ANALYTICAL EVALUATION OF SHEAR FAILURE BEHAVIOR OF SHCC BEAM BY CONSIDERING SHEAR TRANSFER BEHAVIOR

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ABSTRACT

Shear stress transfer model of SHCC, which is one of the fiber reinforced concrete, was developed by considering the roughness of crack surface, crack distribution behavior and fiber contributions for shear resistance. The applicability of the proposed model was verified by the simulation of the structural analysis of SHCC beam failed in shear. As the results, it was shown that proposed model could simulate the shear failure behavior of SHCC. The influence of fiber contribution was also investigated and it was confirmed that bond strength of fiber in the proposed model affected the failure behavior of SHCC beam failed in diagonal shear rather than that of failed in shear compression.

Keywords: Shear stress transfer model, fiber contribution, shear failure, UHP-SHCC.

1. INTRODUCTION

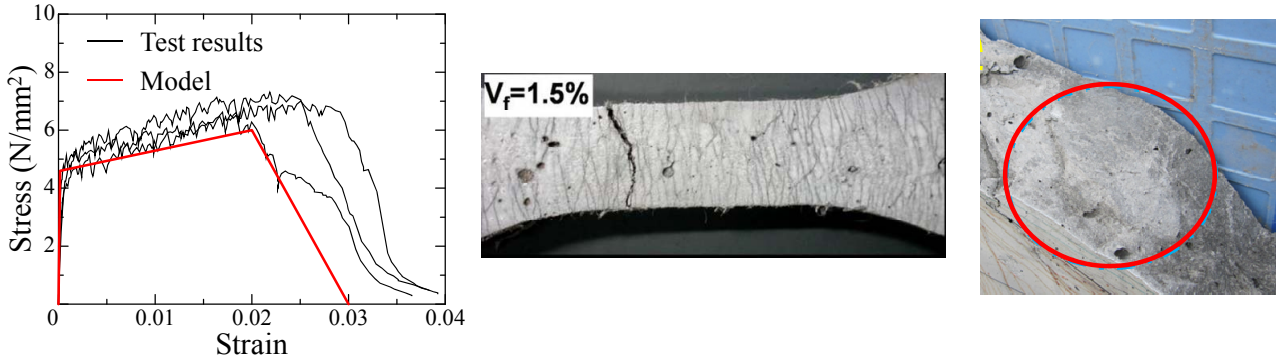
Recently, Strain Hardening Cementitious Composites (SHCC), which is one of the fiber reinforced concrete, has been tried to use as the structural members. For structural use, safety evaluation against the shear failure is most important issue and experimental investigation has been conducted for various SHCC members. Some researchers have also tried to evaluate shear failure behavior of SHCC beam by finite element analysis. It is recommended to conduct the FE analysis in order to clarify the shear failure mechanisms since the analysis can make clear the influences of the mechanical behavior on the shear failure.

In this study, for evaluating the shear failure behavior of SHCC beams, shear stress transfer model of SHCC was developed by considering the geometry and mechanisms of fracture process. Especially, the roughness of crack surface, crack distribution behavior and fiber contributions for shear resistance, which are the typical character of SHCC, were considered. Then, applicability of the proposed model was verified by the simulation of the structural analysis of SHCC beam failed in shear. Moreover, influence of fiber contribution in the proposed model was discussed.

2. FEATURE OF SHCC (UHP-SHCC)

In this study, as the SHCC material, Ultra High Performance Strain Hardening Cementitious Composite, that is UHP-SHCC, developed by Kunieda et al. (Kunieda et al. 2011), was selected for the material. UHP-SHCC has high compressive strength (about 100MPa) and also has significantly higher tensile strain hardening behavior at peak strength under tension. Figure 1(a) shows the one example of stress-strain relationship obtained from uniaxial tensile tests of dumbbell specimens, in which the strains were obtained from the elongation rate in measurement length of 100 mm. Figure 1(b) shows the crack pattern of a specimen after test. The strain hardening behavior arises from propagation of multiple fine cracks, and the uniaxial tensile test specimens failed due to one of the multiple fine cracks localized.

