Oxygen isotope fractionation during the freezing of sea water

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ABSTRACT. The dependence of oxygen isotope fractionation on ice growth rate during the freezing of sea water is investigated based on laboratory experiments and field observations in McMurdo Sound, Antarctica. The laboratory experiments were performed in a tank filled with sea water, with sea ice grown under calm conditions at various room temperatures ranging from −5°C to −20°C. In McMurdo Sound, the ice growth rate was monitored using thermistor probes for first-year landfast ice that grew to ~2 m in thickness. Combining these datasets allows, for the first time, examination of fractionation at a rate of 10−7 to 9.3 × 10−7 ms−1. A case study on sea ice in the Sea of Okhotsk, where the growth rate is modeled by coupling the thermodynamic properties of the sea ice with meteorological data, demonstrates the utility of the fitted models.

1. INTRODUCTION

Since variations in stable oxygen isotopic compositions (δ18O) are a good indicator of past surrounding environments, these have been used for climate research (Hoefs, 2009). For example, the vertical profiles of δ18O preserved in ice cores from Greenland and Antarctica have revealed a detailed temperature record for the past 700 ka from the relationship between δ18O and surface air temperature (Dansgaard and others, 1993; Johnsen and others, 1995), and the geographical distribution of the δ18O values of precipitation provides implications for the hydrological cycle on a global scale (Joussaume and others, 1984; Koster and others, 1993). In the case of sea ice, δ18O has been used to reveal growth processes mainly related to snow. Since snow usually has much lower values of δ18O (~−20‰ to −5‰) than sea water (~1‰ to 0‰), it is a useful tool for distinguishing snow ice from ice of sea-water origin and quantifying the fraction of snow (Lange and others, 1990; Jeffries and others, 1994, 1997; Worby and Massom, 1995; Granskog and others, 2004; Toyota and others, 2007). In addition to its relevance to snow, δ18O also holds promise as a tool for deducing columnar ice growth rate (Souchez and others, 1987, 1988; Eicken, 1998; Smith and others, 2012).

During the freezing of sea water, the heavier 18O-bearing isotopomer of water (H218O) is preferentially entrapped in the sea ice. Thus sea ice takes a somewhat larger value of δ18O than the underlying sea water. The degree of fractionation depends significantly upon the freezing conditions, mainly the freezing rate, and is usually represented by a fractionation factor (ε eff, si), defined by 1000 ln(α eff, si), which is approximately equal to Δ18Oice – Δ18Owater. If the relationship between ε eff, si and the freezing rate is derived, it is expected that the growth rate history of sea ice can be retrieved from the vertical δ18O profiles of the sea-ice core, referenced to the δ18O of the sea water from which the sea ice formed. That would allow for the estimation of the seasonal variation of ice growth rate, providing useful information on the timing and magnitude of salt fluxes to the deep ocean. Since such salt fluxes drive the global thermohaline circulation, the investigation of δ18O of sea ice can contribute to climate research in the polar region. In addition, ε eff, si is a critical parameter for estimating snow contribution quantitatively (Jeffries and others, 1997; Toyota and others, 2007) and examining water-mass formation processes from stable-isotopic ratios in the polar oceans (Macdonald and others, 1995; Melling and Moore, 1995; Yamamoto and others, 2001). In the Arctic Ocean there has been an attempt to reconstruct the ocean surface δ18O field using δ18O profiles from sea-ice cores (Pfirman and others, 2004).

At the interface between continuous sea-ice cover and water, the fractionation coefficient between pure water and ice has been estimated at 2.91% under equilibrium laboratory conditions (Lehmann and Siegenthaler, 1991). More widely varying values have been reported from observations at sea under varying freezing conditions. For example, Macdonald and others (1995) estimated a fractionation coefficient of 2.57 ± 0.10% at slow growth rates in spring for ~2 m thick ice. They studied the offset of δ18O between the bottom of the sea ice and the underlying water in the Canadian Beaufort Sea. Melling and Moore (1995) found a fractionation coefficient of 2.09 ± 0.38% for ~1 m thick ice in nearly the same season and region. From wintertime
observations, Toyota and others (2007) made a statistical estimate of the average $\varepsilon_{\text{eff}, \text{si}}$ of 1.73 ± 0.23% from the difference in the $\delta^{18}O$ distribution between sea ice (granular ice) and sea water for 0.1–2.5 m thick sea ice from the Sea of Okhotsk. Slightly higher values (1.91 ± 0.30%) were obtained using a similar method for 0.6–2.4 m thick sea ice from the Weddell Sea, Antarctica (Toyota and others, 2008). Thus the range of $\varepsilon_{\text{eff}, \text{si}}$ is ~1%, depending on the freezing rate. It is therefore important to correlate $\varepsilon_{\text{eff}, \text{si}}$ with ice growth rates.

Since during sea-ice formation brine is partly entrapped in the sea ice and the expelled highly saline water induces convection underneath sea ice during freezing, the experimental results on the $\varepsilon_{\text{eff}, \text{si}}$–growth-rate relationship obtained with pure water and ice by Lehmann and Siegenthaler (1991) cannot be simply applied. Although Eicken (1998) theoretically derived the formula that predicts the sea-ice growth rate from $\varepsilon_{\text{eff}, \text{si}}$ based on the stagnant boundary-layer model (Burton and others, 1953; Weeks and Lofgren, 1967), the parameters used in the model still need optimization from direct measurement. In this model, the key parameters are the boundary-layer thickness ($z_b$) and equilibrium fractionation coefficient ($\varepsilon_{\text{eq}}$), and Eicken (1998) assumed these parameters to be $z_b=1.3\text{ mm}$ and $\varepsilon_{\text{eq}}=2.91\%$. However, these values have not yet been validated over a full range of observational data. In McMurdo Sound, Antarctica, Smith and others (2012) showed from in situ measurements of $\delta^{18}O$ and measured growth rates typical of sea ice >1 m thick that the Eicken (1998) formula underestimated the real growth rate and they suggested some modification of the parameters used in Eicken’s model. However, their data contained incorporated platelet ice in addition to columnar ice.

One of the keys to resolving this issue is to collect $\delta^{18}O$ data for sea ice over a wide range of growth rates. In the field, a large growth rate can be achieved only for relatively thin ice, making it dangerous to collect such ice samples, while slow growth rate data can be obtained safely at a site with relatively thick ice but long-term monitoring is required. From laboratory experiments, for technical reasons it is not easy to obtain very slow growth rates. Therefore, in this study relatively large growth rates (>2.2 × 10^{-7} m s^{-1}) were obtained from laboratory tank experiments, while relatively small growth rates (<2.5 × 10^{-7} m s^{-1}) were provided by in situ observations in McMurdo Sound and the southern Sea of Okhotsk. By combining these datasets, a wide range of growth rates was obtained. Only data from ice of columnar crystal structure from the McMurdo Sound observations are included here to simplify the analysis by removing possible confounding influences of crystal structure. Our purpose is to examine the theoretically derived formula that correlates fractionation of $\delta^{18}O$ with growth rate based on real data, in order to find the best-fit parameters for use in the theoretical model. In addition, an empirical formula is derived that correlates fractionation of $\delta^{18}O$ with growth rate over a wide range of growth rates. Since this issue is closely related to the microphysics of sea ice during freezing, we believe that our study might also provide some insights into freezing processes at the interface.

