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VIRTUAL WORK SENSITIVITY METHOD FOR THE OPTIMIZATION DESIGN OF TALL BUILDINGS

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ABSTRACT

The stiffness of structure is a very important design issue, and the main index to ensure the stiffness is the story drift. Due to the super height and flexible structure, this index is usually the key factor to determine the size of the structure members of tall and super tall buildings. However, this work used to be a “trial and error” procedure, which was time-consuming and not so economical. This paper introduced a fast optimization method based on the principle of virtual work, which can help the designer to find the most efficient member to control the story drifts, and the automate resize procedure can provide the optimum size of the structure member to minimize the total cost. The virtual work of the structural members was calculated first by multiplying the joint displacement of the real load case by the joint force of the virtual load case. Then, the virtual work of unit cost of each member was defined as the Virtual Work Sensitivity (VWS) which indicate the contribution of each member to the certain displacement. Under the acceptable assumption that the changing of the member size affects little of its inner force, the design problem was converted to be a numerical optimization problem and many good algorithms are available in the mathematical step. The generalized reduced gradient method (GRG) method was adopted in this paper, as it was proved to be efficient to solve the optimization problem and integrated in the add-in tools of the office software that can be easily mastered by the engineers. After a few optimization cycles, the convergence of the total cost and the member size was obtained. Finally, the virtual work sensitivity method was adopted in a case study of a simplified structure and the best design scheme was obtained based on the minimum total cost by changing the design variables such as the thickness of the sheer wall and the size of the column. The results showed that the contribution of each member to the reducing of story drifts was great different which was not directly proportional to the member size and the virtual work sensitivity method was proved to be very efficient in the optimization design.

Keywords: tall buildings, virtual work sensitivity, optimization design

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1. INTRODUCTION

Due to the super height and flexible structure, modern tall and super tall buildings are very sensitive to the horizontal force such as the wind load and the earthquake. The stiffness is usually a main design issue, to meet the needs of the stiffness (story drifts), the structure members should have a comparatively large modulation. Usually this work will relatively significant affects the cost of the building. So the structural engineers have to cope with the challenge of not only designing safe and functional tall buildings in a relatively short time, but also ensuring that the design produced must be optimal in the sense of minimum structural cost. The traditional structural design of tall buildings is a trial-and-error procedure that relies heavily on engineers’ experience and intuition. Although the final design may be feasible, it does not guarantee optimality (Chan 2001).So it is of great importance to find a way to guide the engineers to optimize the structure.

However, with the complexity and scale of tall building structures, the simultaneous consideration of all system design consideration is incomprehensible using the traditional trial-and-error analysis–design procedure (Chan 2001). Many researcher have done a lot research to the optimal design, among them the optimality criteria (OC) has been shown to be very effective for large system and was successful used in the optimization of sever tall buildings (Zhou and Rozvany, 1992 ;Chan et al ., 1992; Chan, 2001) and usually the fundamental theory is the principle of virtual work. The principle of virtual work is the basis for a method used for calculating displacements in a structure when it is loaded under real loading conditions. Any displacement of some certain point can be expressed as the summary of the virtual work under the virtual force in the corresponding position and direction of each member. It relies on the internal energy in a system so it is very versatile in its application, such as the total displacement and the contributions of each member can be calculated (Christopher Douglas Basrrar, 2009; Flnley A. Charney,1993; Lin hai,2009). This paper will introduce a new way to calculate the virtual work and suggest a convenient way to optimize the structure based on the virtual work sensitivity of each structure member.