Figure 1(c) shows one example of crack surface of UHP-SHCC beam failed in diagonal shear. It can be seen that crack surface was quite smooth (circled in red area). The roughness of the crack surface was measured by laser displacement meter and it was suggested that the feature of crack surface can be assumed as a series of triangle with inclination of 15 degree and height of 1.2mm (Zhang 2012).



(a) Stress – strain relationship **(b) Crack pattern** **(c) Crack surface**

Figure 1: Characteristics of UHP-SHCC.

3. CONSTITUTIVE MODEL

Constitutive model used in the analysis is Lattice Equivalent Continuum Model (LECOM) (Tanabe and Ishtiag 1999), which is one of the fixed smeared crack models and consists of combination of tension, compression and shear lattices. For each lattices, uniaxial stress-strain relationships are adopted.

3.1. Modeling of strain hardening behavior under tension

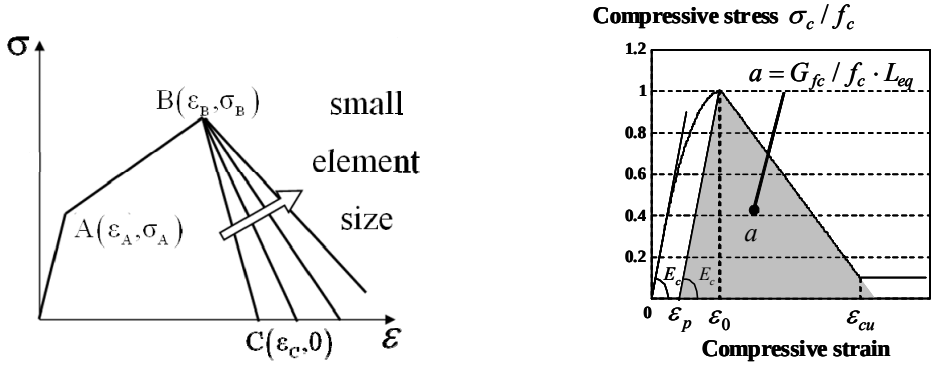
The stress-strain relationship of UHP-SHCC under tension is modeled by tri-linear curve as shown in Figure 2(a). In the model, Point A and B are defined by the results of uniaxial tensile tests such as in Figure 1(a). The strain of point C is represented by equation (1) considering the absorbed energy in localized element.

$$\varepsilon_C = \varepsilon_B + \frac{2G_F}{\sigma_B \cdot L_{elm}} \tag{1}$$

where, σ_B , ε_B are stress and strain at point B, ε_C is strain at point C. L_{elm} is element size (mm) and G_F is the absorbed energy after peak in uniaxial tensile test (N/mm). By using the stress-strain relationship with element size dependency in softening area considering the absorbed energy, a unique load-displacement relationship can be obtained independent on element size.

3.2. Modeling of compressive behavior

The stress-strain relationship of UHP-SHCC under compression can be modeled by compression softening model. In the model, Saenz equation is used up to the compressive strength and a linear softening branch is assumed as shown in Figure 2(b). The slope of linear softening branch is defined by considering the compressive fracture energy (G_{fc}) in order to avoid element size dependency as well as tensile behavior (Nakamura and Higai 2001). Since UHP-SHCC shows more ductile behavior than normal concrete under compression, compressive fracture energy of UHP-SHCC was assumed to be twice larger than that of proposed equation of Nakamura and Higai.

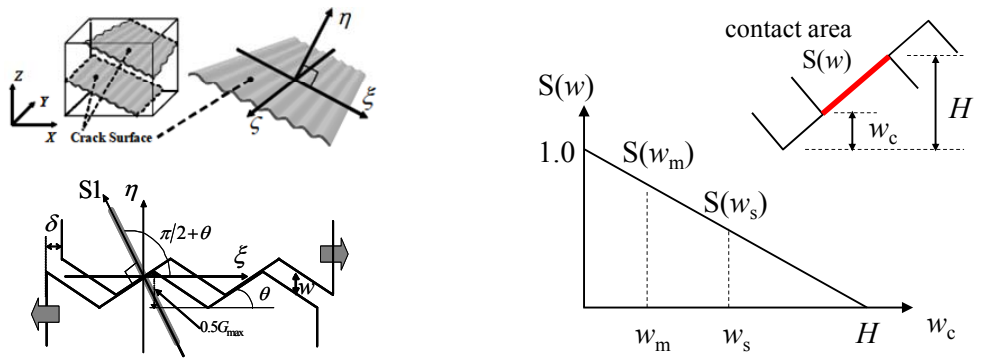


(a) Under tension **(b) Under compression**

Figure 2: Schematics of uniaxial stress - strain relationship.