2. LABORATORY EXPERIMENT

2.1. Apparatus

In order to collect samples for relatively large growth rates, we conducted a laboratory experiment with a thermally insulated rectangular tank (inner dimensions of 0.3 m depth × 0.3 m width × 0.65 m height) in a cold room (Fig. 1). The tank was made from transparent acrylic 0.01 m thick and was covered on all surfaces except the upper surface with a 0.1 m thick layer of Styrofoam™ to prevent freezing on the side walls and the bottom of the tank. Natural sea water ($S=32.5$ on the Practical Salinity Scale 1978 (UNESCO, 1981); $\delta^{18}O=–0.8\%$) collected from the Pacific Ocean near the coastal region of Hokkaido, Japan, was poured into the tank up to 0.60 m depth, and sea ice was grown to ~5 cm thickness under calm and steady conditions. We had to stop freezing at this thickness to minimize the change in the freezing conditions caused by the expelled brine, i.e. the decrease in the freezing-point temperature due to the increase in water salinity within the tank. Growth rates were controlled by changing the room temperature from –5°C to –20°C. The temperature of the air 5 cm above the ice surface and in the ice or water at tank depths of 0.5, 2.5, 5, 15, 35 and 55 cm was monitored using copper–constantan thermocouples mounted on the side wall (Fig. 1). These
temperature data were logged on a data logger (YOKOGAWA, model DR230) at 1 min intervals. Ice thickness was measured visually at (mostly) 3 hour intervals with a measuring scale attached to the side wall. In order to monitor the salinity and $\delta^{18}$O of sea water, we made small holes on a side wall of the tank at depths of 5, 15, 25, 35, 45 and 55 cm and fitted them with a septum through which water samples were taken with a syringe before and after each experiment.

### 2.2. Experiments and processing

Before starting this experiment, the tank was cooled uniformly to a room temperature of –1°C, with occasional stirring. After the water temperature had settled to about –1°C, we set the room temperature at –5°C, –7.5°C, –10°C, –12.5°C, –15°C or –20°C and began to grow sea ice at each temperature. When the ice thickness reached ~5 cm, we used a saw to cut out a rectangular ice block with a basal area of 0.2 m × 0.2 m. Immediately after collecting the ice block, we transferred it to another cold room set to –15°C. It remained there until ready for ice structure analysis.

In the cold room, the ice block was divided into four sections with a basal area of 0.1 m × 0.1 m. One section was used for thick-/thin-section analysis of textural structure, two sections were used to measure ice density, salinity and $\delta^{18}$O and the fourth section was archived. Thin (<1 mm) and thick (5 mm) sections were analyzed to observe, respectively, the crystallographic alignments with crossed linear polarizers and the structure of inclusions with scattered light. One example grown at a room temperature of –15°C is shown in Figure 2. As shown in Figure 2b, ice texture is composed mainly of four types: (1) a vertical $c$-axis layer (0.0–0.5 cm depth), (2) a transition layer (0.5–2.0 cm), (3) an inner columnar layer (2.0–3.3 cm) and (4) a bottom columnar layer (3.5–5.0 cm). The ice section was sliced into these four segments to measure density, salinity and $\delta^{18}$O. A bottom columnar layer was categorized separately because it exhibited different properties (e.g. significantly higher salinity and brine volume fraction than the inner columnar layer, as shown in Table 1). The ice density was determined from the weight and dimensions of each segment, with an estimated accuracy of ±1%. The salinity and $\delta^{18}$O were measured for the melted ice segments with a salinometer (TOWA Electronic Industry, model SAT-210) and an isotope ratio mass spectrometer (Thermo Finnigan DELTA plus), respectively. The measurement method of $\delta^{18}$O is dual inlet based on Epstein and Mayeda (1953). The instrumental accuracy of salinity is <0.05. The analytical precision of $\delta^{18}$O is estimated to be 0.02‰ from the root mean square of the differences in values of two measurements on the same sample for all samples. The brine volume fraction was calculated from ice temperature, density and salinity from the formula of Cox and Weeks (1983) at each layer.

An example set of results, obtained at a room temperature of –15°C, is shown in Table 1. This table shows that each layer has characteristic properties: the first layer is characterized by low density, low salinity and high $\varepsilon_{eff, si}$, the second layer by low $\varepsilon_{eff, si}$, the third layer by high density and the fourth layer by low density and significantly high salinity. In this study, particular attention is paid to the inner columnar layer (layer 3) as a commonly observed sea-ice type. Another noticeable result in Table 1 is the increase in salinity (2.14) and the decrease in $\delta^{18}$O (–0.15‰) of sea water during the

### Table 1. One example of the vertical profiles obtained for a room temperature of –15°C. See Figure 2 for the depths of layers 1–4

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density kg m⁻³</th>
<th>Salinity psu</th>
<th>Brine volume fraction</th>
<th>$\delta^{18}$O</th>
<th>Difference*‰</th>
<th>$\varepsilon_{eff, si}$ ‰</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea water</td>
<td>–</td>
<td>32.02</td>
<td>–</td>
<td>–0.81</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Before</td>
<td>–</td>
<td>34.16</td>
<td>–</td>
<td>–0.96</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Layer 1</td>
<td>851</td>
<td>7.89</td>
<td>6.9</td>
<td>1.51</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td>Layer 2</td>
<td>876</td>
<td>9.75</td>
<td>10.2</td>
<td>0.56</td>
<td>1.37</td>
<td>1.40</td>
</tr>
<tr>
<td>Layer 3</td>
<td>890</td>
<td>11.5</td>
<td>11.5</td>
<td>0.75</td>
<td>1.56</td>
<td>1.64</td>
</tr>
<tr>
<td>Layer 4</td>
<td>866</td>
<td>11.59</td>
<td>13.3</td>
<td>0.73</td>
<td>1.54</td>
<td>1.61</td>
</tr>
<tr>
<td>Bulk mean</td>
<td>875</td>
<td>9.94</td>
<td>12.7</td>
<td>0.73</td>
<td>1.54</td>
<td>1.59</td>
</tr>
</tbody>
</table>

*The difference of $\delta^{18}$O between initial sea water and sea ice.

$\varepsilon_{eff, si}$ is the effective fractionation coefficient using the Rayleigh equation.

Before and after indicate the values of the averages, taken for the full depth of sea water, just before and after the freezing experiment.
experiment. Regarding the increase in salinity, the resultant decrease of the freezing point is estimated to be only 0.1 °C, so its effect on the result is assumed to be negligible. On the other hand, the decrease in $\delta^{18}O$ corresponds to about one-tenth of $\varepsilon_{eff, si}$. Thus the effect is non-trivial and careful analysis is required to estimate $\varepsilon_{eff, si}$.

2.3. Estimation of effective fractionation coefficient ($\varepsilon_{eff, si}$)

In general, when we conduct a freezing experiment in a closed system, the isotope ratio of the remaining sea water ($R_{SW}$) changes due to the formation of sea ice. To derive a relation for the effective fractionation coefficient in terms of the original sea-water $\delta^{18}O$ and the $\delta^{18}O$ of ice, the evolution of $R_{SW}$ should be calculated from the following Rayleigh equation (Hoesf, 2009):

$$R_{SW} = f^{n-1}$$

where $R_{SW}$ is the isotope ratio of the initial sea water and $R_{SW0}$ is the instantaneous isotope ratio of the remaining sea water, $f$ is the fraction of the remaining sea water ($V_{SW0}$) to the initial volume ($V_{SW0}$) and $n$ is the fractionation factor given by $R_{ice}/R_{SW}$.

