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Nobuhito Suga and Makoto Hisanaga

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Graduate School of Economics &
Business Administration
Hokkaido University
Kita 9 Nishi 7, Kita-Ku, Sapporo 060-0809, JAPAN

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Nobuhito Suga*

Graduate School of Economics and Business Administration, Hokkaido University

Makoto Hisanaga

Graduate School of Economics and Business Administration, Hokkaido University

Abstract

This paper investigates trade patterns and the gains from trade in a Chamberlinian-Ricardian model with a CES type of upper-tier utility function. It is shown that a strong tendency toward complete specialization emerges under free trade and that free trade is preferable to autarky from the viewpoint of each country's welfare. This paper also considers the trade regime called *semi-autarky*, in which one sector is under free trade, while the other is closed. The analysis demonstrates that free trade does not necessarily attain higher welfare in all countries relative to semi-autarky if cross-sector substitution in consumption is elastic.

Keywords: Chamberlinian-Ricardian model; Trade patterns; Gains from trade

JEL classification: F12

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* *Address for Correspondence*: Graduate School of Economics and Business Administration, Hokkaido University, Kita9 Nishi7, Kita-ku, Sapporo 060-0809, Japan. *E-mail Address*: suga@econ.hokudai.ac.jp.

1. Introduction

Chamberlinian monopolistic competition models of trade have become a standard analytical tool in the modern trade theory. To highlight the role of imperfect competition and increasing returns to scale, they usually assume technical homogeneity across countries[1]. In addition to this, there is another traditional assumption that is often made in the Chamberlinian framework: a fixed share of consumers' expenditure on each sector. Over the last fifteen years, several attempts have been made to incorporate a Ricardian aspect (i.e., cross-country technical heterogeneity) into the Chamberlinian framework (e.g., Ricci, 1999; Venables, 1999; Montagna, 2001; Forslid and Wooton, 2003; Kikuchi et al., 2006; Kikuchi and Shimomura, 2007; Demidova, 2008; Kikuchi et al., 2008; Huang, 2013). However, these studies all assume a Cobb-Douglas upper-tier utility function, so the share of consumers' expenditure on each sector is constant. Little investigation has so far been made into the implications of variable expenditure shares for international trade in the Chamberlinian-Ricardian framework.

The purpose of this paper is to explore trade patterns and the gains from trade in a two-country, two-sector and one-factor (labor) Chamberlinian-Ricardian model with variable expenditure shares. In our model, consumers' preferences over the composite goods made up of products in each monopolistically competitive sector are given by a CES (constant elasticity of substitution) function. This implies that the share of consumers' expenditure on each sector varies in response to a change in the price ratio of the two composite goods. This paper also assumes a gradual cross-sector adjustment

[1] This assumption has been one of the traditions in the literature on intra-industry trade since the seminal work of Krugman (1979).

process in which labor moves between sectors according to the wage gap[2]. Under these assumptions, we investigate specialization patterns achieved as a result of labor reallocation between sectors and the associated welfare changes.

All the results in this paper are premised on the assumption that the elasticity of cross-sector substitution in consumption is smaller than that of substitution between every pair of products in each sector. If no restriction is imposed on the sensitivity of expenditure shares to a change in the price ratio of the two composite goods, we are bothered with instability and/or multiplicity of the equilibrium. To avoid this problem, we assume throughout the paper that the above condition on the elasticity of substitution is satisfied.

Under the condition for stability, our model exhibits a strong resemblance to the classical Ricardian model in regard to the pattern of *inter-industry* trade: Each country specializes and hence has positive net exports in its relatively productive sector. At least one county completely specializes in the free-trade equilibrium, and intra-industry trade is less likely compared to the case where countries are identical in technologies. Which type of specialization pattern arises depends on both fixed and marginal costs in each sector. This suggests that unlike the classical Ricardian model, each country's relative price in autarky, which is proportional only to its relative marginal cost, is no longer a reliable predictor of trade patterns.

In the analysis of trade gains, we shed light on the effects of entering into free trade from two types of imperfectly open situation: autarky and *semi-autarky*. The latter

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[2] This kind of adjustment process is often assumed in the literature on geography and trade (see e.g. Fujita et al, 1999).

represents the situation in which one sector is under free trade, while the other is closed. In our model, free trade attains higher welfare in both countries relative to autarky, whereas bilateral gains are not always realized in the case of moving from semi-autarky to free trade. Indeed, it is shown that one country can lose from the opening of free trade from semi-autarky if the elasticity of cross-sector substitution in consumption exceeds unity. This finding is one of the major contributions in this paper. From the empirical viewpoint, a state of semi-autarky is more plausible as a situation before the opening of free trade, compared to autarky (i.e., a completely self-sufficient state). To our knowledge, however, there is no existing research addressing a shift in the trade regime such as movement from semi-autarky to free trade.