3.3. Shear stress transfer model

In LECOM, shear stress transfer model is constructed by considering the mechanism of shear stress transfer (Phamavanh et al. 2005), such as, shear deformation and aggregate interlocking action on the crack surface, as shown in Figure 3(a). In the model, crack surface is modeled as a simple geometry with series of triangle and contact stress perpendicular to the incline surface is considered. Parameters which express the shape of crack surface in the model are the angle (θ) and the asperity height (H). For high strength concrete, the suggested values of them are 35 degree and 2.5mm, respectively, considering the smooth crack surface (Phamavanh et al. 2005). Moreover, additional coefficient representing the contact rate related with crack width is also taken into account as shown in Figure 3(b), that is, an increase of crack width decrease contact area.



(a) Assumed cracked surface **(b) Contact area**

Figure 3: Schematics of Shear stress transfer models (Phamavanh et al. 2005).

4. DEVELOPMENT OF SHEAR STRESS TRANSFER MODEL OF SHCC

4.1. Roughness of crack surface of UHP-SHCC

From the experimental investigation described in previous chapter, the shape of the crack surface of UHP-SHCC are obviously different from that of supposed in the shear stress transfer model for high strength concrete. As described in previous chapter, the shapes of crack surface, θ and H, measured by laser displacement meter were estimated as 15 degree and 1.2mm, respectively.

4.2. Crack distribution behavior

As described before, crack width is one of the important parameter for shear stress transfer. In LECOM, 1 crack is assumed in 1 element in the estimation of crack width. However, Cracking in UHP-SHCC under tension is explained as follows; multiple cracking until peak stress and localization of one of the multiple cracks after peak stress. Therefore, this cracking behavior is modeled in the estimation of crack width in the constitutive model. In the model, number of crack is assumed to be saturated and constant from the first cracking even though multiple cracks actually occur in sequence. Thus, crack width of UHP-SHCC, w_c, was modeled by Equation (2), as shown in Figure 4.

$$w_c = \begin{cases} 0 & \varepsilon < \varepsilon_A \\ (\varepsilon - \varepsilon_A)\cdot l_c & \varepsilon_A \le \varepsilon \le \varepsilon_B \\ (\varepsilon_B - \varepsilon_A)\cdot l_c + (\varepsilon - \varepsilon_B)\cdot L_{elem} & \varepsilon > \varepsilon_B \end{cases} \tag{2}$$

where, ε is strain and ε_A and ε_B are corresponding to the strain drawn in Figure 4. l_c is crack spacing. In the Equation (2), the crack width in the range of $\varepsilon_A < \varepsilon < \varepsilon_B$, is defined by the width of a crack of multiple cracks, and in the range of $\varepsilon > \varepsilon_B$, the crack width is for a localized crack developed from one of multiple cracks. In this study, crack spacing, l_c, of UHP-SHCC used in experiment was assumed as 3 mm based on the crack pattern obtained by uniaxial tensile test.

Moreover, shear deformation of multiple cracks was assumed to become smaller in the element during multiple cracking. Shear strain was made small according to the same manner as consideration of crack width.