Since $\ln \left( \frac{R_{SW}}{R_{SW0}} \right) \approx \frac{\delta^{18}O_{SW} - \delta^{18}O_{SW0}}{1000}$, Eqn (1) can be expressed as

$$\delta^{18}O_{SW} - \delta^{18}O_{SW0} = \varepsilon_{eff, si} \cdot \ln \left( \frac{V_{SW}}{V_{SW0}} \right)$$

where $\varepsilon_{eff, si}$ is equal to $\delta^{18}O_{ice} - \delta^{18}O_{SW}$. Then the following equation can be derived:

$$\varepsilon_{eff, si} = \frac{\delta^{18}O_{ice} - \delta^{18}O_{SW0}}{1 + \ln \left( \frac{V_{SW}}{V_{SW0}} \right)}$$

The value of $\varepsilon_{eff, si}$ at each layer can be obtained by substituting into Eqn (3) $V_{SW}/V_{SW0} = 59.8 \text{ cm/60 cm}$ for the first layer, $58.8 \text{ cm/60 cm}$ for the second layer, $57.3 \text{ cm/60 cm}$ for the third layer and $55.8 \text{ cm/60 cm}$ for the fourth layer. The result applied to a room temperature of –15°C is presented in Table 1. By taking the thickness-weighted average, the bulk mean $\varepsilon_{eff, si}$ of the total layer is estimated as 1.59‰. If this value is substituted into Eqn (2), the change in $\delta^{18}O$ of sea water during the experiment is expected to be –0.14‰, which coincides approximately with the observed value of –0.15‰, obtained by taking the average of the $\delta^{18}O$ of sea water at depths of 5, 15, 25, 35, 45 and 55 cm, within the measurement accuracy (Table 1). Therefore our method of estimating $\varepsilon_{eff, si}$ seems reasonable.

2.4. Estimation of growth rates

The growth rates in each layer were calculated from the slope of the thickness-versus-time curve, within the layer, by a least-squares method. For example, in the case of an inner columnar layer we estimated the slopes and the confidence interval at the 95% level by using the thickness data between 2.0 and 3.5 cm (Fig. 3). The results for various room temperatures for each layer are listed in Table 2, which shows that the growth rates range from (2.25 ± 0.50) × 10^{-7} ms^{-1} for –5°C to (9.28 ± 2.08) × 10^{-7} ms^{-1} for –20°C for layer 3. For other layers, the growth rates ranged from 1.81 × 10^{-7} ms^{-1} (–5°C) to 6.75 × 10^{-7} ms^{-1} (–20°C) (layer 4) and from 3.17 × 10^{-7} to 9.16 × 10^{-7} ms^{-1} (layer 2) and from 1.81 × 10^{-7} to 9.31 × 10^{-7} ms^{-1} (layer 4) (Table 2). Whereas the growth rate was relatively slow for layer 1, it was comparable in the other layers, indicating that the sea ice grew at a near-constant rate except at the early stage. The effect of ocean heat flux transferred from sea water to sea ice was not taken into account here since the sea water underlying sea ice was kept at ~2.0°C, somewhat below the freezing temperature (~1.7°C), in each case while sea ice was grown. Consequently a number of thin platelet ice crystals with a height of a few centimeters extended from the bottom of the sea ice into water that appears to be supercooled. Therefore, the ocean heat transfer from sea water to ice is considered not to be positive.

3. OBSERVATIONS

3.1. McMurdo Sound, Antarctica

We have analyzed samples of columnar landfast sea ice from McMurdo Sound, Antarctica. Lying at the southwestern end of the Ross Sea, McMurdo Sound is bounded to the south by the McMurdo Ice Shelf, to the west by the Victoria Land coast and to the east by Ross Island (Fig. 4a).

The landfast sea ice in McMurdo Sound has been the subject of intensive scientific investigation since the early 20th century (Wright and Priestley, 1922). Most papers reporting on McMurdo Sound sea-ice structure have noted

<table>
<thead>
<tr>
<th>Room temperature</th>
<th>Ice layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>°C</td>
<td>(0.0–0.5 cm)</td>
</tr>
<tr>
<td>10^{-7} ms^{-1}</td>
<td></td>
</tr>
<tr>
<td>–5.0</td>
<td>1.81 ± 0.00</td>
</tr>
<tr>
<td>–7.5</td>
<td>5.33 ± 0.26</td>
</tr>
<tr>
<td>–10.0</td>
<td>3.11 ± 0.09</td>
</tr>
<tr>
<td>–12.5</td>
<td>4.08 ± 1.71</td>
</tr>
<tr>
<td>–15.0</td>
<td>4.28 ± 0.62</td>
</tr>
<tr>
<td>–20.0</td>
<td>6.75 ± 1.45</td>
</tr>
</tbody>
</table>
the presence of incorporated platelet ice, at least in the latter part of the growth season (e.g. Paige, 1966; Crocker and Wadhams, 1989; Jeffries and others, 1993; Gow and others, 1998; Purdie and others, 2006; Gough and others, 2012a). This paper focuses on columnar ice only; therefore, data from the latter part of the growth season when incorporated platelet ice was observed are not included in our analysis.

3.1.1. 1999 data
Two 1999 McMurdo Sound measurements of sea-ice growth rates for columnar ice were included in the analysis, along with the corresponding values of \( \varepsilon_{\text{eff}, \text{si}} \), calculated from the \( \delta^{18}O \) sea-ice measurements in conjunction with the end-of-season \( \delta^{18}O \) sea-water measurement. Core sampling was conducted on 24 August and sea-water sampling was conducted on 23 August. Full details of the equipment used, location and methodology are given by Smith and others (2012). The other growth-rate and \( \varepsilon_{\text{eff}, \text{si}} \) data reported in that paper are not included here due to the presence of incorporated platelet ice.

3.1.2. 2009 data
Seven 2009 McMurdo Sound measurements of columnar sea-ice growth rates were included in the analysis, along with the corresponding values of \( \varepsilon_{\text{eff}, \text{si}} \), calculated from the mean of \( \delta^{18}O \) sea-ice measurements at each depth in cores taken throughout the growth season, in conjunction with the \( \delta^{18}O \) sea-water measurement taken close to the end of the observation period. Sea-ice cores were collected on 22 June, 10 August, 21 September and 27 September. Full details of the equipment used, location and methodology are given by Gough and others (2012a). Samples in which incorporated platelet ice was observed are not included in the present analysis.

The sea-water sample used in the analysis was from 10 m depth below the water surface at the Erebus Bay site on 23 July 2009 (\( \delta^{18}O = -0.61 \% \)). This sample was selected as it was collected when only columnar ice was present (i.e. it is not affected by the presence of platelet ice) (Fig. 4a; Gough and others, 2012a). When sampling sea water, much care was taken to ensure that freezing did not occur. Sea-water samples were collected sporadically from June to September in 2009 (\( N = 12 \)) while the ice-core samples were collected. The sample statistics show the mean to be \( -0.61 \pm 0.06 \% \). A Niskin bottle (NIO configuration) was used to sample water. Some water was allowed to drain from the bottle when it was brought to the surface before the sample water was sieved and used to double-rinse the 100 mL glass vials before two vials were filled (leaving a small air gap) and then sealed with lids that had rubber inserts. The vial lids were then taped tightly shut. The vials were stored in darkness at \( \sim 15 \^\circ \text{C} \) at Scott Base over the 2009 winter and transported by airplane to New Zealand for measurement of \( \delta^{18}O \).