This paper is closely related to Kikuchi et al. (2006, 2008), which clarified trade patterns in a two-country, one-factor and multi-industry Chamberlinian model with cross-country technical heterogeneity. Their analyses suggest that the emergence of intra-industry trade is very unlikely in the Chamberlinian-Ricardian framework. They constructed the technology index made up of fixed and marginal costs in each sector and illustrated that the division of industries between countries is determined based on the relative productivity defined by the technology index. We depend heavily on their technique in our analysis of trade patterns. However, our paper is distinguished from them by our focus on the normative aspects of trade they did not address.

This paper is also related to Montagna (2001) and Demidova (2008). They developed a two-country Chamberlinian-Ricardian model and demonstrated that trade liberalization is not necessarily beneficial to both countries. However, they used a framework with heterogeneous firms[3], and their focus was on the interaction between cross-country technical heterogeneity and intra-industry reallocation of resources via selection of firms. In contrast, we abstract from such a selection effect and focus on the implications of variable expenditure shares for international trade in the presence of cross-country technical heterogeneity.

The remainder of the paper is organized as follows. The next section formulates the model. Section 3 describes the autarky equilibrium and Section 4 studies trade patterns in the free-trade equilibrium. Section 5 presents the semi-autarky equilibrium and explores each country's welfare changes induced by the opening of free trade. Section 6 provides concluding remarks.

2. The Model

The economy comprises two countries, the home country and the foreign country. The home (foreign) country is endowed with L (L^*) units of labor, which is the only primary factor of production. The two countries are identical in regard to consumers' preferences but not in regard to size and production technologies. There are two sectors, sector A and sector B. Each sector is modeled as a Dixit-Stiglitz (1977) monopolistically competitive sector.

All consumers have the same preferences represented by CES (constant elasticity of

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[3] Strictly, the two studies are different in the manner of formulating heterogeneous firms. Demidova's (2008) formulation is based on Melitz (2003) in which firm-specific productivity is stochastically determined. In Montagna (2001), on the other hand, the distribution of firms' productivity is provided exogenously.

substitution) utility function. In the home country, the social utility function is given by

$$U = \left[\alpha_A C_A^{(\sigma-1)/\sigma} + \alpha_A C_B^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \ \sigma > 0, \ \sigma \neq 1,$$

where C_k is the sub-utility function and takes the form

$$C_k = \left[\int_0^{n_k} c(i_k)^{(\sigma_k-1)/\sigma_k} di_k + \int_0^{n_k^*} c(i_k^*)^{(\sigma_k-1)/\sigma_k} di_k^* \right]^{\sigma_k/(\sigma_k-1)}, \ \sigma_k > 1, \ k = A, B. \quad (1)$$

This can be interpreted as the consumption of a single good composed of differentiated products in sector k ($k = A, B$). In the upper-tier utility function, α_A and α_B are the positive parameters showing the intensity of preferences for composite goods A and and B, respectively, and satisfying $\alpha_A + \alpha_B = 1$. σ is the elasticity of substitution between two composite goods, say C_A and C_B. In the sub-utility function given in (1), n_k (n_k^*) is the number of products in sector k in the home (foreign) country, $c(i_k)$ ($c(i_k^*)$) is the consumption of product i_k (i_k^*) in the home country, and σ_k is the elasticity of substitution between every pair of products in sector k. The foreign country's social utility function is expressed by the same equation with the corresponding foreign variables.

Let us derive the home country's demand functions. Under the present utility function, it is convenient to decompose the consumer's utility maximization problem into two stages. In the first stage, taking C_k ($k = A, B$) and the prices of the varieties as given, the consumer chooses the consumption level of each variety so as to minimize its expenditure on each sector. In the second stage, the consumer chooses the utility-maximizing level of C_k ($k = A, B$) under a given income. Solving this utility maximization problem yields the following demand functions:

$$c(i_k) = \alpha_\kappa^\sigma p(i_k)^{-\sigma_k} P_k^{\sigma_k - \sigma} [E(P_A, P_B)]^{\sigma - 1} I \,, \tag{2}$$

$$c(i_k^*) = \alpha_\kappa^\sigma p(i_k^*)^{-\sigma_k} P_k^{\sigma_k - \sigma} [E(P_A, P_B)]^{\sigma - 1} I \,, \tag{3}$$

where $c(i_k)$ ($c(i_k^*)$) is the home country's demand for product i_k (i_k^*), $p(i_k)$ ($p(i_k^*)$) is the price of product i_k (i_k^*), and I is the home country's national income. $E(P_A, P_B)$ expresses the unit-expenditure function and is given by

$$E(P_A, P_B) = (\alpha_A^\sigma P_A^{1-\sigma} + \alpha_A^\sigma P_B^{1-\sigma})^{1/(1-\sigma)} \,, \tag{4}$$

where P_k represents the price index of sector k, which takes the form

$$P_k = \left[\int_0^{n_k} p(i_k)^{1-\sigma_k} di_k + \int_0^{n_k^*} p(i_k^*)^{1-\sigma_k} di_k^* \right]^{1/(1-\sigma_k)} . \tag{5}$$

The foreign country's demand functions, $c^*(i_k)$ and $c^*(i_k^*)$, are obtained by replacing I with the foreign country's national income, I^*, in (2) and (3).