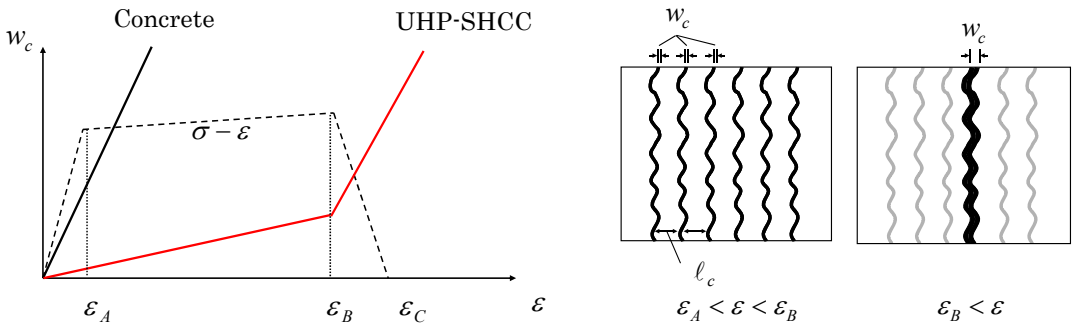


Figure 4: Relationship between crack width and strain of UHP-SHCC.

4.3. Fiber contribution for shear stress transfer

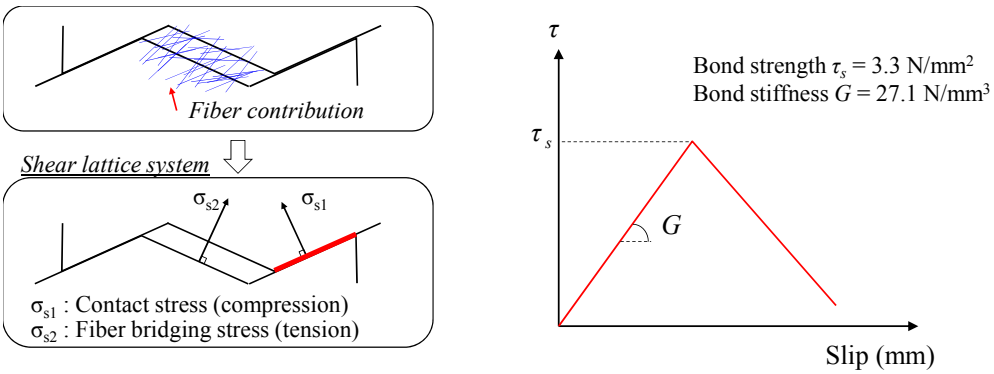
When shear deformation occurs along the crack surface of UHP-SHCC, bridging forces of the fiber in the material are expected to have an ability of resistance. Fiber contribution against the shear stress transfer was considered in the shear stress transfer model. Figure 5(a) shows the outline of the fiber effects on the crack surface. In proposed model, tensile stress along the perpendicular direction to the inclined plane of crack due to fibers, fiber bridging stress, was considered as well as contact stress in concrete shear lattice system.

Kunieda et al. conducted fiber pullout test, in which 1 PE fiber was pulled out from mortal of UHP-SHCC. From their experiment, bond stress - slip relationship for 1 PE fiber was defined as shown in Figure 5(b) (Kunieda et al. 2010). Here, initial bond stiffness G and bond strength τ_s were proposed 27.1 N/mm^3 and 3.3 N/mm^2, respectively. At the crack surface, there was not 1 PE fiber but a number of fibers. Therefore, bond strength and bond stiffness in tension region of shear lattice were assumed by considering fiber contents of UHP-SHCC, as Equation (3) and (4).

$$G^* = (V_f)^{\frac{2}{3}} \cdot G \tag{3}$$

$$\tau_s{}^* = (V_f)^{\frac{2}{3}} \cdot \tau_s \tag{4}$$

where V_f is fiber contents of UHP-SHCC, G^* and $\tau_s{}^*$ are initial bond stiffness and bond strength of shear lattice, respectively. Note that since the pullout resistance of fiber is changing by the orientation of fiber, which is arranged randomly in the matrix, bond stiffness and bond strength may be slightly different from the proposed value.



(a) Fiber contribution on crack surface **(b) Bond stress – slip relationship of 1 PE fiber**

Figure 6: modeling of Fiber contribution.

5. ANALYTICAL EVALUATION OF SHCC BEAM FAILED IN DIAGONAL SHEAR

5.1. Analytical model

Figure 6 shows the analytical models of the UHP-SHCC beams. The effective depth d of both are 150mm and the shear spans lengths a are 300 mm and 450 mm, that is a/d = 2 and 3 respectively The mesh size of the models was 30mm in longitudinal direction and 25mm in vertical direction. Rebar was model as consist of truss elements and perfect bond was assumed. Boundary condition was set as simple beam and incremental displacement was applied at the center of loading plate. Material properties of the UHP-SHCC are summarized in Table 1.These values were determined by reference to the experimental result.