For sea ice, stable-isotope samples were retained from salinity samples after the salinity of the melted samples had been measured (Gough and others, 2012b). Sections of ice 10 cm in vertical thickness were cut from freshly extracted ice cores and quickly placed in plastic pots with lids. The pots were transported cold to Scott Base and melted over a 2 day period. Once melted, samples were opened, stirred and measured with a salinometer. The sample was then used to prepare two glass vials for later isotopic analysis using a similar procedure to that described above for the sea-water sample. Only one sample was retained from each segment of each core sampled, but on most occasions two cores taken at the same location and time were used to prepare independent stable-isotope samples.

Oxygen isotope analysis was carried out at the Department of Chemistry, University of Otago. Oxygen isotope ratios in the samples were analysed by continuous flow mass spectrometry. Aliquots (0.5 mL) of sample were equilibrated with \( \text{CO}_2 \) (0.3% in He) for at least 18 hours at 25°C. After equilibration, the \( \text{CO}_2 \) was analysed utilizing a GasBench (Thermo Finnigan, Bremen, Germany) interfaced to a Delta Plus Advantage mass spectrometer (Thermo Finnigan, Bremen, Germany). Three water standards of known value
were run with each set of samples, and the isotope ratios of the samples were normalized to the Vienna Standard Mean Ocean Water (VSMOW) scale. Mean precision (the repeatability of a measurement on a single sample) for all ice and ocean samples was ±0.05‰.

3.2. Sea of Okhotsk
The southern Sea of Okhotsk is at one of the lowest latitudes in the world where a sea-ice cover exists. The sea-ice extent is at its largest at the end of February and the level ice thickness is mostly <1 m, with the mean being 0.3–0.5 m (Toyota and others, 2004), although ridged ice more than a few meters thick occasionally appears (Fukamachi and others, 2006). In this region, sea-ice observations have been carried out in collaboration with the Japan Coast Guard on board the icebreaker P/V Soya in early February every winter since 1996, corresponding to the ice growth season, to survey the ice and oceanographic conditions and to tackle ad hoc topics related to sea ice.

During 1996–99, focus was placed on the estimation of the surface heat budget to reveal the transformation process of the air mass that originates over Siberia (Toyota and Wakatsuchi, 2001). For this purpose, we conducted measurements of meteorological data (air temperature, pressure, relative humidity, wind, incident and reflected solar radiation, cloud) and of ice conditions (ice concentration, types, thickness) along the ship’s track. Ice concentration and types were observed visually according to the ASPeCt (Antarctic Sea Ice Processes and Climate) protocol, which was designed for ship-based observations in Antarctic seas (Worby and Allison, 1999), and ice thickness was monitored using a video monitoring system (Toyota and others, 2004). While the observation period was limited to ~1 week, the ship’s tracks covered a wide area of the southern Sea of Okhotsk. Therefore, we examined the representative diurnal variation of the surface heat budget in this area from hourly averaged meteorological data. In our calculations, we used a thermodynamic ice model similar to that of Maykut (1978, 1982) based on the obtained ice thickness distribution. As a result, it was found that sea ice grows only at night-time due to abundant daytime solar radiation at low latitudes at that time of year. The averaged night-time growth rates were estimated to be 2.78, 1.61, 3.58 and 2.19 × 10^{-7} m s^{-1} for the years 1996–99, respectively (Toyota and Wakatsuchi, 2001). Taking the average of this result, we use (2.54 ± 0.83) × 10^{-7} m s^{-1} as a representative growth rate in this region.

During 2003–05, our major topic of investigation was the properties of sea ice and overlying snow in this region (Toyota and others, 2007). For this purpose, we collected 27 ice samples, ranging from 6 to 225 cm in thickness, with a core auger and then analyzed the crystallographic alignment and the vertical profiles of density, δ^{18}O and salinity at 3 cm depth intervals. The sampling locations are plotted in Figure 4b. δ^{18}O of columnar ice was shown to be 0.9 ± 0.3‰ (mean ± 1 std dev.; N = 301) (Toyota and others, 2007). In addition, the δ^{18}O of the surface sea water was shown to be −1.0 ± 0.2‰ (mean ± 1 std dev.) from the samples collected over a wide area during the same cruises. Since the δ^{18}O distributions of both sea ice and sea water follow an almost normal distribution, the representative effective fractionation coefficient (ε_{eff, i}) of columnar ice is estimated statistically to be 1.68 ± 0.10‰ from the difference of the distributions. Although the observation period is different from that for the growth rate estimation, there was no significant difference in the meteorological and ice conditions between these two periods: for example the mean air temperatures and level ice thicknesses were −5.2, −4.1 and −10.1°C and 0.42, 0.60 and 0.42 m for 2003–05 and −5.0, −5.4, −8.1 and −5.0°C and 0.18, 0.55, 0.30 and 0.29 m for 1996–99, respectively. Therefore we regard (2.54 ± 0.83) × 10^{-7} m s^{-1} and 1.88 ± 0.10‰ as the representative values of ice growth rate and ε_{eff, i} in winter in this region. Since the estimation of growth rate is not a direct measurement, these data are to be used mainly to provide some support for the derived parameterizations.

4. SUMMARY OF THE STAGNANT BOUNDARY-LAYER MODEL
Prior to showing the results, we briefly describe the stagnant boundary-layer (SBL) model. The SBL model was introduced by Burton and others (1953) to explain theoretically the incorporation of impurity into single crystals frozen from melt. The model divides the sea water into a fixed-depth boundary layer adjacent to the bottom of the sea ice and an underlying well-mixed layer (Fig. 5). Within the boundary layer, δ^{18}O is taken to decrease toward the ice as it is affected by fractionation due to bottom freezing, hence a purely diffusive process is dominant in this layer. Within the well-mixed layer, δ^{18}O is kept constant by convection and advection. ε_{eff, i} is defined as the difference between the δ^{18}O of the sea ice near the ice–water interface and the δ^{18}O of the well-mixed layer. According to this model, the equation of the solute concentration (C_i) within the boundary layer is described as (Burton and others, 1953)

\[ \frac{\partial C_i}{\partial t} = D \frac{\partial^2 C_i}{\partial z^2} + v_i \frac{\partial C_i}{\partial z}, \]

where D is a molecular diffusivity, z is the vertical coordinate and v_i is growth rate. In our case, C_i corresponds
to the isotope ratio normalized by VSMOW, i.e.
\[ C_i = \frac{(^{18}\text{O}/^{16}\text{O})_{\text{seawater}}}{(^{18}\text{O}/^{16}\text{O})_{\text{VSMOW}}} = 1 + \frac{\delta^{18}\text{O}_{\text{seawater}}}{1000}, \]
and \( D \), the diffusivity of \( H_2^{18}\text{O} \) at \(-2^\circ \text{C} \), is set to \( 1.20 \times 10^{-9} \text{m}^2 \text{s}^{-1} \) (Levine, 1983). The boundary condition is given by
\[ -D \frac{\partial C_i}{\partial z} \bigg|_{z=0} = \nu_l \cdot (1 - \alpha_{\text{eq}}^* - \alpha_{\text{eq}}^0) C_0 \]
where \( \alpha_{\text{eq}}^* \) is the equilibrium fractionation factor, equal to \( 1 + \varepsilon_{\text{eff,s}} \cdot \exp(-z_{\text{bl}} \nu_l / D) \) at \( z = 0 \).

