STABILITY DIAGRAM AIDED MULTIVARIATE AR ANALYSIS FOR IDENTIFYING THE MODAL PARAMETERS OF A STEEL TRUSS BRIDGE SUBJECTED TO ARTIFICIAL DAMAGE

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ABSTRACT

This study is intended to investigate the validity of the stability diagram (SD) aided multivariate autoregressive (MAR) analysis for identifying modal parameters of a real truss bridge subjected to two artificial damage scenarios. The MAR models are adopted to fit the time series of the dynamic accelerations recorded from a number of observation points on the bridge; then the modal parameters are extracted from the MAR model coefficient matrix. The SD is adopted to determine physically meaningful modes. Furthermore, in plotting the SD, a number of stability criteria are further adopted for filtering out those modes with unstable modal parameters. The first five modal frequencies and mode shapes are identified with very high precision, while the damping ratios are identified with acceptably high precision for the 1st mode but with less precision for higher modes. It is observed that those modal parameters changed due to the artificial damage. Moreover, the ability of the SD in selecting structural modes without getting involved in any model-order optimization problem is highlighted through a comparison study.

Keywords: automatic mode selection; system identification; field dynamic experiment; damage detection.

1. INTRODUCTION

The modal frequencies, damping ratios and mode shapes of a bridge are important parameters in realizing its dynamic characteristics, which have served as useful indices in many applications such as the structural health monitoring (SHM) [Doebling et al. 1998; Carden and Fanning 2004], model updating [Mottershead and Friswell 1993] and vibration control. To identify the modal parameters, multivariate autoregressive (MAR) models can be adopted to fit the time series of the dynamic responses recorded from a number of observation points on the bridge [Ljung 1999]; then the modal parameters are extracted from the MAR model coefficients.

An open problem for real world application in SHM is how to determine physically meaningful modes efficiently and accurately. One way to determine physically meaningful modes is aided by

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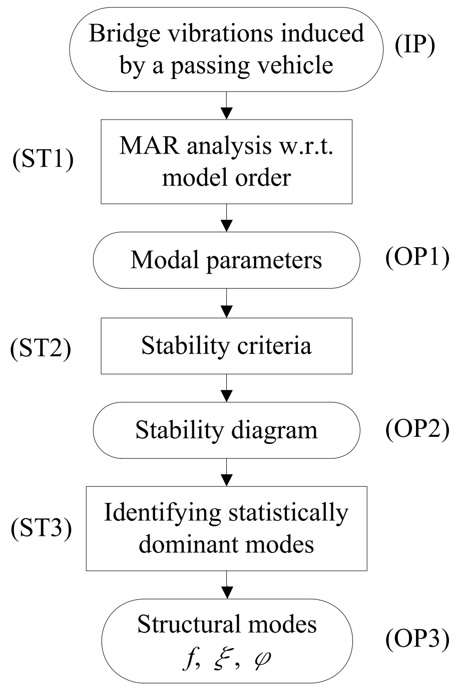
the stability diagram (SD) [Allemang and Brown 2010], which is known as a model order vs. eigenfrequency diagram covering a wide range of model order. It is believed that a physically meaningful mode may stably yield close eigenfrequencies while the model order is over-specified and therefore display a nearly vertical line in the SD, but the spurious ones may not. Applying such a mode-selection approach avoids the tough task in model-order selection. Although an optimal model order can be determined by certain existing criteria, it may offer more than physical modes. This feature is demonstrated in a comparison study in this paper.

Furthermore, in plotting the SD, a number of stability criteria are further adopted for filtering out the modes with unstable dynamic parameters: only the modes with their identified modal parameters close (within a pre-defined tolerance) to those of several adjacent model orders are identified as stable and thus retained.

