeh 2002 ;Lu and Tow 2004) describe the mechanical behaviors of reinforced concrete members using the model. Present paper modifies the endochronic model to describe the mechanical behaviors of concrete under cyclic loading. A modified endochronic model is derived. The theoretical results have been compared with the experimental data from published papers (Sinha et al. 1964; Bahn and Hsu 1998).

† Presenter: Email: sh.wu@msa.hinet.net
* Corresponding author: Email: julu@mail.npust.edu.tw

2. A SUMMARY OF THE ENDOCHRONIC MODEL FOR CONCRETE

In the endochronic model of Lu and Wu (Lu and Wu 1998), the mechanical responses of concrete are simplified into two parts: the hydrostatic response and the deviatoric response.

$$\mathrm{d}\varepsilon_{ij} = d\varepsilon_{kk}\delta_{ij} + de_{ij} \tag{1}$$

$$d\varepsilon_{kk} = d\sigma_{kk}/3K + 3\mathrm{B}_o d\gamma_{kk} \tag{2}$$

$$de_{ij} = dS_{ij}/2G + C_2 dP_{ij} \tag{3}$$

where ε_{ij} represents the strain; σ_{ij} represents the stress; γ_{ij} and P_{ij} denotes the internal variable related to hydrostatic deformation and deviatoric deformation, respectively; e_{ij} and S_{ij} are the deviatoric components of ε_{ij} and σ_{ij}; B_o, E_o, M_o, C_2, F_2 and N_2 are the material constants; Z_H and Z_D represent the hydrostatic and the deviatoric intrinsic time scale. The evolution equation of internal variables for the hydrostatic response and deviatoric response are

$$d\gamma_{kk} = \left((B_o/M_o)\sigma_{kk} - (E_o/M_o)\gamma_{kk}\right)dZ_{\mathrm{H}} \tag{4}$$

$$dP_{ij} = \left((C_2/N_2)S_{ij} - (F_2/N_2)P_{ij}\right)dZ_D \tag{5}$$

The bulk modulus K and the shear modulus G are defined in terms of the compressive strength of concrete.

$$K = (K_0 + K_1 f_c^{'}) \tag{6}$$

$$G = (G_0 + G_1 f_c^{'}) \tag{7}$$

where k_1, k_2, K_0, K_1, G_0 and G_1 are material constants and $f_c^{'}$ represents compressive strength of concrete. The intrinsic time and intrinsic time scale increment $\mathrm{d}\zeta_H$, $d\zeta_D$, dZ_H and dZ_D are

$$\mathrm{d}\zeta_H = |d\varepsilon_{kk} - k_1 \cdot d\sigma_{kk}/3K| \tag{8}$$

$$d\zeta_D = \left|de_{ij} - k_2 \cdot dS_{ij}/2G\right| \tag{9}$$

$$dZ_H = \mathrm{d}\zeta_H/h(\theta_{kk}) \tag{10}$$

$$dZ_D = d\zeta_D/f \tag{11}$$

The volumetric hardening function $\mathrm{h}(\theta_{kk})$ is defined as:

$$\mathrm{h}(\theta_{kk}) = \left(C_h - (C_h - \theta_m)e^{-\alpha\theta_{kk}}\right)/(\theta_m - \theta_{kk}) \tag{12}$$

where C_h, α, and θ_m denote material constants. The deviatoric hardening function $\mathrm{f}(Z_D)$ is redefined in terms of stress invariant and strain invariant by Lu and Wu (Lu and Wu 1998):

$$f = (f_1 \cdot f_2)/(1 + f_3 \cdot f_4) \tag{13}$$

$$f_1 = C_D - (C_D - 1)e^{-\beta \cdot ZD} \tag{14}$$

$$f_2 = b_1 + b_2 f_c^{'} + b_3 J_{2d} f_c^{'b_4} \tag{15}$$

$$f_3 = a_1 - a_2 f_c^{'} \tag{16}$$

$$f_4 = \langle J_{2d}^{'} - J_{2d}^{'}(\varepsilon_y)\rangle \cdot \left(d_1 + d_2 \cdot J_{2d}/{I_{1h}}^2\right) \tag{17}$$

where C_D, β, b_i, a_i and d_i are material constants; ${I_{1h}}^2$, J_{2d} and $J_{2d}^{'}$ represent the first invariant of the hydrostatic stress, the second invariant of the deviatoric stress and the second invariant of the deviatoric strain

3. THE APPLICATION MODEL OF CONCRETE FOR CYCLIC LOADING

A modified endochronic model is derived for the mechanical behaviors of concrete under cyclic loading. In present model, the shear modulus has been modified. Also, hardening function and the internal variable's evolution equations of the deviatoric response have been redefined.