Furthermore, by taking into account the entrapment of brine into sea ice, Eicken (1998) derived the following formula correlating \( \varepsilon_{\text{eff,i}} \) with \( \nu_l \) for sea ice:
\[ \varepsilon_{\text{eff,i}} = \left[ \frac{\alpha_{\text{eq}}^* + (1 - \alpha_{\text{eq}}^0) \cdot \exp(-z_{\text{bl}} \nu_l / D)}{\alpha_{\text{eq}}^0 - \alpha_{\text{eq}}^*} - 1 \right] \cdot 1000 \]
where \( \varepsilon_{\text{eff,i}} \) is the effective fractionation coefficient between pure ice and sea water and is described as \( \varepsilon_{\text{eff,i}} = \varepsilon_{\text{eff,S}}(0) - C_{\text{li}} \cdot (C_i - C_{\text{li}}) \), where \( \varepsilon_{\text{eff,S}}(0) \) is the effective fractionation coefficient at \( z = 0 \).

The application of the formula of Cox and Weeks (1975) to our experimental data started to deviate from the line for growth rates less than \( 2.0 \times 10^{-7} \text{m}^{-1} \) (fig. 32 in Cox and Weeks, 1975). They inferred that the change in slope is probably due to a change in the morphology of the ice-water interface. However, as shown in Figure 6, there is some discontinuity at the threshold. To smooth this discontinuity, Cox and Weeks (1988) introduced a second formula with a new threshold of \( 3.6 \times 10^{-7} \text{m}^{-1} \). In this paper, we adopt the formula and threshold of Cox and Weeks (1975) because we place higher priority on the underlying physical processes than on obtaining a smooth curve fit at the threshold and these two predicted curves almost coincide except around the threshold (Fig. 6).

Cox and Weeks (1975) set this threshold because their experimental data started to deviate from the line for growth rates less than \( 2.0 \times 10^{-7} \text{m}^{-1} \) (fig. 32 in Cox and Weeks, 1975). They inferred that the change in slope is probably due to a change in the morphology of the ice-water interface. However, as shown in Figure 6, there is some discontinuity at the threshold. To smooth this discontinuity, Cox and Weeks (1988) introduced a second formula with a new threshold of \( 3.6 \times 10^{-7} \text{m}^{-1} \). In this paper, we adopt the formula and threshold of Cox and Weeks (1975) because we place higher priority on the underlying physical processes than on obtaining a smooth curve fit at the threshold and these two predicted curves almost coincide except around the threshold (Fig. 6).

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5. RESULTS
The columnar ice data obtained from field observations and the laboratory experiment are plotted in Figure 7a, which shows that although the data from Antarctica and the experiment cover separate ranges of growth rates, they can be connected at a growth rate of about \( 2.0 \times 10^{-7} \text{m}^{-1} \) (the threshold of Cox and Weeks, 1975) and that the data from the Sea of Okhotsk lie approximately on the line of the experimental data. We therefore combined the Antarctic field observations dataset and the laboratory experiment dataset in order to investigate processes over the full range of growth rates.

We then applied the model of Eicken (1998), which is based on the SBL model, to our results to examine the optimal values of the physical parameters used in the model.
The curve of Eicken (1998) is superimposed on Figure 7a. It is found that overall the model of Eicken (1998) (1) and (2) were drawn based on Antarctic field data, where \( z_{2d} = 0.90 \text{ mm} \) and \( a_{eq}^i = 1.00311 \), and on laboratory experiments, where \( z_{2d} = 0.10 \text{ mm} \) and \( a_{eq}^i = 1.00266 \), respectively, whereas \( z_{2d} = 1.3 \text{ mm} \) and \( a_{eq}^i = 1.00291 \) in Eicken (1998). (b) Fitting curve of Eqn (9) obtained for practical use from modified Eicken (1998) (1) for \( \nu < 2.0 \times 10^{-7} \text{ m s}^{-1} \) and (2) for \( \nu > 2.0 \times 10^{-7} \text{ m s}^{-1} \) plotted with data from the field observations and laboratory experiments.

The curve of Eicken (1998) is superimposed on Figure 7a. It is found that overall the model of Eicken (1998) underestimates \( \varepsilon_{eff, si} \), as pointed out by Smith and others (2012). The laboratory experiment and Sea of Okhotsk observational data presented here show significant deviation from the Eicken (1998) model at growth rates larger than about \( 2.0 \times 10^{-7} \text{ m s}^{-1} \).

To improve this model, we first attempted to use the form of the Eicken (1998) equation and performed regression analysis using least-squares fitting to select values of the parameters \( z_{2d} \) and \( a_{eq}^i \) that fit all the growth rate data. However, that attempt proved unsuccessful because the slope of the curve at \( \sim 2.0 \times 10^{-7} \text{ m s}^{-1} \) increased significantly for decreasing growth rates. Hence we separately examined the optimal parameters for field observations in Antarctica (\( \nu < 1.7 \times 10^{-7} \text{ m s}^{-1} \)) and the laboratory experiment (\( \nu > 2.2 \times 10^{-7} \text{ m s}^{-1} \)) to minimize the root-mean-square error (RMSE). As a result, we obtained \( \varepsilon_{eq}^i = 3.11 \pm 0.59 \%) and \( z_{2d} = 0.9 \pm 1.9 \text{ mm} \) at the 95% confidence level for \( \nu < 2.0 \times 10^{-7} \text{ m s}^{-1} \), and \( \varepsilon_{eq}^i = 2.66 \pm 0.13 \%) and \( z_{2d} = 0.1 \pm 0.1 \text{ mm} \) at the 95% confidence level for \( \nu > 2.0 \times 10^{-7} \text{ m s}^{-1} \). The curves for both cases are drawn in Figure 7a. Slight gaps seen at \( \nu = 2.0 \times 10^{-7} \text{ m s}^{-1} \) come from the discontinuity in \( k_{eff,5} \) of Cox and Weeks (1975), as mentioned before. Thus significantly different sets of parameters were obtained for the two ranges of growth rates. It is especially noticed that while the values of \( z_{2d} \) and \( \varepsilon_{eq}^i \) are close to those of Eicken (1998) (1.3 mm and 2.91\%) and to Smith and others (2012) (2.2 mm and 3.4\%), which were estimated for lower growth rates (<1.3 \times 10^{-7} \text{ m s}^{-1}) but including platelet ice, for higher growth rates \( z_{2d} \) is an order of magnitude smaller and \( \varepsilon_{eq}^i \) is also significantly smaller. Relatively significant errors of \( z_{2d} \) for both regimes may suggest the limitations of the SBL model.

We next considered what may have led to the two regimes. One possibility might be that various factors could affect the fractionation process in field observations, whereas the laboratory experiment was much simpler. However, the fact that the datum from the Sea of Okhotsk lies approximately on the curve derived primarily from the experimental data in Figure 7a suggests that the growth rate may be a controlling factor in determining \( \varepsilon_{eff, si} \), at least for \( \nu > 2.0 \times 10^{-7} \text{ m s}^{-1} \). The detail is discussed further in Section 6.

Finally, for practical use we sought to derive an empirical formula that predicts \( \varepsilon_{eff, si} \) from \( \nu \) in m s\(^{-1}\) with good accuracy over the range of velocities of both the Antarctic and laboratory experiment datasets. Although there were different regimes in the modified Eicken (1998) model at a threshold of \( 2.0 \times 10^{-7} \text{ m s}^{-1} \), for simplicity we obtained a single regression formula covering the whole range of \( \nu \) by fitting the output of the modified Eicken (1998) model at intervals of \( 0.1 \times 10^{-7} \text{ m s}^{-1} \) with a least-squares method as follows.