This study investigates the validity of the SD aided MAR analysis for identifying modal parameters of a real truss bridge subjected to different damage scenarios. Within this scope, firstly the algorithm of modal-parameter identification is given, followed by a brief description on the field dynamic experiment. Then the modal parameters of the bridge are identified by the present analysis method, with focus on their identification precision, feature in physical-mode selection, and the changes due to the artificial damage.



(ST*n*: step *n*; IP: input; OP*n*: output *n*)

Figure 1: Flow chart.

2. ALGORITHM OF MODAL-PARAMETER IDENTIFICATION

Given a set of discrete time series $X(n)=\{x_1(n), x_2(n), \ldots, x_k(n)\}^T$ of length N (n= 1, 2, …, N) measured at k observation points on the bridge, the algorithm for identifying the modal parameters of the bridge can be illustrated as in Figure 1 and briefly described as follows.

2.1. Multivariate Autoregressive (MAR) Analysis

In the first step (ST1), fit $X(n)$ with MAR models with respect to (w.r.t.) a wide range of model order. For each model order, modal parameters, including modal frequencies f's, modal damping ratios ξ's, and mode shapes φ's, can be calculated from the estimated coefficients of MAR model. A brief operational algorithm of such an MAR analysis is given as follows, while more details are available in many other works (e.g. Huang 2001; Kim et al. 2012).

The time series $X(n)$ can be fitted with an MAR model of order M as

$$X(n)=\sum_{m=1}^{M}A(m)X(n-m)+U(n),\tag{1}$$

where $A(m)$ denotes the m-th AR coefficient matrix and $U(n)$ the white-noise vector. To estimate the model coefficients, one can utilize the Yule-Walker equation, which is expressed as

$$GT_1 = T_2\,,\tag{2}$$

with $G=\left[A(1),A(2),...,A(M)\right],\ T_2=-\left[R(1),R(2),\cdots,R(q)\right],$ (2a, b)

$$T_1=\begin{bmatrix} R(0) & R(1) & \cdots & R(M-1) & \cdots & R(q-1) \\ R(-1) & R(0) & \cdots & R(M-2) & \cdots & R(q-2) \\ \vdots & \vdots & & \ddots & & \\ R(-M+1) & R(-M+2) & \cdots & R(0) & \cdots & R(q-M) \end{bmatrix},\tag{2c}$$

where $R(r)$ is the covariance matrix with time lag r (r= 1, 2, …, q). The maximum time lag q should be selected greater than k×M. The matrix G, assembled by the 1st to M-th AR coefficient matrices, can be solved using pseudo-inverse technique as

$$G=(T_2T_1^{T})(T_1T_1^{T})^{-1}\,.\tag{3}$$

After obtaining M coefficient matrices (from G), one can assemble the observable transformed system matrix S as follows

$$S=\begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & \cdots & I \\ -A(M) & -A(M-1) & -A(M-2) & \cdots & -A(1) \end{bmatrix}.\tag{4}$$

The complex conjugated pairs of eigenvalues μ's and μ^*'s of S have been proved to relate to ω's ($=2\pi f_i$) and ξ's as (taken the i-th eigenvalue conjugates μ_i and μ_i^* for example)

$$\mu_i,\mu_i^*=\exp\left(\left(-\xi_i\omega_i\pm j\omega_i\sqrt{1-\xi_i^2}\right)\Delta t\right),\tag{5}$$

where Δt is the sampling time interval and j the imaginary unit; with equation (5), ω_i's and ξ_i's can be solved. As for the corresponding eigenvectors v_i's of S, they are exactly φ_i's.