Differentiated products are supplied by monopolistically competitive firms. The amount of labor required to produce the quantity $x(i_k)$ ($x(i_k^*)$) of product i_k (i_k^*) is given by

$$l(i_k) = b_k x(i_k) + f_k \quad (l(i_k^*) = b_k^* x(i_k^*) + f_k^*),$$

where b_k ($b_k^{(*)}$) is the marginal labor requirement and f_k (f_k^*) the fixed labor requirement.

With the number of firms being very large, all firms take the price index P_k as given. Then, profit maximization implies that the price of product i_k (i_k^*) is

$$p(i_k) = \sigma_k b_k w_k / (\sigma_k - 1) \quad (p(i_k^*) = \sigma_k b_k^* w_k^* / (\sigma_k - 1)), \tag{6}$$

where w_k (w_k^*) is the wage rate in sector k in the home (foreign) country. There is no barrier to entry or exit. Hence, from the zero-profit condition, the equilibrium output of any active firm in the home (foreign) country is

$$x(i_k) = (\sigma_k - 1) f_k / b_k \quad (x(i_k^*) = (\sigma_k - 1) f_k^* / b_k^*), \tag{7}$$

and the associated equilibrium labor input in the home (foreign) country is

$$l(i_k) = \sigma_k f_k \quad (l(i_k^*) = \sigma_k f_k^*).$$

Then, the number of firms in sector k in the home (foreign) country is

$$n_k = L_k / \sigma_k f_k \quad (n_k^* = L_k^* / \sigma_k f_k^*), \tag{8}$$

where L_k (L_k^*) is labor supply in sector k in the home (foreign) country. Using (6) and (8), the price index (5) can be rewritten as

$$P_k = \frac{\sigma_k^{\sigma_k/(\sigma_k - 1)}}{\sigma_k - 1} \left(\frac{w_k^{1-\sigma_k} L_k}{a_k^{\sigma_k}} + \frac{w_k^{*1-\sigma_k} L_k^*}{a_k^{*\sigma_k}} \right)^{1/(1-\sigma_k)}, \tag{9}$$

where

$$a_k \equiv f_k^{1/\sigma_k} b_k^{(\sigma_k - 1)/\sigma_k}, \tag{10}$$

and

$$a_k^* \equiv (f_k^*)^{1/\sigma_k} (b_k^*)^{(\sigma_k - 1)/\sigma_k}. \tag{11}$$

In each country, labor gradually moves toward the sector that offers a higher wage rate. Then, the dynamic adjustment process can be depicted by the following differential equations:

$$\dot{L}_A = g(w_A / w_B - 1); \tag{12}$$

$$\dot{L}_A^* = g^*(w_A^* / w_B^* - 1), \tag{13}$$

where dot denotes a time derivative and $g^{(*)}(z)$ is a strictly increasing, differentiable function of $z \in R$ with $g^{(*)}(0) = 0$.

3. Autarky Equilibrium

Before turning to the trading equilibrium, let us consider the autarky equilibrium in the home country. (Although we direct our attention to the home country, a similar argument applies to the foreign country.) In this section, we find that under the adjustment process given by (10), the stability of the long-run equilibrium crucially depends on the elasticity of substitution between any pair of varieties, σ_k, and the elasticity of cross-sector substitution, σ. Our analysis shows that that the long-run equilibrium is unique and globally stable if both σ_A and σ_B exceed σ.

To determine the long-run equilibrium, it is necessary to derive the short-run equilibrium wage ratio of sector A to sector B, which clears all goods markets for a given L_A. This wage ratio is given by

$$\frac{w_A}{w_B} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L/a_B - L_A/a_B)^{(\sigma_B-\sigma)/\sigma(\sigma_B-1)}}{(L_A/a_A)^{(\sigma_A-\sigma)/\sigma(\sigma_A-1)}}, \tag{14}$$

where L denotes the home country's labor endowment, and

$$\Theta \equiv \frac{\alpha_A}{\alpha_B}\left[\frac{\sigma_B{}^{\sigma_B/(\sigma_B-1)}}{(\sigma_B-1)} \Big/ \frac{\sigma_A{}^{\sigma_A/(\sigma_A-1)}}{(\sigma_A-1)}\right]^{(\sigma-1)/\sigma}.$$