For \( 0.8 \times 10^{-7} \text{ m s}^{-1} < \nu < 9.3 \times 10^{-7} \text{ m s}^{-1} \)

\[
\varepsilon_{eff, si} = a_1 + b_1 \exp(-\nu/c_1) + d_1 \exp(-\nu/e_1)
\]

where \( a_1 = 1.2280 \%), \( b_1 = 0.7311 \%), \( c_1 = 8.0100 \times 10^{-8} \text{ m s}^{-1} \), \( d_1 = 0.8441 \%) \) and \( e_1 = 0.7800 \times 10^{-6} \text{ m s}^{-1} \).

Equation (9) is drawn in Figure 7b. In fitting the curve, we used modified Eicken (1998) model output instead of the measured values to ensure that all parts of the curve were equally biased. Using a similar procedure to that above, an inverse function for \( \nu \) as a function of \( \varepsilon_{eff, si} \) was obtained from Eqn (9):

For \( 1.47\% < \varepsilon_{eff, si} < 2.38\% \)

\[
\nu = a_2 + b_2 \exp(-\varepsilon_{eff, si}/c_2) + d_2 \exp(-\varepsilon_{eff, si}/e_2)
\]

where \( a_2 = 2.7570 \times 10^{-8} \text{ m s}^{-1} \), \( b_2 = 1.1000 \times 10^{-4} \text{ m s}^{-1} \), \( c_2 = 0.3226\% \), \( d_2 = -1.5430 \times 10^{-6} \text{ m s}^{-1} \) and \( e_2 = 0.6800\% \). Equations (9) and (10) are discussed further in a case study in the Sea of Okhotsk in Section 6.3.

6. DISCUSSION

6.1. Effects of other factors

So far we have examined the dependence of \( \varepsilon_{eff, si} \) on \( \nu \) using Eicken’s model, which was derived from the SBL model. In general, it is known that fractionation is also
affected by other factors such as grain size and grain shape due to differences in crystal structure (Hoefs, 2009). Here we examine the effect of these factors on $\varepsilon_{\text{eff, si}}$. This may provide some insight into how two regimes are apparent when $\varepsilon_{\text{eff, si}}$ is plotted with respect to $v_t$. To approach this issue, first we show the results of non-columnar ice types obtained from our laboratory experiment. As shown in Table 1, layer 1 (2) has significantly higher (lower) isotope fractionation than the inner columnar layer, layer 3. These layers, corresponding to vertical c-axis and transition layers, have significantly larger and smaller grain sizes, respectively, than the columnar ice layer. Therefore, it is likely that grain size and shape also affect $\varepsilon_{\text{eff, si}}$. However, since $v_t$ varied depending on each layer, we need to separate out the effect of $v_t$. To do so, in Figure 8 we plot the data obtained from all the layers. Figure 8 clearly shows that even at the same growth rate, layers 1 and 2 tend to have higher and lower values of $\varepsilon_{\text{eff, si}}$, respectively, while layers 3 and 4, which are composed of similar columnar ice, have almost comparable $\varepsilon_{\text{eff, si}}$. This indicates that grain size and type (i.e. granular or columnar ice) affect $\varepsilon_{\text{eff, si}}$ significantly and should be taken into account in the determination of $\varepsilon_{\text{eff, si}}$.

Next we consider if the substructure of sea ice can affect $\varepsilon_{\text{eff, si}}$ for the same columnar ice. The characteristic size in the substructure of columnar ice is represented by brine layer spacing ($a_0$). The results obtained from field observations in the Arctic by Nakawo and Sinha (1984) and Sinha and Zhan (1996) showed that $a_0$ tends to be inversely proportional to $v_t$ within the range $0.8 \times 10^{-7} < v_t < 2.0 \times 10^{-7} \text{ m}^{-1}$ in accordance with the theoretical prediction by Bolling and Tiller (1960). According to the observational results of Nakawo and Sinha (1984) and Sinha and Zhan (1996), $a_0$ doubled from 0.5 to 1 mm with a decrease in $v_t$ from $2.0 \times 10^{-7}$ to $0.8 \times 10^{-7} \text{ m}^{-1}$. Bolling and Tiller (1960) and Lofgren and Weeks (1969) suggested that the relationship becomes inversely proportional between $a_0$ and $\sqrt{v_t}$ for large growth rates, which means less dependence of $a_0$ on $v_t$ for large $v_t$. Therefore it may be possible that such a significant increase in grain size contributed to producing a different regime for small growth rates ($v_t < 2.0 \times 10^{-7} \text{ m}^{-1}$). Besides, observational results show that $a_0$ also depends somewhat on ocean current direction relative to the c-axis (Nakawo and Sinha, 1984) and ocean current speed (Eicken, 2003), indicating that ocean current can also affect $\varepsilon_{\text{eff, si}}$. Consequently, various factors tend to affect the values of $\varepsilon_{\text{eff, si}}$ especially for small $v_t$. This may also explain the large variability of $\varepsilon_{\text{eff, si}}$ for the observation in the Antarctic observations in Figure 7a.

Another possible effect is the enhancement of the interface area ($A_i$) between sea ice and seawater and the groove structure of the bottom surface. With an increase in the interface area, $\varepsilon_{\text{eff, si}}$ is expected to increase compared with its value for a planar bottom. Normally the bottom surface of columnar ice is not flat but is composed of numbers of knife-edged cells with spacings ($a_0$) of ~0.5–2.0 mm and depth ($h$) of a few to tens of millimeters (Kovacs, 1996; Eicken, 1998). A schematic is shown in Figure 9b. If we assume the two-dimensional (vertical and horizontal) structures and approximate them by triangles, the enhancement of the interface area is calculated as

$$A_i/A_0 = \sqrt{4 \cdot \left(\frac{h}{a_0}\right)^2 + 1} \quad (11)$$

where $A_i$ is the area of the flat bottom (Fig. 9a). The ratio $A_i/A_0$ is shown as a function of $h$ and $a_0$ in Figure 9c. As mentioned above, $a_0$ tends to decrease with an increase in $v_t$. Besides, although the geometry of the groove structure of the sea-ice bottom surface has not yet been clarified, in metals the tip of the groove structure is known to become sharpened with the increase in $v_t$ (Weeks, 2010), indicating the increase in $h/a_0$ with $v_t$ in Eqn (11). Thus, if this is applicable to sea ice, the ratio $A_i/A_0$ is expected to become enhanced with an increase in $v_t$ from $0.8 \times 10^{-7}$ to $2.0 \times 10^{-7} \text{ m}^{-1}$. Noticeable in Figure 9c is that especially around $a_0 \sim 0.5 \text{ mm}$,
corresponding to \(a_0 = 2.0 \times 10^{-7} \text{m s}^{-1}\), the sensitivity to both \(a_0\) and \(h\) becomes enhanced significantly. This may explain why \(\varepsilon_{\text{eff,si}}\) for the experimental data takes significantly larger values than those predicted by the curve obtained from the Antarctic observations. The fact that the area enhancement is highly dependent on \(\nu_i\) may also contribute to the strong dependency of \(\varepsilon_{\text{eff,si}}\) on \(\nu_i\), especially for \(\nu_i > 2.0 \times 10^{-7} \text{m s}^{-1}\). Eicken’s model included the effect of change in brine volume entrapped in sea ice as a function of \(\nu_i\). Our results suggest that \(\nu_i\) may play an important role in determining \(\varepsilon_{\text{eff,si}}\), not only through a change in brine volume, but also via the change in geometric features such as grain size or brine layer spacing, not limited to the change in brine volume.