2.2. Stability Criteria

In the second step (ST2), apply the following stability criteria on all the modes to filter out unstable modes. Supposed that the modal frequency f_m, damping ratio ξ_m, and mode shape φ_m, of MAR order m have been obtained, they are regarded as locally stable if they satisfy the following stability criteria: 1) for f_m, $f_{m+p} - f_\varepsilon < f_m < f_{m+p} + f_\varepsilon$; 2) for ξ_m, $\xi_{m+p} - \xi_\varepsilon < \xi_m < \xi_{m+p} + \xi_\varepsilon$; 3) for φ_m, $\mathrm{MAC}_l < \mathrm{MAC}_{(m,m+p)}$; $p = -p_s$, …, -1, 1,…, p_s, where p_s is a pre-selected number of MAR order to be evaluated, f_ε a pre-selected frequency deviation tolerance, ξ_ε a pre-selected damping deviation tolerance, $MAC_{(m,m+p)}$ is the modal assurance criteria (MAC) between φ_m and φ_{m+p} , and MAC_l is a pre-selected MAC lower bound. Herein, those parameters are adopted empirically as $f_\varepsilon = 0.1$ Hz, $\xi_\varepsilon = 1\%$, $MAC_l = 0.95$, and $p_s = 3$. A mode with locally stable modal frequency, damping ratio, and mode shape is called a locally stable mode.

2.3. Stability Diagram (SD)

Plotting those frequencies of locally stable modes versus model order yields a SD. It has been observed that meaningful structural-modes show nearly the same frequency value when the model order is over-specified, but the spurious modes do not (Reynders et al. 2012). The statement may also be true for the damping ratios and mode shapes.

Based on the above statements, the structural modes can be identified in the third step (ST3) in a statistical manner: the modes appearing frequently throughout a wide range of model order, i.e., the statistically dominant modes, are identified as structural modes. The modal parameters such identified are expressed in terms of their means, i.e., the mean frequencies, mean damping ratios, and mean mode shapes, of the corresponding locally stable modes. For simplification, the term "mean" is omitted hereinafter.

With the aid of SD, one can identify the modal parameters without getting involved in any model-order optimization problem. On the other hand, without the aid of SD, one has to select a proper model order so that the time series are well fitted. Several criteria may help in selecting an optimal model order, with which the estimated information loss is minimized while a certain large-order penalty is introduced, from a number of candidate model orders. This study adopts the Akaike information criterion (AIC) (Akaike 1974), defined as AIC = -2(ML) + 2(NP), where ML denotes the maximum logarithm likelihood and NP the number of independently adjusted parameters within the model. For a MAR model of order M, ML can be expressed as ML= $-(k\log(2\pi) + \log|V_M| + k) \times N/2$, where V_M is the cross-covariance matrix of the estimated residual, and NP as NP= $Mk^2+k(k+1)/2$, leading the AIC to the following expression $AIC_M = Nk\log(2\pi) + N\log|V_M| + Nk + 2Mk^2 + k(k+1)$. Among a number of candidate model orders, the one that yields the minimum AIC value is selected as the optimal model order M_o.

Although, with the aid of AIC, the optimal model can be determined, not all the identified modal parameters based on such a model are physically meaningful. Some of the estimated coefficients of the model are merely for a better fitting of the mathematical model to the real time series; some of them are related to the structural modal parameters of engineers' interest. The AIC offers no clue on deciding physically meaningful modes, which will be illustrated below.

3. EXPERIMENT DESCRIPTION

The experiment bridge was a simply-supported through-type steel Warren truss bridge, as shown in Figure 2(a). It was 59.2 m in span length, 8.2 m in maximum height, and 3.6 m in width, designed for single lane. During the experiment, the bridge had been closed to traffic, and therefore no vehicles besides an experiment vehicle were allowed. Eight uni-axial accelerometers were installed vertically on the experiment bridge, five at the damage side (A1-A5) and three at the opposite side (A6-A8). The sampling frequency was set as 200 Hz for all sensors.