The wage ratio shown in (14) is derived as follows. Since all active firms make no profits, each sector's revenue is equal to its total cost, so it follows that

$$w_k L_k = P_k C_k. \tag{15}$$

In autarky, n_k^* can be set at zero in (1). In view of this and combining (1) with (2), the consumption ratio of two composite goods is given by

$$C_A / C_B = (\alpha_A / \alpha_B)^{\sigma} (P_A / P_B)^{-\sigma}. \tag{16}$$

We can also set L_k^* at zero in (9). Taking account of this and making use of (9), (15), (16) and the full employment condition $L_A + L_B = L$, we obtain (14).

From (14), it is immediately shown that the long-run equilibrium is unique and globally stable if the following conditions are satisfied:

$$\sigma_k \geq \sigma, \quad k = A, B \tag{17}$$

and

$$\max\{\sigma_A, \sigma_B\} > \sigma. \tag{18}$$

(17) and (18), together whit (14), imply that the short-run equilibrium wage ratio is decreasing in L_A, as shown by the downward sloping curve AA' in Figure 1. Hence, the conditions ensure uniqueness and global stability of the long-run equilibrium, where the wage rates in both sectors are equalized, or equivalently, $w_A / w_B = 1$. If there are no assumptions on σ_k and σ, the subsequent analysis is more complicated due to instability and/or multiplicity of the equilibrium. To avoid this problem, we assume throughout the paper that (17) and (18) are satisfied.

(Figure 1)

4. Trading Equilibrium

We are in a position to examine trade patterns in the long-run equilibrium under free trade. As we shall see below, if the technology indices defined in (10) and (11) satisfy

$$a_A / a_B > a_A^* / a_B^*, \tag{19}$$

the home (foreign) country tends to specialize and hence has positive net exports in sector B (sector A). Thus our model exhibits a strong resemblance to the classical Ricardian model in regard to the pattern of *inter-industry* trade. At least one country completely specializes in its relatively productive sector in the free-trade equilibrium. This tendency toward complete specialization implies that the opening of trade does not always lead to the emergence of *intra-industry* trade. In addition, we find that each country's relative price in autarky is no longer useful for predicting trade patterns. In what follows, it is assumed that (19) is satisfied.

As in the previous section, we first derive both countries' wage ratios in the short-run equilibrium. Under free trade, the wage ratios are determined so as to clear all goods markets for given L_A and L_A^*. By carrying out similar calculations to those in the case of autarky, the home country's wage ratio w_A / w_B is given by

$$\frac{w_A}{w_B} = \Theta \cdot \frac{[L - L_A - (w_B^* / w_B)(L^* - L_A^*)]^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{[L_A + (w_B^* / w_B) L_A^*]^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}}. \tag{20}$$

To obtain a completely explicit form for the home country's wage ratio, the term w_k^* / w_k in the right-hand must be determined. This can be calculated as

$$w_k^* / w_k = a_k / a_k^*, \tag{21}$$

where the market-clearing conditions for products i_k and i_k^* are used with (6), (7) and the definitions of technology indices a_k and a_k^*. Inserting (21) into (20) yields

$$\frac{w_A}{w_B} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L / a_B + L^* / a_B^* - L_A / a_B - L_A^* / a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A / a_A + L_A^* / a_A^*)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}}. \tag{22}$$

In a similar way, the foreign country's wage ratio is obtained as

$$\frac{w_A^*}{w_B^*} = \Theta \cdot \frac{a_B^*}{a_A^*} \cdot \frac{(L / a_B + L^* / a_B^* - L_A / a_B - L_A^* / a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A / a_A + L_A^* / a_A^*)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}}. \tag{23}$$

From (22) and (23), we find that both countries' wage ratios are decreasing in L_A and L_A^* under conditions (17) and (18). This decreasingness of both countries' wage ratios in L_A and L_A^*, together with (21), implies that under the adjustment process given by (12) and (13), the phase diagram can be depicted as Figure 2. In the figure, schedule HH' (FF') illustrates the locus of (L_A, L_A^*) for which $\dot{L}_A = 0$ ($\dot{L}_A^* = 0$).

(Figure 2)

As in the traditional Ricardian model, three types of equilibrium arise, depending on the relative size of labor endowment, L^* / L. In the case where the difference between L and L^* is sufficiently large, one country completely specializes in the sector where it has higher relative productivity, while the other diversifies (i.e., has positive outputs in both sectors). On the other hand, if L^* / L is at an intermediate level, both countries completely specializes in the sector where they are relatively productive.