2. VIRTUAL WORK SENSITIVITY METHOD

The displacement d_j of a structure at the concern point j can be expressed as follows:

$$d_j = \frac{1}{Q}\sum_{i=1}^{n}\int_0^L \left(\frac{Mm}{EI_i} + \frac{Ff}{EA_i} + \frac{Vv}{EA_{v,i}} + \frac{Tt}{GJ_i}\right)dx \tag{1}$$

Where Q is the virtual force, M,F,V,T , m, f, v, t are the real and virtual inner force of the member respectively. The displacement can be expressed as the total virtual work $g_{0,}$ and they are consist of each member’s virtual work $g_{i..}$which represents its contribution to the whole structure. Assuming the cost of *ith* member is C_i and the virtual work sensitivity index b_i can be defined as follows:

$$b_i = \frac{g_i}{C_i} \tag{2}$$

It can be proved that to minimize the total cost, the index b_i of each active member should be the same. The calculation of the virtual work of each member by the equation 1 is not so easy since it needs numerical integration of the inner force at each point. Chan (2001) derived the concise form of the virtual work as the function of the member size. However, to calculate the exact virtual work force is still a laborious work. Considering the principle of virtual work, it is easy to be proved that:

$$W_i = W_e = \sum_{j}^{N_{joint\,s}} F_{joint} \Delta_{joint} \tag{3}$$

Since the joint force and joint displacement are very convenient to get from the FEM software, the virtual work can be calculated fast and accurately by the equation 1. The method will be proved by a case study in the next part of this paper.

To simplify the analysis, the design variable of each member can be the area A_i, and assuming that the moment of inertia I is proportional to the area A_i,. Considering the change of the member size affects little of the inner force of the tall buildings since they can be seen as a cantilever. Then, change the A_i, by α_i times , one can get the new virtual work $g'_{i.}$ and the new cost $C'_{i.}$:

$$g'_i = \frac{g_i}{a_i}; C'_i = a_i C_i \tag{4}$$

To find the suitable α_i to minimize the total cost $C_{Tot} = \sum C'_i = \sum a_i C_i$ with the constraint to its limit: $g = \sum g'_i = \sum \frac{1}{a_i} g_i = \overline{g}$,the Lagrangian method can be used, and it can be easily to prove ：

$$a_i = \sqrt{b_i \left. \sum g_i C_i \middle/ \overline{g} \right.} \tag{5}$$

The equation 5 suggests that the member with higher virtual work sensitivity index is more efficient to the optimal design and it provided the adjustment coefficient, one can easily get the optimized member sizes. However, the design variables always have boundaries to meet the strength requirements and architectural effect needs, which means as long as the design variable was modified to its boundaries, it cannot be changed any further. The members that are within the boundaries are called “active members”, and the equation 5 cannot be indirectly used any more. To solve this problem, the numerical optimization method should be used. Using the equation 4, the structure optimization problem turned to be a math program problem, and many numerical methods can be used. The generalized reduced gradient method (GRG) method was proved to be one of the efficient tools for optimization. What’s more, the method was integrated in the add-in tools of the office software, which can be easily mastered by the engineers. For the further information of the GRG method can refer to the related reference, such as Daniel Fylstra (1998).

3. CASE STUDY

A simplified model of a tall building is present at here as an example (Fig1). The material of the structure member is concrete with the strength of C30. The load case is a horizontal force of 1000KN at the top, and there are two different initial member sizes as listed in the table 1. The story drifts of the of the certain floor of the structure is also calculated and listed in the table, without loss of generality the limit of the story drift is 1/550, it can be seen from the table that the two initial design was either too stiffness or too flexible. The virtual work sensitivity method will be used to optimize the structure to minimize the total cost. The design variables are the size of the column and the thickness of the shear wall. The members are divided into five groups vertically and their boundaries are listed in the table 1.

