Calculation Formulas of the Design Bending Moments on Boundaries of Slabs

Part1: Application of the Mean-width from RC Standard to Other Shaped Slabs

T. TSUJII[1*], Y. IMAI[1], S. MOROOKA[2] and K. NOMURA[3]

[1] *Graduate Student, Department of Architecture, School of Engineering, Tokai University, Japan*

[2] *Prof., Department of Architecture, Faculty of Engineering, Tokai University, Japan, Dr. Eng.*

[3] *Graduate Student, School of Science and Technology, Tokai University, Japan, M. Eng.*

ABSTRACT

The purpose of this study is to propose calculation formulas for design bending moments on boundary sides of a fixed end triangular slab and a fixed end tangential quadrilateral slab subjected to uniform load. With these formulas, you can calculate the design bending moment of these slabs more simply.

Keywords: Calculation formula for bending moment, Mean-width, Triangular slab, Tangential quadrilateral slab, Decreasing ratio

1. INTRODUCTION

The purpose of this study is to propose calculation formulas for design bending moments of arbitrary shaped slabs. Some of the authors have proposed calculation formulas for maximum bending moments on boundary sides of a fixed end triangular slab and a fixed end tangential quadrilateral slab (as shown in Figure 1) subjected to uniform load. On the other hand, the design bending moments of a rectangular slab given by RC standard formula of Architectural Institute of Japan are less than the analytical solutions by a theory of elasticity. Therefore, in the structural design of triangular slabs and tangential quadrilateral slabs, we don't think we need to use the calculation formulas for maximum bending moments proposed in previous papers by some of the authors.

In this paper, the authors assume a certain width in which the average bending moment is equal to the design bending moment given by RC standard formula. We will call this certain width a mean-width. Firstly, we calculate the mean-width, which is not described in RC standard but supposed to be existed in its idea. Then, we apply this idea of the mean-width to the triangular slab and the tangential quadrilateral slab. The value of design bending moments of these slabs comes to be an average bending moment in the mean-width. And we introduce a decreasing ratio of average bending moment in the mean-width to the maximum bending moment in each side. The

[*] Corresponding author and presenter: Email: 2bcbm016@mail.tokai-u.jp

calculation formulas for design bending moments are formulated in the form of multiplication of the decreasing ratio and the maximum bending moments.

In this paper, we consider only bending moments on the fixed boundary side. And the vertex angle of the triangular slabs and the tangential quadrilateral slabs is from 30 degree to 120 degree. Furthermore, elasticity solutions of the slabs are obtained by the finite element method (FEM) solutions. The analytical model of these slabs is divided by triangular elements (DKT) and quadrilateral elements (DKQ).

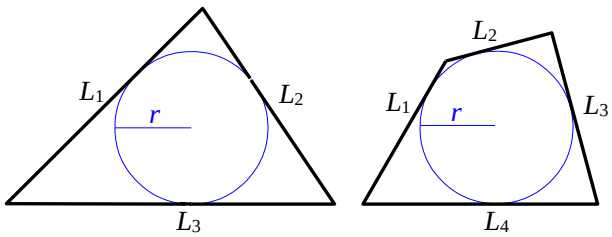

Figure 1: Example of triangular slabs and tangential quadrilateral slabs

2. THE MEAN-WIDTH OF RECTANGULAR SLABS

A divided width of the FEM model for the rectangular slab is 1/80 length of a short boundary side. Figure 2 shows an example of a distribution of bending moments on a side of a rectangular slab. A solid line indicates a distribution of bending moments, and a broken line shows the value given by RC standard formula. In this paper, the mean-width is calculated in the way that the area A and B are the same.

In order to investigate the mean-widths in rectangular slabs, we analyzed 161 rectangular slabs whose ratio of long side to short side, λ, is from 1.0 to 5.0, and calculated the mean-widths in the long side and the short side of the slabs. Figure 3 shows the results. The horizontal axis indicates λ, and the vertical axis represents a ratio of the mean-width to the boundary length, a. And marker ○ shows the ratio a of the short side, marker × represents a of the long side. We found that the ratio λ changes the mean-width and the ratio a of the long side and a of the short side are different.