The above result is proved with the aid of Figure 2. By allowing L^* / L to vary while keeping $L / a_B + L^* / a_B^*$ constant at $\bar{L}$, we can explore all possible specialization patterns without shifting schedules HH' and FF' (see (22) and (23)). In the figure, the line WW' represents the locus of (L, L^*) for which $L / a_B + L^* / a_B^* = \bar{L}$.

Let us first consider the case in which L^* / L is so large that L^* exceeds the level implied by F. Then, the labor endowments in both countries are as indicated by E. According to the dynamic behaviors shown by arrows, the long-run equilibrium is given by F. In this equilibrium, the foreign country diversifies and the home country completely specializes in sector B.

Let us turn to the case in which L^* / L is small enough for L^* to fall short of the level implied by H. Then, the long-run equilibrium is attained at point on schedule HH', as shown by T. Evidently, in this case the home country diversifies, while the foreign country completely specializes in sector A.

Finally, let us consider the case in which L^* / L takes an intermediate value, so that L^* is at the level implied by E”. Then, the long-run equilibrium is attained at point such as T’. In this equilibrium, complete specialization by both countries arises, that is, the home country completely specializes in sector B, while the foreign country completely specializes in sector A.

Consequently, under conditions (17), (18) and (19) the home (foreign) country has positive net exports in sector B (sector A) for all possible specialization patterns under free trade.

Proposition 1: *Suppose that (17), (18) and (19) are satisfied. Then, in the long-run equilibrium under free trade, the home (foreign) country becomes a net exporter of the differentiated products in sector* B *(sector* A*).*

Obviously, it is only when either the home country or the foreign country diversifies that intra-industry trade emerges. As in the traditional Ricardian model, such a specialization pattern tends to occur if the two countries greatly differ in the absolute level of each sector’s productivity as well as the size of labor endowment. The same tendency is also found in the case where the cross-country difference in the relative productivity of one sector to the other is sufficiently small or consumers’ preferences have a strong bias toward one composite good. It is easy to confirm these results from (22), (23) and Figure 2.

Noteworthy is that each country’s relative price in autarky does not serve as a reliable predictor of trade patterns. As Kikuchi and Shimomura (2007) pointed out, this is intrinsic in the Chamberlinian-Ricardian model. In the present model, the pattern of specialization is determined based on the technology indices a_k and a_k^*, which are made up of marginal as well as fixed labor inputs, as shown in (10) and (11). However, from (6) and $w_k^{(*)} = w^{(*)}$, each country's autarky relative price is proportional only to its relative marginal labor input. This implies that unlike the classical Ricardian model, autarky relative prices are not useful for predicting trade patterns.

5. Gains from Trade

Let us turn to the welfare effects of trade. In this section, we consider the following two types of trade regime: free trade, in which both sectors are under free trade; and what we call *semi-autarky*, in which one sector is under free trade, while the other is closed[4]. We can show that moving from autarky to free trade is always welfare-improving for both countries. However, the welfare effects of moving from semi-autarky to free trade is shown to be crucially dependent on the elasticity of cross-sector substitution, σ: If the substitution parameter exceeds unity, it is possible that a country is better off under semi-autarky than under free trade.

5.1. Welfare Effects of Trade: Autarky vs. Free Trade

This subsection investigates the welfare effects of moving from autarky to free trade. Our analysis reveals that the production shifts induced by the opening of trade increase the real wage rates measured in terms of the two composite goods and hence lead to an

[4] The term 'semi-autarky' is also used in Deardorff (2011), which explored whether or not the principle of comparative advantage is theoretically valid as it is applied to trade in services.

improvement in welfare.

To analyze the impacts of the trade-induced production shifts on welfare, we need to determine each country's welfare level in the long-run equilibrium. In the current model, consumers' preferences are represented by CES utility function, so the home country's welfare level is captured by the wage rate divided by the unit expenditure function:

$$\frac{w}{E(P_A, P_B)} = \left[\alpha_A^{\sigma} \left(\frac{w}{P_A} \right)^{\sigma-1} + \alpha_B^{\sigma} \left(\frac{w}{P_B} \right)^{\sigma-1} \right]^{1/(\sigma-1)}, \tag{24}$$

where w is the home country's wage rate in the long-run equilibrium and $E(\cdot)$ is the unit expenditure function given in (4). A similar expression showing the foreign country's welfare level is obtained by substituting w^* for w in (24), where w^* is the foreign country's wage rate in the long-run equilibrium. Therefore, (24) implies that each country's welfare increases with the real wage rates measured in terms of the two composite goods.