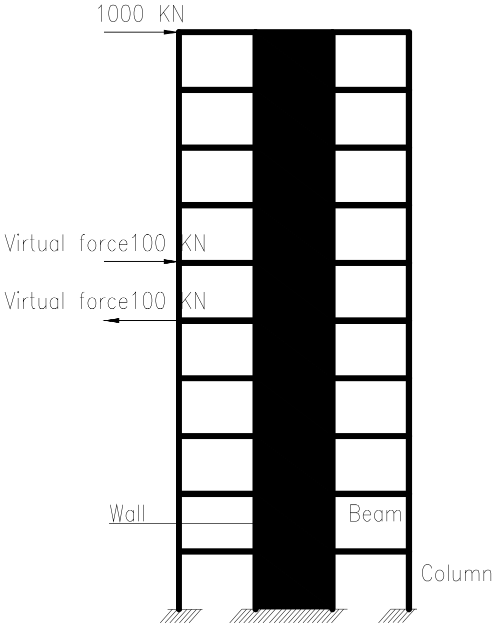

Figure 1: The simplified structure system of the example building.

. Table 1 Initial design of the structure of two different cases

Initial design	Column 1~5	Beams 1~5	Shear wall 1~5	Story drifts	Unit cost	Total cost
Boundary	0.25~1m	0.125~0.5m	0.125~0.5m	- -	- -	- -
Case 1	0.5×0.5m	0.25×0.5m	0.25m	1/370(6F)	300 CNY	16500 CNY
Case 2	0.5×1.0m	0.4×0.5m	0. 50m	1/747(6F)	300 CNY	33600 CNY

Note: To simply the analysis and to make construct, the heights of the cross section of the structure members are restricted to consistent and the area changing is reflected on the width of the cross section. The members are divided into five groups vertically, Column 1 respect the columns of the first and second floor of the structure, Column 2 respect the columns of the third and fourth floor of the structure and so on.

A 100KN force was loaded in the opposite direction at the top and bottom of the sixth floor as the virtual force. As introduced earlier, the virtual work of it can be obtained by multiplying the joint displacement of the real case by the joint force of the virtual case despite the shape and size of the member which is very convenient to access in the FEM software. Take the initial design case 1 for example, the virtual work of the shear wall of the second floor was calculated and listed in Table 2. The results are same with those provided by the FEM software as shown in Figure 2.

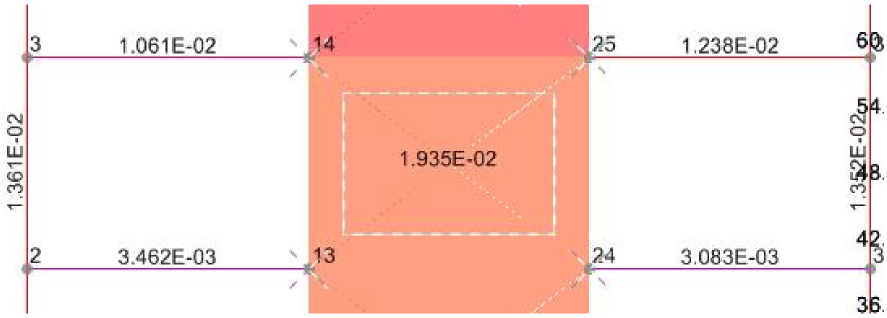

Figure 2 The relative virtual work per unit volume provided by the FEM software (SAP 2000)

. Table 2 Calculation of the virtual work

Joint	F1	dx	gdx	F3	dz	gdz	M2	r2	gr2
13	0.030	0.001395	-21.860	-21.860	0.001578	-0.034495	-3.8579	0.000800	-0.0031133
14	-0.470	0.004893	23.005	23.005	0.002816	0.0647821	0.3911	0.001410	0.000550
24	0.768	0.001395	21.863	21.863	-0.001580	-0.034544	-1.5094	0.000800	-0.001214
25	-0.328	0.004894	-23.008	-23.008	-0.002820	0.0649286	2.7892	0.001410	0.003936
Sum and per unit volume of the virtual work						0.058039/0.019346			

Note: the unit of the force and the moment is KN and KN.m respectively ; the units of the displacement are m and rad; The force and displacement in the direction of dy, r1 and r3 are zeros and not listed in this table.