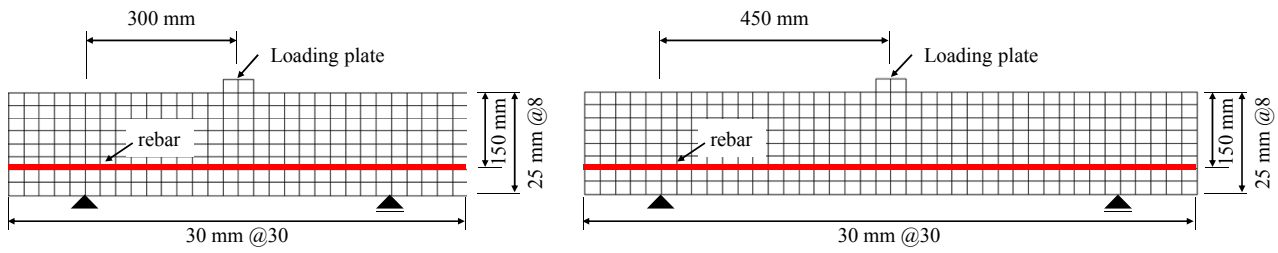


Figure 6: Analytical model (left: *a/d* = 2, right: *a/d* = 3).

Table 1: Material property of UHP-SHCC

	unit	*Value*
Yong's modules, E_c	kN/mm^2	29.0
Initial cracking stress, σ_A	N/mm^2	4.6
Peak stress, σ_B	N/mm^2	6.0
Strain at peak stress, ε_B	-	0.002
Fracture energy, G_F	N/mm	3.00
Compressive strength, f_c'	N/mm^2	91.0
Strain at compressive strength, ε_0	-	-0.004
Compressive fracture energy, G_{fc}	N/mm	168.0

Four analyses were conducted in order to clarify the influence of shear stress transfer model. Table 2 shows the parameter of each analysis. Shear transfer model of Case 1 is the original shear stress transfer model proposed by Phamavanh et al. and that of case 4 is the proposed model in this study.

Table 2: Analytical case

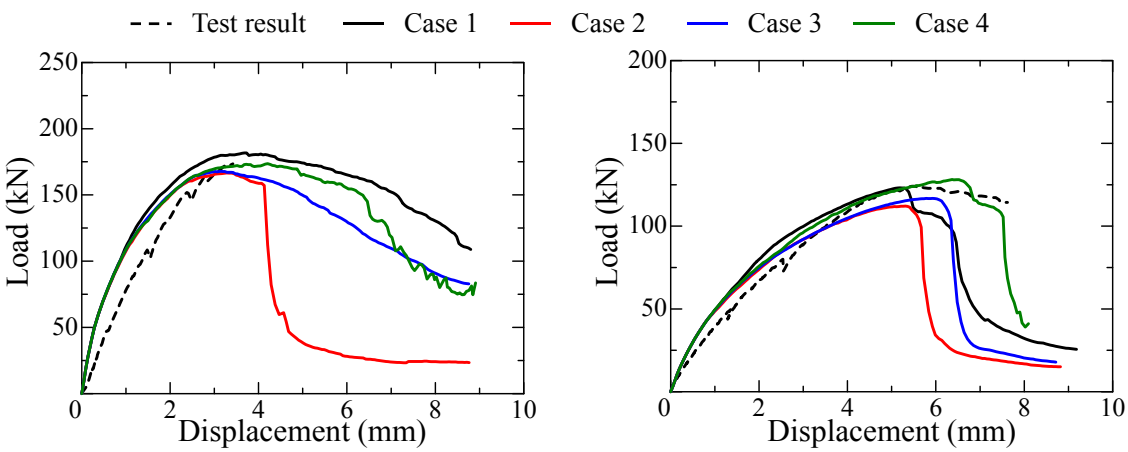
	Angle (degree)	*Asperity height* (mm)	*Cracking*	*Fiber*	*Note*
Case 1	35	5.0	Single	-	Original model
Case 2	15	1.2	Single	-	-
Case 3	15	1.2	Multi-single	-	-
Case 4	15	1.2	Multi-single	○	Proposed Model

5.2. Analytical result and discussion

Figure 8 shows the load-displacement relationship for a/d = 2 and 3 case, in which color lines show the analytical results and dotted line shows experimental result. Black, red, blue and green lines are the results of case 1, 2 3 and 4, respectively.