By making use of (9), we can derive the real wage rates before and after the opening of trade. Setting L_k^* (L_k) at zero and replacing w_k (w_k^*) with w (w^*) in (9), the home (foreign) country's real wage rate measured by composite good k in the autarky equilibrium can be calculated as

$$\frac{w^{(*)a}}{P_k^{(*)a}} = \frac{\rho_k}{a_k^{(*)}} \left[\frac{L_k^{(*)a}}{a_k^{(*)}} \right]^{1/(\sigma_k-1)}, \tag{25}$$

where superscript a denotes values of variables in the long-run equilibrium under autarky and

$$\rho_k \equiv (\sigma_k - 1)\sigma_k^{\sigma_k/(1-\sigma_k)}.$$

Similar calculations yield each country's real wage rate in the free-trade equilibrium:

$$\frac{w^t}{P_k^t} = \frac{\rho_k}{a_k}\left[\frac{L_k^t}{a_k} + \frac{L_k^{*t}}{a_k^*}\left(\frac{a_k w^t}{a_k^* w^{*t}}\right)^{\sigma_k - 1}\right]^{1/(\sigma_k - 1)} ; \tag{26}$$

$$\frac{w^{*t}}{P_k^t} = \frac{\rho_k}{a_k^*}\left[\frac{L_k^t}{a_k}\left(\frac{a_k^* w^{*t}}{a_k w^t}\right)^{\sigma_k - 1} + \frac{L_k^{*t}}{a_k^*}\right]^{1/(\sigma_k - 1)} , \tag{27}$$

where superscript t denotes values of variables in the long-run equilibrium under free trade. (26) represents the home country's real wage rate and (27) the foreign country's real wage rate.

As has been mentioned, the production shifts brought about by moving from autarky to free trade result in increases in the real wage rates. This can be proved from (25), (26) and (27). Therefore, we have the following proposition.

Proposition 2: *Suppose that (17), (18) and (19) are satisfied. Then, both countries become better off by moving from autarky to free trade.*

Proof: See Appendix A.

After the opening of trade, each country benefits from more varieties in the sector where it has positive net exports. To see this, let us take the case of the home country. As shown in Appendix A, the home country allocates more labor to sector B in response to the opening of trade, or $L_B^t > L_B^a$. Thus it can consume more varieties of its domestic products in sector B after trade opens. In addition, it can also consume the foreign products in the sector via import if the foreign country diversifies in the free-trade equilibrium, or $L_B^{*t} > 0$. (25) and (26) imply that these increases in the number of consumable varieties in sector B lead to an increase in the real wage rate measured by composite good B in the home country[5]. In parallel fashion, we find that after the opening of trade, the foreign country can consume a wider range of products in sector A and hence has a higher real wage rate measured by composite good A.

In contrast, it is a little bit complicated to explain the mechanism behind a rise in the real wage rate measured by the other composite good in each country. For example, an increase in the real wage rate measured by composite good A in the home country is decomposed into the following two ingredients:[6]

$$a_A w^t \geq a_A^* w^{*t}; \tag{28}$$

$$\frac{L_A^t}{a_A} + \frac{L_A^{*t}}{a_A^*} > \frac{L_A^a}{a_A}. \tag{29}$$

(28), together with (26), implies that the cost difference between countries in sector A tends to have a positive effect on the home country's real wage rate measured by composite good A. Moreover, the home country allocates more labor to sector A under autarky than under free trade, so the number of its domestic varieties in the sector falls in response to the opening of trade. However, (29), together with (25), (26) and (28), implies that the import from the foreign country more than offsets this negative effect on the home country's real wage rate. Analogously, we can explain an increase in the real wage rate measured by composite good B in the foreign country.

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[5] With 'love-of-variety' preferences, an increase in the number of varieties allows consumers to gain higher utility with less consumption of each variety. As a result, the increased number of varieties lowers the price index and hence results in a higher real wage rate.

[6] For the proof of (28) and (29), see Appendix A.

5.2. Welfare Effects of Trade: Semi-Autarky vs. Free Trade

The welfare effects of moving from semi-autarky to free trade are more complex. If the elasticity of cross-sector substitution, σ, falls short of unity, the production shifts induced by the opening of free trade from semi-autarky increase the real wage rates in both countries. In contrast, if σ exceeds unity, the production shifts do not always favor the real wage rates. Actually, it is shown that the production shifts can reduce both countries' real wage rates measured by the composite good of the sector which is opened under semi-autarky. Such a harmful effect on the real wage rate gives rise to the possibility of welfare losses.