Since the virtual work of all concerned members can be calculate by this method, the virtual work sensitivity can be obtained simultaneously. The results are listed in the table 3, as the virtual work sensitivity suggested the most efficient member to control the story drifts is the Beam 3, that is to say, to reduce the story drifts of the sixth floor the most economical way is to increase the size of the beam of the fifth and sixth floor. One can also read form the table 3 that the beams have much greater effects to control the story drifts of sixth floor and some members will even increase the story drifts such as the shear walls of the seventh to tenth floor whose virtual work sensitivity is negative. The characteristic of deformation shape of the shear wall and the moment resistant frame can help explain this phenomenon since they are flexure type and shear type respectively.

. **Table 3 Virtual Work sensitivity method of the optimization design**

Member	Virtual Work	Cost	Virtual work Sensitivity		Adjustment Coefficient	New virtual Work	New Cost
W1	0.1284	1800	7.13E-05	28.59%	1.464007167	0.087704	2635.213
W2	0.0949	1800	5.27E-05	21.13%	1.258814346	0.075388	2265.866
W3	0.0594	1800	0.000033	13.23%	0.995923804	0.059643	1792.663
W4	-0.0162	1800	-9E-06	-3.61%	0.5	-0.0324	900
W5	-0.00028	1800	-1.6E-07	-0.06%	0.5	-0.00056	900
Col1	0.0415	900	4.61E-05	18.48%	1.177184163	0.035254	1059.466
Col2	0.038	900	4.22E-05	16.92%	1.126507612	0.033733	1013.857
Col3	0.037574	900	4.17E-05	16.73%	1.120187417	0.033543	1008.169
Col4	0.0096	900	1.07E-05	4.28%	0.566261907	0.016953	509.6357
Col5	0.0128	900	1.42E-05	5.70%	0.653790533	0.019578	588.4115
BEAM1	0.0147	600	2.45E-05	9.82%	0.858479107	0.017123	515.0875
BEAM2	0.0593	600	9.88E-05	39.61%	1.723933789	0.034398	1034.36
BEAM3	0.1497	600	0.00025	100.00%	2	0.07485	1200
BEAM4	0.1021	600	0.00017	68.20%	2	0.05105	1200
BEAM5	0.0784	600	0.000131	52.37%	2	0.0392	1200

Note: the adjustment coefficient means the changing of area (width) of member's cross section.

As discussed in the second part, the virtual work sensitivity can be used in the optimization procedure. As long as the current virtual work was obtained, the new virtual work can easily calculated by the equation 4. The only task is to determine the adjustment coefficient. Since the GRG method has a considerable effect to solve this problem and it is integrated in add-in tools of the office software, engineers can utilize this tool to do the next complex numerical optimization procedure. After setting the object and the limits well, the mathematical optimization is very fast and the optimum size of the member will be obtained. The first optimization results of the initial design case 1 is listed in the Table 3, it can been seen that to minimize the total cost, the sizes of beams are increased and the thickness of the shear wall of seventh to tenth floor are decreased.

However, the assumption adopted in the virtually work sensitivity method that the inner force of the member changing little with its size is not exactly true, so the “optimum size” obtained in this optimization cycle should be checked in the model analysis and do the optimization procedure again. Usually, the .convergence of the total cost and the member size will be obtained after a few iterations. Using the virtual work method mentioned above, the optimum sizes are fast obtained as depicted in the figure 3.