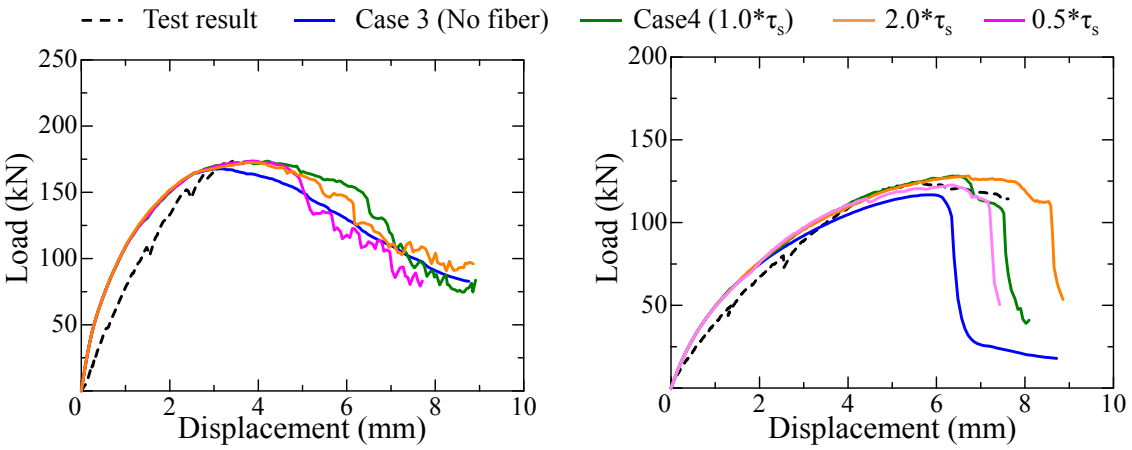
It is shown that the shear load carrying capacities obtained from the original model (Case 1) were the similar with test results. However, these results were not clearly explained based on mechanisms but it might be just only similar, because there were not any considerations for SHCC in shear stress transfer model. For Case 2 and 3, the stiffness became smaller than Case 1 after cracking (120kN for a/d= 2, 70kN for a/d=3) and shear load carrying capacities also became small. The result of Case 3 showed larger peak load compared to case 2 and it was the result from the consideration of multiple cracking.

The behavior of Case 4 showed more ductile behavior compared to case 3, especially, for a/d= 3. It means that the shear behavior of SHCC beam was affected by fiber contributions, and its influence was dominantly shown in a/d= 3 because the shear deformation along the diagonal crack was large. The result of case 4 showed good agreement with test results.



(a) UHP-SHCC beam with *a*/*d* = 2 **(b) UHP-SHCC beam with *a*/*d* = 3**

Figure 8: Load – displacement relationship of analytical results.



(a) UHP-SHCC beam with *a*/*d* = 2 **(b) UHP-SHCC beam with *a*/*d* = 3**

Figure 9: Influence of bond strength of fiber (fiber contribution).

Figure 9 shows the comparison of the shear behavior of SHCC beams with different bond strength of fiber, double and half, in proposed model. For *a*/*d*= 2, differences of bond strength of fiber did not affect the shear behavior. This result indicates that the shear compression failure occurred rather than the diagonal shear in *a*/*d*= 2. On the other hand, since the diagonal shear was dominant, more large bond strength of fiber led more ductile behavior in *a*/*d*= 3. This result implies that proposed model can evaluate the fiber contribution to the shear failure behavior of SHCC beam.

6. CONCLUSIONS

Shear stress transfer model for SHCC was developed by considering the geometry and mechanisms of fracture process of SHCC. The proposed model could simulate shear failure behavior of SHCC beam reasonably. The applicability for other fiber reinforced concrete will be verified in the future.

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