3.1. The shear modulus of concrete

Experimental data (Sinha et al. 1964; Bahn and Hsu 1998) showed that the shear modulus of concrete is increased with respect to the increase of strain. But in the stress softening procedure, the shear modulus will be decreased following strain increasing. In present paper, the shear modulus be treated as a function of intrinsic time scale ZD^*. G_I present the shear modulus under reloading condition. G_{II} present the shear modulus under unloading condition. The G_{II} and G_I are redefined in terms of the compressive strength and intrinsic time scale ZD^* in present paper.

$$G_I = 0.9\bar{\alpha}(G_0 + G_1 f_c^{'}) \tag{18}$$

$$G_{II} = (10/9)G_I \tag{19}$$

$$\bar{\alpha} = 1.0 + \left(\alpha_1/\alpha_y\right){Z_D}^* \qquad \text{for } {Z_D}^* < 1 + \alpha_1 \tag{20}$$

$$\bar{\alpha} = \alpha_2 + \alpha_3 \exp\left(-\alpha_4({Z_D}^* - \alpha_y)\right) \qquad \text{for } {Z_D}^* \geq 1 + \alpha_1 \tag{21}$$

$${Z_D}^* = \sum_{i=1}^{n} {Z_D}_I^{(i)} \tag{22}$$

where the α_1, $\alpha_2, \alpha_3, \alpha_4$ and α_y are material constants; the "(i)" symbol present that material sample was suffering the i-th cycle of cyclic loading.

3.2. The internal variable's evolution equations

The internal variable's evolution equations of the deviatoric response have been modified in this paper as:

$$dP_{ij}^{II} = \left((C_2/N_2)S_{ij} - \left(F_{2II}^{(i)}/N_2\right)P_{ij}\right) dZ_{DII}^{(i)} \tag{23}$$

$$F_{2II}^{(i)} = \bar{\beta}_1 \cdot F_{2I}\exp(-\beta_1) \tag{24}$$

$$\beta_1 = \bar{\beta}_2 \cdot \left\{(J_{2d}^{'})_{ui}^{(i)}\right\}^{\bar{\beta}_3} - \bar{\beta}_4 \tag{25}$$

$$dP_{ij}^{I} = \left((C_2/N_2)S_{ij} - (F_{2I}/N_2)P_{ij}\right) dZ_{DI}^{(i)} \tag{26}$$

where P_{ij}^{II} and P_{ij}^{I} denotes the internal variable related to deviatoric deformation under unloading and reloading situation respectively; F_{2_I}, $F_{2_{II}}^{(i)}$ and $\bar{\beta}_i$ are the material constants; $(J_{2d}^{'})_{ui}^{(i)}$ present the second invariant of the deviatoric strain in the initial point of the i-th cycle of cyclic loading. The new deviatoric intrinsic time scales $dZ_{D_{II}}, dZ_{D_I}$ and new deviatoric intrinsic time ζ_D^I, ζ_D^{II} under unloading and reloading conditions are redefined as following:

$$dZ_{D_{II}}^{(i)} = d\zeta_D^{II}/f_{II} \tag{27}$$

$$dZ_{D_I}^{(i)} = d\zeta_D^{I}/f_{I} \tag{28}$$

$$d\zeta_D^{II} = \left|de_{ij} - k_2 \cdot dS_{ij}/2G_{II}\right| \tag{29}$$

$$d\zeta_D^{I} = \left|de_{ij} - k_2 \cdot dS_{ij}/2G_{I}\right| \tag{30}$$

3.3. The deviatoric hardening function

To account for unloading and reloading situation, a new unloading and a reloading deviatoric hardening function $f_{1I}^{(i)}$ and $f_{II}^{(i)}$ have been defined as:

$$\mathrm{f}_{II}^{(i)} = 0.3(\mathrm{C}_D - (C_D - 1))\exp\left(-\beta^{(i)} \cdot \mathrm{Z}_{\mathrm{D}_{II}}^{(i)}\right) \tag{31}$$

$$\beta^{(i)} = \beta \cdot \{\ \tilde{\beta}_1 f_c^{'\bar{\bar{\beta}}_1} + \tilde{\beta}_2 e^{(\bar{\bar{\beta}}_2 - \bar{\bar{\beta}}_3 \cdot [(J_{2d}^{'})_{ui}^{(i)}]^{\bar{\bar{\beta}}_4})} - \tilde{\beta}_3 \cdot < (J_{2d}^{'})_{ui}^{(i)} - \tilde{\beta}_4 >\} \tag{32}$$

$$f_{1I}^{(i)} = \left(r_1 \cdot \bar{f}_1^{(i)} \cdot f_2\right)/(r_2 + r_3 \cdot f_3 \cdot f_4) \tag{33}$$

$$\bar{f}_1^{(i)} = C_D - (C_D - 1)\exp(-\beta^{(i)} \cdot Z_{DI}^{(i)}) \tag{34}$$

$$\gamma_1 = \bar{r}_1 \times \{\tilde{r}_1 - \tilde{r}_2[(J_{2d}^{'})_{ui}^{(i)}]^{\bar{\bar{r}}_1} + \tilde{r}_3[(J_{2d}^{'})_{ui}^{(i)}]^{\bar{\bar{r}}_2}\} \tag{35}$$

$$\gamma_2 = \bar{r}_2 \times \exp\{1 - \tilde{r}_4 \cdot [(J_{2d}^{'})_{ui}^{(i)}]^{\bar{\bar{r}}_3}\} \tag{36}$$

$$\gamma_3 = \bar{r}_3 \times [1 - \tilde{r}_5 \cdot (J_{2d}^{'})_{ui}^{(i)}] \tag{37}$$

where β, $\tilde{\beta}_i, \bar{\beta}_i, \bar{r}_i$ and $\tilde{r}_i$ are the material constants.