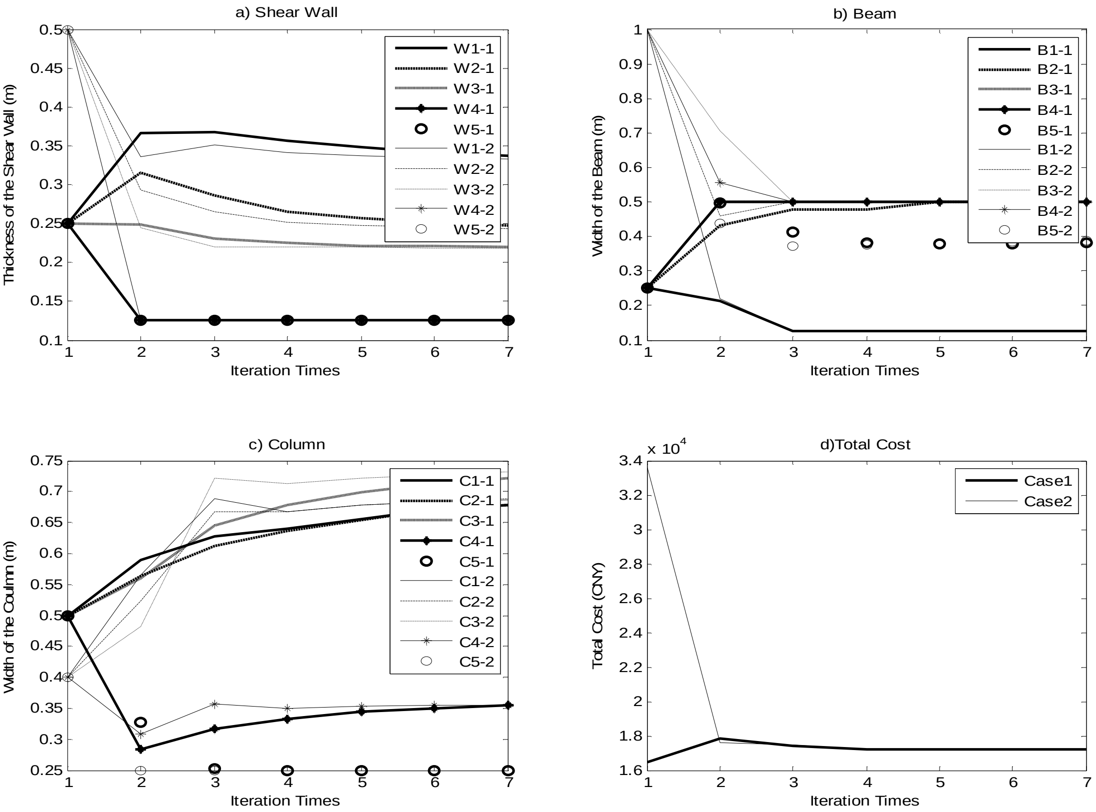

Figure 3 Variations of the design variables and the total cost during the optimization procedure

Note: W1-1 means W1 of the initial design case1, and so on.

It can be seen form the figure 3 that: the virtual work sensitivity method is very efficient in the optimization design, which is more evident in the first few iteration times. With 3 or 4 times reanalysis the structure, all the design variables converged to the optimum values. What’s more the two different design cases converged to the same values, which mean this algorithm is robust and applicable. The final optimization results also suggest that the contribution of each member to the reducing of story drifts is great different which is not directly proportional to the member size. To minimize the total cost, the materials should be transferred form the low virtual work sensitivity members to the high virtual work sensitivity members.

4. CONCLUSIONS

The stiffness of the structure is an important design issue, and the controlling of the story drifts is one of the most critical requirements of tall and super tall buildings. The virtual work sensitivity method introduced in this paper was prove to be an efficient tool for the engineers to optimize the structure form the time-consuming “trial and error” procedure. This paper discussed the virtual work calculating method, and recommended a method that multiplying the member joint force by the joint displacement. This method was proved to be convenient and applicable and can be widely used despite of the member type or shape. Then the utilizing of the GRG method that integrated into the add-tools of office software was proved to be user friendly and easy-to-handle for engineers.

The example adopted in this paper suggested that the contribution of each structure member is not directly proportional to the size, and to minimize the total cost the materials should be transferred to make the active members having the same virtual work sensitivity. It should also be mentioned that this method can also be used in the optimization problem with frequency constrains. And the stiffness constrain in this paper is the story drifts of sixth story, however, the actual structure has many other constrain, the multi-constrained optimization problem will be studied in the further research.

5. ACKNOWLEDGMENTS

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