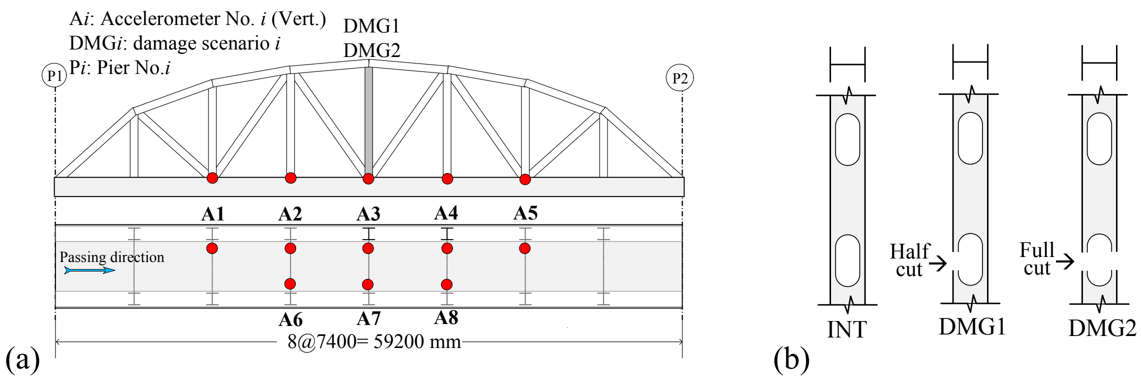


Figure 2: Sketch of (a) experiment bridge and sensor layout; (b) artificial damage.

Three scenarios are considered in this study, as sketched in Figure 2(b). Initially, the INT scenario denotes the intact bridge with no artificial damage. Then, two damage scenarios were applied consecutively. The first damage (DMG1) denotes a half cut applied by a cutting torch in the vertical member at the mid-span; the second damage (DMG2) denotes a full cut applied in the same member. Those applied cuts were designed to imitate real damage patterns that had been inspected previously, probably caused by corrosion or overloading.

A moving vehicle experiment was performed on the experiment bridge. During the experiment, the bridge was travelled by the experiment vehicle, which was a two-axle commercial van of 21kN in weight, and the bridge vertical acceleration responses were measured by the accelerometers. In this study, taken for analysis were 30-sec free-vibration (FrV) intervals about 5 seconds after the vehicle exited the bridge. A 5-sec spacing was adopted for securing the bridge responses fully free from vehicle-bridge interactions, although theoretically the bridge shows FrV just after the vehicle leaves the bridge.

4. EXPERIMENT RESULTS AND DISCUSSIONS

4.1. Identified Modal Parameters and Their Precision

To illustrate the algorithm of SD aided MAR analysis and its ability in choosing physically meaningful modes, Run 1 of the INT scenario is considered for example. Performing the MAR analysis on the bridge FrVs measured with all eight sensors and then applying stability criteria to every candidate mode, one can plot a SD with modes of locally stable frequency, damping ratio, and mode shape, as shown in Figure 3(a), or a SD with locally stable modes, as shown in Figure 3(b). From the SD, the structural modes, i.e., the statistically dominant modes that appear steadily throughout a wide range of model order, can be identified, as those marked in Figure 3(b) in vertical green virtual lines. In this example, the modal parameters thus identified are given in Figure 4.