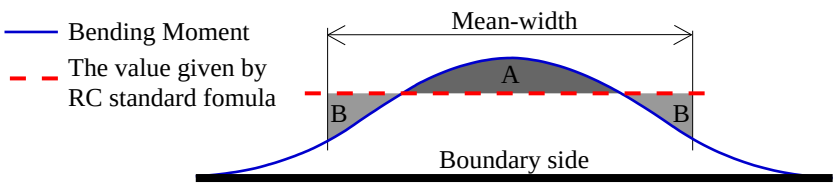

Figure 2: Example of a distribution of bending moments and the mean-width on the boundary side of rectangular slabs

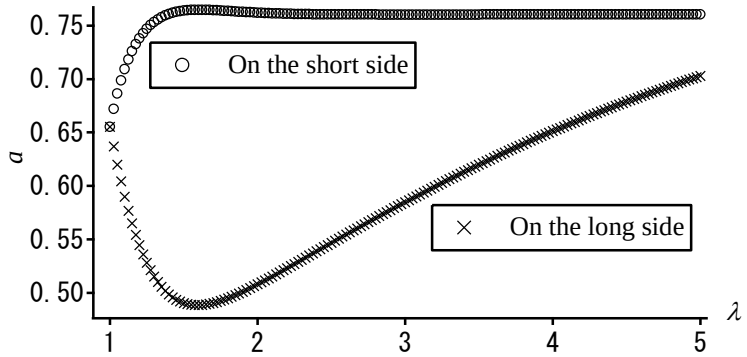

Figure 3: The mean-width of rectangular slabs

3. PROPOSISION OF CALCULATION FORMULAS FOR DESIGN BENDING MOMENTS

In this chapter, we determine the mean-width which might be applied to triangular slabs and tangential quadrilateral slabs. And we make the equations that give the average bending moment in the mean-width, and we propose it as the calculation formulas for design bending moments.

In the previous paper, some of the authors have already proposed calculation formulas for maximum bending moments of the triangular slab and the tangential quadrilateral slab. In this paper, we introduce the decreasing ratio D_a of the average bending moment in the mean-width to the maximum bending moment in each side. Then, the calculation formulas for design bending moments are expressed in the form of multiplication of D_a and the maximum bending moment.

3.1. Calculation formulas of maximum bending moments

Equation (1) shows the calculation formulas for maximum bending moments of triangular slabs proposed in the previous paper.

$$ {}_tM_i = 0.198\left(\frac{L_i}{r}\right)^{0.175} \times wr^2 \tag{1} $$

Where ${}_tM_i$ is the maximum bending moment in the i-th boundary side, L_i is the length of the i-th boundary side, w is the uniform load, and r is the radius of the inscribed circle. Solutions of this equation are bigger than all FEM solutions of 37 triangular slabs, whose vertexes are from 30 degree to 120 degree. A maximum difference between the solutions by this equation and FEM solutions is about 4 %.

Equation (2) represents the calculation formulas for maximum bending moments of tangential quadrilateral slabs proposed in the previous paper.

$$ {}_qM_i = 0.190\left(\frac{L_i}{r}\right)^{0.175} \times wr^2 \tag{2} $$

Where ${}_qM_i$ is the maximum bending moment in the i-th boundary side. Solutions of this equation are bigger than all FEM solutions of 313 tangential quadrilateral slabs, whose vertexes are from 30 degree to 120 degree. A maximum difference between the solutions by this equation and FEM solutions is about 3 %.

3.2. The mean-width applied to the slabs

We need the mean-width a ,but as previously, the ratio λ changes the mean-width, and the a of the long side and the short side are different. Then, considering the inscribed circle and rectangles, we found squares are common figures that have inscribed circle and are kinds of rectangles. In squares, all the sides are the same length, and the ratio λ is 1.0. In addition, all the shapes considered to be supported by each side are triangles which are the same shape supported by the short side in rectangular slabs. So the value of the mean-width a might be 0.655 obtained from λ=1.0 on the short side in Figure 3. This value is the smallest value on the short side, and is the safest value for design because the smallest value a=0.655 gives the biggest bending moment.