Let us first look at the equilibrium allocation of each country under semi-autarky. Before going into the long-run equilibrium, it is necessary to derive each country's wage ratio in the short-run equilibrium. Without loss of generality, we can assume that sector A is closed before moving into free trade. Then, the home country's wage ratio in the short-run equilibrium under semi-autarky is obtained as

$$\frac{w_A}{w_B} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L/a_B + L^*/a_B^* - L_A/a_B - L_A^*/a_B^*)^{(\sigma_B-\sigma)/\sigma(\sigma_B-1)}}{(L_A/a_A)^{(\sigma_A-\sigma)/\sigma(\sigma_A-1)}} \cdot \lambda^{1/\sigma}, \tag{30}$$

where λ is given by

$$\lambda = \frac{L/a_B - L_A/a_B}{L/a_B + L^*/a_B^* - L_A/a_B - L_A^*/a_B^*}. \tag{31}$$

The wage ratio shown in (20) is derived as follows. Since sector A is closed, for $k = A$ we can set n_A^* at zero in (1) and (15) holds. In view of these and by the use of (1), (2) and (15), we have

$$w_A L_A = \alpha_A^{\sigma} P_A^{1-\sigma} E(P_A, P_B)^{\sigma-1} I. \tag{32}$$

National income is given by $I = w_A L_A + w_B L_B$ and L_A^* can be set at zero in (9) for $k = A$. Keeping these in mind and making use of (4), (9) and (32), we obtain (30). Similar calculations yield the foreign country's wage ratio in the short-run equilibrium under semi-autarky:

$$\frac{w_A^*}{w_B^*} = \Theta \cdot \frac{a_B^*}{a_A^*} \cdot \frac{(L/a_B + L^*/a_B^* - L_A/a_B - L_A^*/a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A^*/a_A^*)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} \cdot (1-\lambda)^{1/\sigma}. \qquad (33)$$

By (30) and (33), the phase diagram can be depicted as Figure 3, where schedule hh' (ff') shows the locus of (L_A, L_A^*) for which $\dot{L}_A = 0$ ($\dot{L}_A^* = 0$)[7]. The intersection of the two curves indicates both countries' labor forces which are allocated to sector A in the long-run equilibrium. As shown in the figure, the long-run equilibrium is unique and globally stable (see Appendix B).

(Figure 3)

Given Figure 3, we can determine the real wage rates in the long-run equilibrium under semi-autarky. The home (foreign) country' real wage rate measured by composite good A in the semi-autarky equilibrium is given by

$$\frac{w^{(*)sa}}{P_A^{(*)sa}} = \frac{\rho_A}{a_A^{(*)}} \left[\frac{L_A^{(*)sa}}{a_A^{(*)}} \right]^{1/(\sigma_A - 1)}, \qquad (34)$$

where superscript sa denotes values of variables in the long-run equilibrium under semi-autarky. Sector A is closed under semi-autarky, so each country's real wage rate measured by composite good A takes the same form as that in the case of autarky (see

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[7] If $\sigma < 1$, the loci of (L_A, L_A^*) for which each country's wage ratio equals unity are depicted by the downward sloping curves as shown in Figure 3. In contrast, if $\sigma > 1$, the loci are given by the upward sloping curves.

(25)). On the other hand, the home (foreign) country's real wage rate measured by composite good B in the semi-autarky equilibrium is given by

$$\frac{w^{(*)sa}}{P_B^{(*)sa}} = \frac{\rho_B}{a_B^{(*)}} \left(\frac{L_B^{sa}}{a_B} + \frac{L_B^{*sa}}{a_B^*} \right)^{1/(\sigma_B - 1)} . \tag{35}$$

Sector B is under free trade, so each country's real wage rate measured by composite good B takes the same form as that in the case of free trade (see (26) and (27)). Since both countries have positive outputs in sector B, each country's wage rate in the semi-autarky equilibrium is given by its wage rate in sector B. Keeping this in mind and recalling (21), we obtain (35).

Free trade attains a higher welfare level in each country relative to semi-autarky if each country's real wage rates rise in response to the opening of free trade. However, the production shifts induced by moving from semi-autarky to free trade do not always improve both real wage rates measured by two composite goods: The real wage rate measured by composite good A rises, while the other can fall. The sufficient condition for an increase in the real wage rate measured by composite good B in each country is that the elasticity of cross-sector substitution is less than unity, or $\sigma < 1$. The following lemmas are useful for establishing that both countries become better off by opening free trade when the above condition is satisfied.

Lemma 1: *Suppose that (17), (18) and (19) are satisfied. Then, the following hold:*

$$\frac{L_A^t}{a_A} + \frac{L_A^{*t}}{a_A^*} > \frac{L_A^{sa}}{a_A} ; \tag{36}$$

$$L_A^{*t} > L_A^{*sa} . \tag{37}$$

Proof. See Appendix C.

Lemma 2: *Suppose that (17), (18) and (19) are satisfied. Then, if the elasticity of cross-sector substitution is less than unity, or* $\sigma < 1$*, the following holds:*

$$\frac{L_B^t}{a_B} + \frac{L_B^{*t}}{a_B^*} > \frac{L_B^{sa}}{a_B} + \frac{L_B^{*sa}}{a_B^*}. \tag{38}$$

Proof. See Appendix D.