6.2. Response depth to the change of a growth rate

When we apply the \(\varepsilon_{\text{eff,si}} - \nu_i\) relationship of Eqn (7), obtained under steady-state conditions, to real sea ice that grows at various growth rates, we need to consider the response time, or response depth in sea ice, required to adjust to the change in \(\nu_i\). Since the change in \(\nu_i\) leads to a gradual adjustment of \(C_{\text{s,0}}\) and subsequently \(\varepsilon_{\text{eff,si}}\), it is expected to take some time before \(\varepsilon_{\text{eff,si}}\) reaches a stable value determined by Eqn (7). If the ice growth amount during the adjustment period (referred to hereafter as response depth) is large enough, Eqn (7) would not necessarily be useful for estimating growth rates from the \(\delta^{18}\text{O}\) profile of sea ice. Thus the response depth is critical to the analysis of \(\delta^{18}\text{O}\) and needs to be examined. Here, as an example of a significant change in \(\nu_i\), we estimate the response depth when sea ice starts to grow. This case is applicable to the Sea of Okhotsk, where ice growth is limited to night-time, as described in Section 3.2.

As for the time-step \(\Delta t\) and grid spacing \(\Delta z\) were set to 1.0 s and 0.05 mm, respectively, so that the computational condition \(\Delta t \leq (\Delta z)^2 / (2D) \approx 1.04 \text{s}\) is satisfied. Physical parameters \(z_{\text{bl}}\) and \(\varepsilon_{\text{eq}}\) were taken to be 2 mm and 2.91%, respectively, as a possible large value for \(z_{\text{bl}}\) and after Eicken (1998) for \(\varepsilon_{\text{eq}}\). Here the \(\delta^{18}\text{O}\) of bulk sea water is assumed to be \(-0.8\%)\), based on the tank experiment. According to Eqn (4), the isotope ratio \(C_{\text{z}}\) at the \(n\)th time-step and the \(i\)th layer within the boundary layer, \(C_{\text{L}(n,i)}\), is calculated by:

\[
C_{\text{L}}(n+1,i) = \left(1 - 2D \frac{\Delta t}{(\Delta z)^2} + \nu_i \frac{\Delta t}{\Delta z}\right) \cdot C_{\text{L}}(n,i) + D \frac{\Delta t}{(\Delta z)^2} \cdot C_{\text{L}}(n,i+1) + \left(D \frac{\Delta t}{(\Delta z)^2} - \nu_i \frac{\Delta t}{\Delta z}\right) \cdot C_{\text{L}}(n,i-1)
\]

(12)

with the initial condition \(C_{\text{L}}(0,0) = (2 - \alpha_{\text{eq}}) \cdot C_{\text{L}B}\) and the boundary condition

\[
C_{\text{L}}(n+1,0) = C_{\text{L}}(n+1,1) + \nu_i (1 - \alpha_{\text{eq}}) \frac{\Delta z}{D} \cdot C_{\text{L}}(n,0) at z = 0
\]

\[
C_{\text{L}} = C_{\text{L}B} at z = z_{\text{bl}}.
\]

The profile of \(\delta^{18}\text{O}\) in sea water was obtained by converting \(C_{\text{L}}\) to \(\delta^{18}\text{O}\) with Eqn (5), while the profile of \(\delta^{18}\text{O}\) in sea ice was obtained by converting \(C_{\text{L}}(0) (= \alpha_{\text{eq}} C_{\text{L}B})\) to \(\delta^{18}\text{O}\) and multiplying \(\nu_i\) by time. In Eicken’s model, including the effect of the entrapped brine, \(\varepsilon_{\text{eff,si}}\) is obtained by multiplying \(\frac{\alpha_{\text{eff}}}{\alpha_{\text{eq}}}\) by \((1 - k_{\text{eff,s}})\), as shown in Eqn (8). However, when a constant growth rate is considered, neglecting this effect will not alter the result since \(k_{\text{eff,s}}\) is kept constant. Numerical simulations were conducted for \(\nu_i = 1.4 \times 10^{-7}\) and \(8.3 \times 10^{-7} \text{m s}^{-1}\), representative of low and high growth rates in real sea ice. The result obtained by integrating Eqn (12) for 5 hours from the beginning of freezing is shown in Figure 10. Note that in this figure ice grows from the left end and the two vertical thick solid lines denote the individual ice-water boundaries. It is confirmed from Figure 10 that the calculation for growth rates of 1.4 \(\times 10^{-7}\) and \(8.3 \times 10^{-7} \text{m s}^{-1}\) is large enough, the ice growth amount during the adjustment period (referred to hereafter as response depth) is large enough, Eqn (7) would not necessarily be useful for estimating growth rates from the \(\delta^{18}\text{O}\) profile of sea ice. Thus the response depth is critical to the analysis of \(\delta^{18}\text{O}\) and needs to be examined. Here, as an example of a significant change in \(\nu_i\), we estimate the response depth when sea ice starts to grow. This case is applicable to the Sea of Okhotsk, where ice growth is limited to night-time, as described in Section 3.2.

By substituting the values, Eqn (13) gives \(R_{\text{B}} \approx 0.46\) and 2.78 mm for 1.4 \(\times 10^{-7}\) and \(8.3 \times 10^{-7} \text{m s}^{-1}\), which almost coincides with the result of the numerical simulation in Figure 9, indicating that Eqn (13) is reasonable for predicting \(R_{\text{B}}\). According to Eqn (13), \(R_{\text{B}}\) is proportional to \(\nu_i\) and \(z_{\text{bl}}^2\). Since our simulation was done with the largest possible values of \(\nu_i\) and \(z_{\text{bl}}\), the estimated \(R_{\text{B}}\) is considered to be close to a maximum value. Therefore, it follows that the effect of the response depth on the profile of \(\delta^{18}\text{O}\) is essentially negligible.

6.3. Case study: application to the Sea of Okhotsk sea ice

As a case study, here we apply Eqns (9) and (10) obtained in Section 5 with a sea-ice sample collected in the seasonal ice zone to reveal the growth history. The ice sample was collected in the southern Sea of Okhotsk (see Fig. 4b for location) on 8 February 2010 as part of a sea-ice observation program in collaboration with the Japan Coast Guard. As shown in Figure 11a, this sample, having a thickness of 23 cm, was composed entirely of columnar ice except in the top 5 cm, where snow ice was present judging from its granular crystal structure and negative \(\delta^{18}\text{O}\) (Toyota and others, 2004). This indicates that the sea ice grew simply by a thermodynamic process for depths between 5 and 23 cm.
The vertical profiles of $\delta^{18}O$ and $\varepsilon_{eff, si}$ are shown in Figure 11b, where $\varepsilon_{eff, si}$ was estimated from the difference in $\delta^{18}O$ between sea ice and sea water ($-1.02 \pm 0.21\%$).