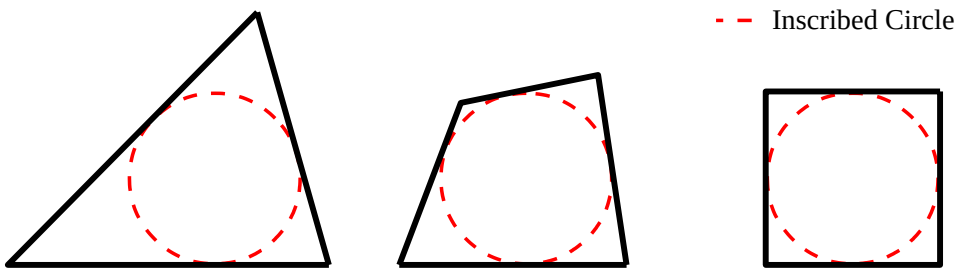

Figure 4: Shape of the slabs with the inscribed circle

3.3. The decreasing ratios and the calculation formulas for design bending moments

The decreasing ratio D_a in each side is the ratio of the average bending moment in the prescribed mean-width to the maximum bending moment. Figure 5 shows an example of a distribution of bending moments on a side of triangular slab. The average bending moment in the mean-width depends on the position along the side. We need to calculate the average bending moment as it become the biggest value. To investigate D_a, we made FEM models of 37 triangular slabs and 313 tangential quadrilateral slabs whose vertexes are from 30 degree to 120 degree. The divided width in each model is 1/30 radius of the inscribed circle.

We compute D_a on all boundary sides of all FEM models. Figure 6 shows results of the decreasing ratio on i-th boundary side of triangular slabs ${}_tD_{ai}$ and of tangential quadrilateral slabs ${}_qD_{ai}$. The horizontal axis indicates a non-dimensional value, L_i/r, and the vertical axis indicates the decreasing ratio. And mark ○ represents ${}_tD_{ai}$, mark × represents ${}_qD_{ai}$. We found that ${}_tD_{ai}$ distributes from 0.75 to 0.60, ${}_qD_{ai}$ from 0.90 to 0.65. And also, as L_i/r increases, the maximum bending moment can be further decreased.

Equations (3) and (4) show calculation formulas for these decreasing ratios. The variable of these equations is L_i/r. We made these formulas as they are bigger than in all FEM solutions.

$$ {}_tD_{ai} = 0.869 - 0.134\log(\frac{L_i}{r}) \tag{3} $$

$$ {}_qD_{ai} = 0.915 - 0.158\log(\frac{L_i}{r}) \tag{4} $$

In Figure 6, a solid line indicates solutions of equation (3), and a broken line represents solutions of equation (4). A maximum difference to the FEM solutions is about 3 %. Then we can have the calculation formulas for design bending moments in the form of the multiplication of equations (3), (4) and equations (1), (2).

$$ {}_tM_{dai} = -(0.869 - 0.134\log(\frac{L_i}{r}))(0.198(\frac{L_i}{r})^{0.175} \times wr^2) \tag{5} $$

$$ {}_qM_{dai} = -(0.915 - 0.158\log(\frac{L_i}{r}))(0.190(\frac{L_i}{r})^{0.175} \times wr^2) \tag{6} $$

Figure 7 shows the values obtained from equations (5) and (6). Solid lines represent the value ${}_tM_{dai}/wr^2$ or ${}_qM_{dai}/wr^2$, the horizontal axis shows L_i/r. To simplify these equations, we assume they are linear with respect to L_i/r, and we make the following equations. The maximum difference is less than 1%.