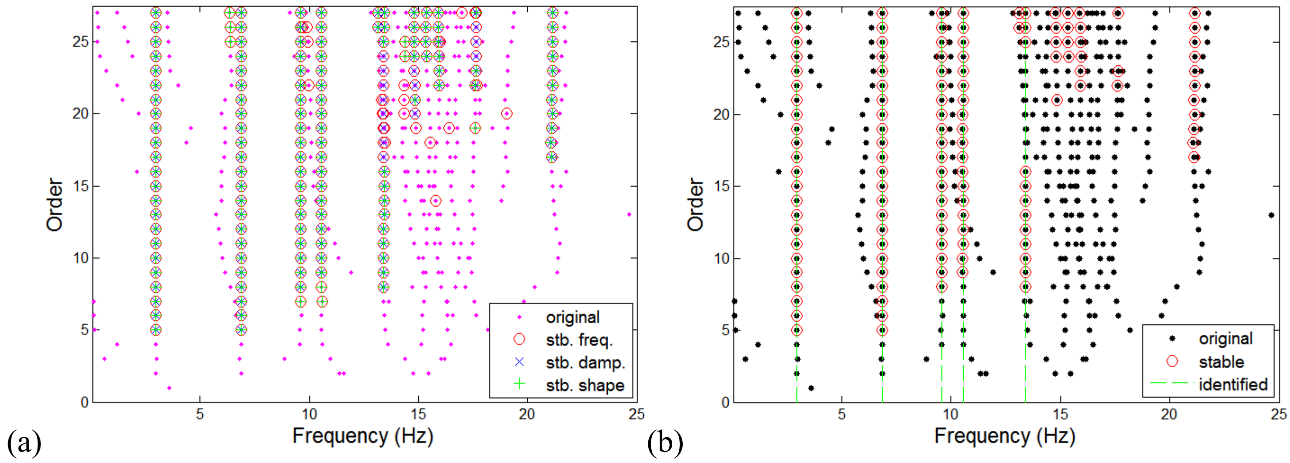


Figure 3: Stability diagram of intact bridge with (a) modes of locally stable frequency, damping ratio, and mode shape; and (b) stable modes.

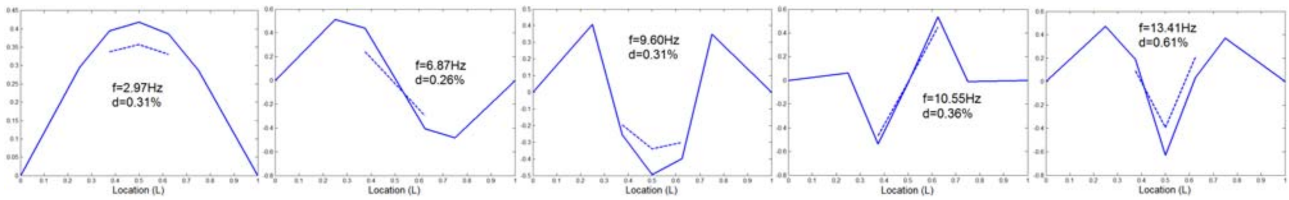


Figure 4: Modal parameters of intact bridge of the 1st to 5th (left to right) modes (solid line: A1-A5 side; dashed line: A6-A8 side).

Table 1: Identified modal frequencies (f, Hz) and damping ratios (ξ) of the first five modes along with their statistical properties.

		INT	DMG1	DMG2
	No.	5/5	12/12	10/10
f_1	M	2.975	2.976	2.885
	STD	0.001	0.002	0.001
	CV	0.05%	0.05%	0.04%
ξ_1	M	0.0032	0.0029	0.0032
	STD	0.0001	0.0001	0.0001
	CV	0.05%	3.96%	2.25%

		INT	DMG1	DMG2
	No.	5/5	12/12	9/10
f_2	M	6.872	6.887	6.876
	STD	0.002	0.003	0.002
	CV	0.03%	0.04%	0.03%
ξ_2	M	0.0028	0.0029	0.0033
	STD	0.0003	0.0005	0.0004
	CV	9.46%	17.24%	11.75%

		INT	DMG1	DMG2
	No.	5/5	12/12	10/10
f_3	M	9.608	9.685	9.663
	STD	0.005	0.007	0.002
	CV	0.05%	0.07%	0.02%
ξ_3	M	0.0036	0.0030	0.0027
	STD	0.0004	0.0005	0.0002
	CV	11.52%	16.69%	8.18%

		INT	DMG1	DMG2
	No.	4/5	11/12	9/10
f_4	M	10.559	10.594	10.568
	STD	0.005	0.015	0.004
	CV	0.04%	0.14%	0.04%
ξ_4	M	0.0063	0.0055	0.0046
	STD	0.0027	0.0026	0.0013
	CV	42.18%	46.71%	29.53%