Ice growth rates were estimated with a one-dimensional thermodynamic model in a similar way to that of Toyota and Wakatsuchi (2001). Meteorological data (air temperature, pressure, wind, cloud, relative humidity) were taken from the European Centre for Medium-Range Weather Forecasts Interim Re-analysis (ERA-Interim) dataset ($1.5^\circ \times 1.5^\circ$) four times per day (03:00, 09:00, 15:00, 21:00 Japan Standard Time (JST)). Ice growth amount was calculated at a gridpoint ($45^\circ N$, $144^\circ E$) near the observation site (Fig. 4b). Comparison between the ERA-Interim dataset at this gridpoint and observational data recorded on the ship in this area ($44.4^\circ -45.5^\circ N$, $142.1^\circ -144.6^\circ E$) during the period 5–9 February 2010 showed that the ERA-Interim dataset reproduced the real meteorological conditions well, with an RMSE of 2.0 hPa for sea-level pressure, 1.5°C for air temperature and 3.8 m s$^{-1}$ for wind speed. We assumed that ocean heat flux is negligible. This assumption is supported by the fact that the water temperature in the surface mixing layer, extending to ~30 m depth, was shown to be nearly at the freezing point in this region from the expendable bathythermograph (XBT) and conductivity–temperature–depth (CTD) observations conducted during the cruise. We assumed snow depth is one-fifth of the ice thickness. This is based on observational results (Toyota and others, 2000). Longwave radiation flux was calculated from the empirical formula of Maykut and Church (1973). Calculation was started on 15 January when the sea-ice cover extended southward to the gridpoint, judging from the sea-ice concentration map of the Advanced Microwave Scanning Radiometer for Earth Observing System (AMSR-E), and was performed only during the night-time (21:00, 03:00 JST) because ice growth usually stops in the daytime due to abundant solar radiation. The estimated ice thickness evolution is shown in Figure 12a. Figure 12a predicts the ice thickness on 8 February as 16 cm, which approximately coincides with the thickness of the columnar ice layer of the sample (Fig. 11a). We confirmed that the error of the ERA-Interim dataset results in differences in the predicted ice thickness of ±1 cm. Therefore, we can safely regard as reasonable the growth rate profile predicted by the model (Fig. 12b).

Based on these datasets, we examine the applicability of the empirical formula, Eqn (9). The vertical profile of $\varepsilon_{eff, si}$ calculated from Eqn (9) is shown in Figure 12c. To compare with the observed profile directly, the values averaged for each 5 cm depth from the bottom are superimposed in Figure 11b. Although $\varepsilon_{eff, si}$ has detailed structure, Figure 11b indicates that overall the predicted $\varepsilon_{eff, si}$ with Eqn (9) reproduced the observed profile well with an RMSE of 0.03%. This result suggests that Eqn (9) is promising for correlating the observed vertical profile of $\varepsilon_{eff, si}$ with the growth rate history for columnar ice.

Finally, we check the applicability of Eqn (10) by predicting the growth time from the $\delta^{18}O$ profile. By estimating $\nu_i$ from the observed $\varepsilon_{eff, si}$ in Figure 11b with Eqn (10), we can calculate how much time the ice took to grow for each layer (~5 cm) except for the top snow-ice layer. By summing up the elapsed time, we can estimate the total number of days of growth before collection. As a result, 11.4 days was obtained. If we consider that the growth period is limited to night-time, the real period would be ~23 days. This estimation shows that the growth period predicted by Eqn (10) almost coincides with the observed period of ice growth (24 days from 15 January to 8 February). Consequently, it is concluded that the empirical formulae, Eqs (9) and (10), are useful in revealing the growth rate history from the vertical $\delta^{18}O$ profile of collected sea-ice samples.

7. CONCLUSION
In this study we examined the relationship between $\varepsilon_{eff, si}$ and $\nu_i$ to apply the SBL model to real sea ice, focusing on columnar ice, and to improve understanding of the stable oxygen isotope fractionation process during the freezing of sea water through laboratory experiments and field observations. Our particular interest was in optimizing the
parameters used in the model of Eicken (1998), which is based on the SBL model. Relatively low growth rates ($\nu _i < 1.7 \times 10^{-7} \text{ m s}^{-1}$) were covered by field observations in McMurdo Sound, Antarctica, while data for relatively high growth rates ($\nu _i > 2.2 \times 10^{-7} \text{ m s}^{-1}$) were obtained from the laboratory experiments. A case study on observational data from the southern Sea of Okhotsk supports the results.

In Antarctica, long-term monitoring observations allowed for the collection of ice samples and the measurement of growth rate data in the winters of 1999 and 2009. In the laboratory experiment, sea ice was grown in a tank under calm and steady conditions. Combining these datasets allowed evaluation of the relationship between $\varepsilon_{\text{eff, si}}$ and $\nu _i$ for a wide range of growth rates for the first time. As a result, the optimum values of equilibrium pure-ice fractionation factor and boundary-layer thickness were estimated. We could not find a single set of parameter values that fit both the laboratory and field data, and the results show that the slope of the curve changed significantly for the growth rate of $\nu _i < 2.0 \times 10^{-7} \text{ m s}^{-1}$, and this change coincides with laboratory/field measurements. Therefore, different sets of physical parameters in the SBL model were estimated:

$$0.8 \times 10^{-7} < \nu _i < 2.0 \times 10^{-7} \text{ m s}^{-1} : \varepsilon_{\text{eq}} = 3.11 \pm 0.59$$
and $z_{bl} = 0.9 \pm 1.9 \text{ mm}$

$$2.0 \times 10^{-7} < \nu _i < 9.3 \times 10^{-7} \text{ m s}^{-1} : \varepsilon_{\text{eq}} = 2.66 \pm 0.13$$
and $z_{bl} = 0.1 \pm 0.1 \text{ mm}$

at the 95% confidence level. It was found that overall the model of Eicken (1998) tends to underestimate $\varepsilon_{\text{eff, si}}$ especially for $\nu _i > 2.0 \times 10^{-7} \text{ m s}^{-1}$. The possible reason for the formation of these two regimes is that the relationship between $\nu _i$ and $\varepsilon_{\text{eff, si}}$ is affected by various factors other than brine entrainment, such as grain size, ocean current and area enhancement due to groove structure at the bottom. Since the response depth required for adjustment to the change of $\nu _i$ was shown to be a few millimeters at most, it was confirmed that the vertical profile of $\varepsilon_{\text{eff, si}}$ in sea ice corresponds approximately to the growth rate history of the sea ice.

For practical use, an empirical formula that correlates $\nu _i$ with $\varepsilon_{\text{eff, si}}$ for the whole range of $\nu _i$ ($0.8 \times 10^{-7} < \nu _i < 9.3 \times 10^{-7} \text{ m s}^{-1}$) was obtained.

$$\varepsilon_{\text{eff, si}} = a_1 + b_1 \exp \left( -\nu _i / c_1 \right) + d_1 \exp \left( -\nu _i / e_1 \right)$$

where $a_1 = 1.2280\%$, $b_1 = 0.7311\%$, $c_1 = 8.0100 \times 10^{-8} \text{ m s}^{-1}$, $d_1 = 0.8441\%$ and $e_1 = 0.7800 \times 10^{-6} \text{ m s}^{-1}$. In the Sea of Okhotsk, an ice sample was collected in early February 2010 and the growth rates were estimated from the heat budget analysis. By applying this formula to a real sea-ice sample from the Sea of Okhotsk, it was shown that the $\varepsilon_{\text{eff, si}}$ profile predicted from $\nu _i$ reproduced the observed profile well and that the $\delta^{18}\text{O}$ profile is useful for estimating the growth rate history. Thus the relationship between $\varepsilon_{\text{eff, si}}$ and $\nu _i$ obtained from our study was revealed to be useful to climate research to some extent through the analysis of ice growth rate history. However, it should be noted that the analysis was focused on columnar ice. Observations show that whereas in the Arctic Ocean columnar ice prevails (e.g. Gow and others, 1987), in a wide area of seasonal sea-ice zones frazil ice has been observed more frequently than columnar ice, for example in the Bellingshausen and Amundsen Seas (e.g. Jeffries and others, 1997), in the Weddell Sea (e.g. Lange and Eicken, 1991) and in the Sea of Okhotsk (Toyota and others, 2007). Therefore, in order to apply the $\delta^{18}\text{O}$ profile of sea ice to climate research in a wide range of regions, future study of the relationship between $\varepsilon_{\text{eff, si}}$ and $\nu _i$ for frazil ice is required.

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