$$ {}_tM_{dai} = -(177 - 0.954(\frac{L_i}{r}))wr^2 \times 10^{-3} \tag{7} $$

$$ {}_qM_{dai} = -(177 - 1.94(\frac{L_i}{r}))wr^2 \times 10^{-3} \tag{8} $$

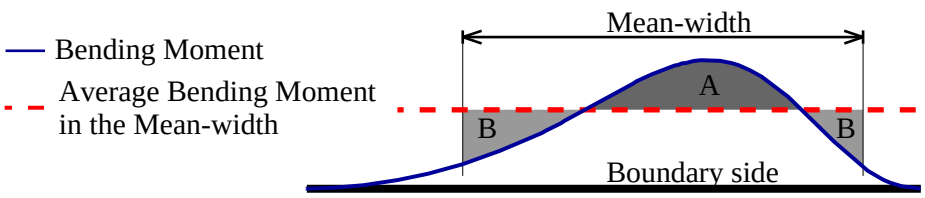

Figure 5: Example of the application of the mean-width to a triangular slab

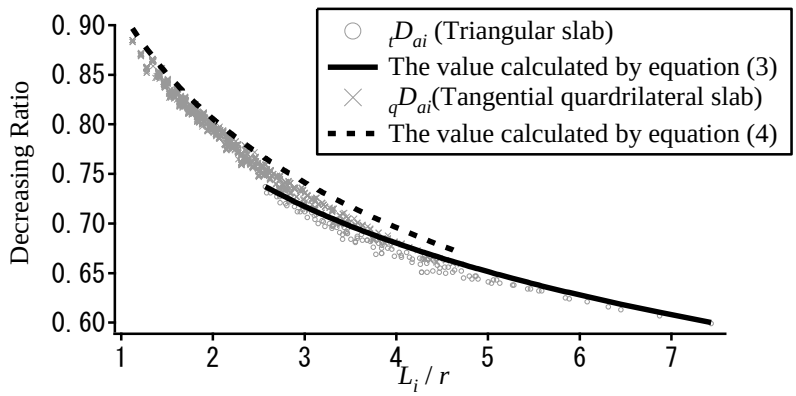

Figure 6: Decreasing ratio

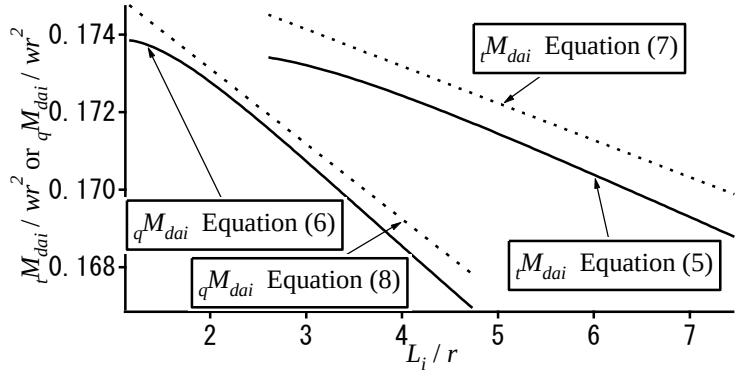

Figure 7: $_tM_{dai}$ and $_qM_{dai}$

4. CONCLUSION

In this paper, from the description in RC standard, the authors imagine the relationship between the maximum bending moment on a boundary side and the value given by RC standard formula. Firstly, we calculate the mean-width, which is not described in RC standard but supposed to be existed in its idea. Then, we apply this idea of the mean-width to the triangular slab and the tangential quadrilateral slab. And we propose the calculation formulas for design bending moments of these slabs. It is clarified that the important variable influencing the bending moment on a side is L_i/r, and increasing this variable makes the maximum bending moments decrease.

REFERENCES

Architectural Institute of Japan (Revised 2010). AIJ Standard for Structural Calculation of Reinforced Concrete Structures, Article 10. Japan. in Japanese.

Nomura K and Morooka S (2012). Calculation Formula for a Maximum Stress and Maximum Deflection of a Fixed End Quadrilateral Slab Subjected to Uniform Load - In the case of tangential quadrilateral, and isosceles trapezoid and right-angled trapezoid -, Journal of Structural and Construction Engineering in Japan. 680, pp.1511-1516. in Japanese.

Nomura K and Morooka S (2012). The Calculation Formula for a Maximum Stress and a Maximum Deflection of the Triangle Slab with Considering Effect of Support Condition Subjected to Uniform Load. Journal of Structural Engineering in Japan. 58B, pp.327-332. in Japanese.