		INT	DMG1	DMG2
	No.	4/5	5/12	6/10
f_5	M	13.418	13.494	13.461
	STD	0.004	0.015	0.008
	CV	0.03%	0.11%	0.06%
ξ_5	M	0.0044	0.0059	0.0055
	STD	0.0011	0.0015	0.0018
	CV	25.41%	26.27%	32.64%

Note
No.: number of runs (success/total)
M: mean
STD: standard deviation
CV: coefficient of variance

Following the same procedure, the modal parameters can be identified for all runs of all scenarios. Table 1 summarizes the number (No.) of successful runs and the mean (M), standard deviation (STD), and coefficient of variance (CV) of the identified modal frequencies and damping ratios of the first five modes for all scenarios. In most runs, the 1st to 3rd modes are successfully identified, but the 4th and 5th modes are identified successfully in less runs. For the modal frequencies, they are identified with very high precision, with CV smaller than 0.2% for all modes and all scenarios. For the damping ratios, they are identified with acceptably high precision (CV smaller than 4%) for the 1st mode, with lower precision (CV up to 17%) for the 2nd and 3rd modes, and with poor precision (CV up to 46%) for the 4th and 5th modes. As for the mode shapes, those in the same scenario are almost identical, despite no relevant statistical property given herein for brevity.

Table 2: Number of non-structural modes identified with the aid of SD and AIC.

	Number of non-structural modes		Remark:
Run	SD	AIC	optimal order
1	0	4	5
2	1	9	8
3	1	9	8
4	0	14	16
5	0	8	7

4.2. Comparison Study: Stability Diagram vs. Optimal Model Order

As has been mentioned in Sec. 2.3, with the aid of SD, one can identify the modal parameters without getting involved in any model-order optimization problem. Without the aid of SD, one has to select an optimal model order with the aid of a certain information criteria, e.g. the AIC herein, so that the time series are well fitted. However, a mathematically well-fitted model generally offers more than structural modes. Such a feature can be illustrated in Table 2, which compares the numbers of modes besides the above five structural modes obtained with the aid of AIC to SD. For brevity, only the results in INT scenario are given, but those in other scenarios show the same feature as well; for discrimination, the modes besides the above five structural modes are called non-structural modes, even though some of them might be related to true structural modes but not included above.

In Table 2, SD always offer few non-structural modes, e.g. at most 1 for Runs 2 and 3, verifying its validity in filtering out non-structural modes. Contrarily, the optimal models (i.e. the models of an optimal order) determined by AIC always offer many non-structural modes, up to 14 for Run 4. If no further criterion is introduced, it is generally difficult to differentiate structural modes from non-structural modes.

4.3. Effect of Artificial Damage

From Table 1, the change of the modal frequency and damping ratio due to different artificial damage scenarios can be observed. As DMG1 was applied, the first five (mean) modal frequencies increased slightly. As DMG2 was applied, the 1st modal frequency decreased obviously from those in INT and DMG1 scenarios, but the 2nd to 5th modal frequencies decreased slightly from those in DMG1 scenario and increased from those in INT scenario. Similar results were observed by means of FDD and FFT. As for the damping ratios, they did not show specific trend as DMG1 and DMG2 was applied. The change of the modal parameters may serve as damage sensitive features in SHM, even though the mechanisms of the change in modal parameter are still under investigation.

5. CONCLUDING REMARKS

This study validates the SD aided MAR analysis for identifying modal parameters of a real truss bridge subjected to two artificial damage scenarios. By the analysis method, the first five modal frequencies and mode shapes are identified with very high precision; the damping ratios are identified with acceptably high precision for the 1st mode but with lower precision for higher modes. It is observed that those modal parameters changed due to the artificial damage, but the mechanisms of the change are still under investigation. Moreover, the ability of the SD in selecting structural modes without getting involved in any model-order optimization problem is highlighted through a comparison study.

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