4. THE COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

After the modified endochronic model has been derived. The theoretical results of present model are compared with the experimental data (Sinha et al. 1964；Bahn and Hsu 1998). There are two sets of curves with compression strength of concrete 21 MPa from Sinha et al. (Sinha et al. 1964): one for unloading condition and another for reloading condition. The figure 1 shows the cyclic loading loops predicated by present model and the experimental data for unloading condition. Theoretical results match the experimental data quiet well. The initial slope of each unloading curve have a descend trend following to increasing strain. The figure 2 shows the cyclic loading loops predicated by present model and the experimental data for reloading condition. The theoretical results are consisting with experimental data. Except for the unloading and reloading curves, the new model shows the ability of describing with envelope curves and transition part between envelope curves and reloading curve set.

The theoretical results of cyclic loading hysteresis loops compared with the experimental data of Bahn and Hsu (Bahn and Hsu 1998) are showed in Figure 3. For a stress softening material, its subsequent yield stress will be reduced with respect to strain path increasing and the residual strains have opposite situation with strain path increasing. In the Figure 3, the phenomenon of subsequent yield stress reducing ($\sigma_y^{'} > \sigma_y^{''} > \sigma_y^{'''}$) and the trend of residual strains increasing ($\varepsilon_r^{''} > \epsilon_r^{'}$) could be described by the present model. It is seen that the present model works well.

5. CONCLUSIONS

A model of endochronic constitutive equations accounting for cyclic loading effects has been developed in present paper. The present model is used to describing the mechanical behaviors of concrete material under cyclic loading conditions. The effects of cyclic loading have been included in the shear modulus, the hardening function and internal variable evolution equations. After the constitutive equations have been developed, they are applied to describe the mechanical behaviors of concrete material. The theoretical results are compared with the experimental data. A reasonable consistency between theory and experiment are achieved.

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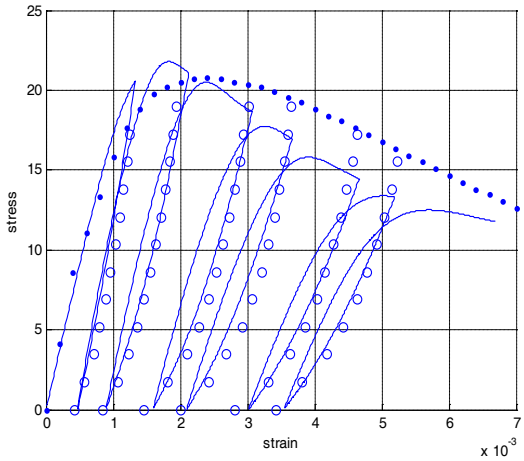

Figure 1: The predicated cyclic loading loops of concrete compare with unloading curves

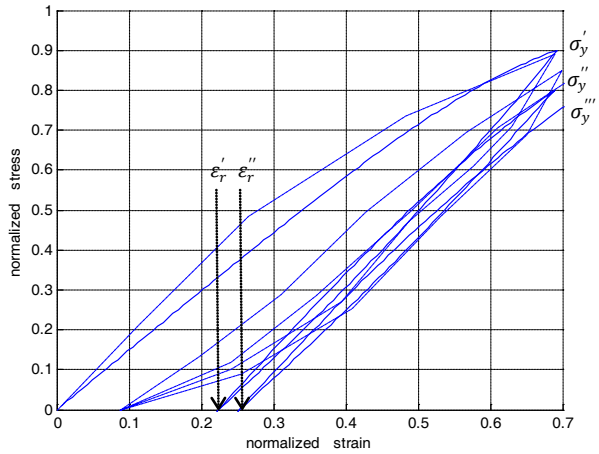

Figure 3: The predicated hysteresis loops compare with cyclic loading loops

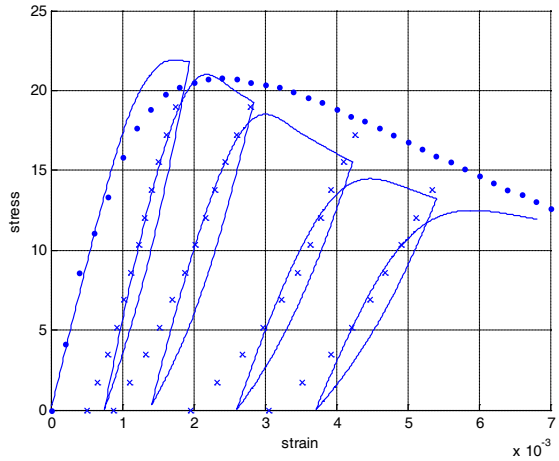

Figure 2: The predicated cyclic loading loops of concrete compare with reloading curves