From (26), (27), (34), (35), Lemmas 1 and 2, it can be shown that under $\sigma < 1$, the opening of free trade from semi-autarky raises each country's real wage rates. Hence, we have the following proposition.

Proposition 3: *Suppose that (17), (18) and (19) are satisfied. Then, if the elasticity of cross-sector substitution is less than unity, or* $\sigma < 1$*, both countries become better off by moving from semi-autarky to free trade.*

Proof. See Appendix E.

The mechanism behind a rise in the real wage rate measured by composite good A in each country is the same as that in the case of moving from autarky to free trade. The home country allocates more labor to sector A under semi-autarky than under free trade and hence suffers fewer varieties of its domestic products in the sector after the opening of free trade[8]. However, (36), together with (26), (28) and (34), implies that this

[8] In the case where $\sigma < 1$, the amount of labor allocated to sector A in the home country falls in response to the opening of free trade from semi-autarky, or $L_A^t < L_A^{sa}$.

negative effect on the real wage rate is dominated by the positive effects associated with the import from the foreign country. As for the foreign country, from (27), (34) and (37) we find that its higher real wage rate measured by composite good A reflects more varieties due to the increased amount of its labor forces allocated to sector A and the import from the home country.

The way in which the opening of free trade affects each country's real wage rate measured by composite good B differs from that in the case of moving from autarky to free trade. Under semi-autarky, each country consumes the other country's products in sector B via intra-industry trade. Thus the real wage rate measured by composite good B in a country is affected by a change in the number of varieties that arises in the other country after the opening of free trade. In the case where the economy is under autarky before the opening of free trade, however, the trade-induced change in the number of varieties in a country has no direct effect on the real wage rate in the other country. In this case, each country can obtain a higher real wage rate by lifting the ban on import in sector B, irrespective of whether the foreign variety expands after the opening of free trade.

In the foreign country, a rise in the real wage rate measured by composite good B is decomposed into (38) and[9]

$$a_B w^t \leq a_B^* w^{*t} . \quad (39)$$

The foreign country reduces the amount of labor allocated to sector B in response to

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This can be shown by making use of (22), (30) and (31) and with the aid of Figure 3. The proof is simple and thus left to the reader.

[9] (39) can be proved in the same manner as (A6) in Appendix A.

the opening of free trade (see (36)) and hence suffers fewer varieties of its domestic products in the sector. However, (38), together with (27), (35) and (39), implies that this negative effect on the real wage rate is dominated by the positive effects associated with more varieties in the home country due to the increased amount of its labor forces allocated to sector B.

In the home country, its higher real wage rate measured by composite good B reflects only the net effects of both countries' labor reallocation given by (38). That is, the cost difference between countries in sector B plays no role in improving the home country's real wage rate. This is because (39) holds with inequality only if the foreign country completely specializes in sector A, or $L_B^{*t} = 0$[10]. This, together with (27), implies that the net effects of both countries' labor reallocation are the only sources of a rise in the real wage rate measured by composite good B in the home country.

It should be noted that the real wage rate measured by composite good B can fall if the amount of labor allocated to sector B in the economy is reduced[11]. This possibility arises when the elasticity of cross-sector substitution σ exceeds unity. A large σ implies that the demand for composite good B drastically decreases in response to a decline in the relative price of composite good A. Other things being

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[10] In the case where the foreign country diversifies, since both countries have positive outputs in sector B, it follows from (21) that $a_B w^t = a_B^* w^{*t}$. In this case, it is obvious from (27) and (35) that the net effects of both countries' labor reallocation given by (38) immediately leads to $w^t / P_B^t > w^{sa} / P_B^{sa}$.

[11] Precisely, each country's real wage rate measured by composite good B can go down if $L_B / a_B + L_B^* / a_B^*$ decreases.

equal, the movement from semi-autarky to free trade lowers the price index of sector A in both countries by enabling each country to consume the other country's products in this sector. Therefore, if cross-sector substitution is elastic (i.e., $\sigma > 1$), the opening of free trade causes the large consumption shifts from composite good B to composite good A and its resultant production shifts can give rise to a fall in the number of varieties in sector B by reducing the amount of labor allocated to this sector. Hence, entering into free trade leads to the possibility of a lower real wage rate measured by composite good B.

In the case where $\sigma > 1$, we can establish the following proposition regarding the impacts of the opening of free trade on each country's real wage rates.

Proposition 4: *Suppose that (17), (18) and (19) are satisfied. Then, if the elasticity of cross-sector substitution is greater than unity, or $\sigma > 1$, the following statements apply to the changes in each country's real wage rates induced by moving from semi-autarky to free trade: (I) Each country's real wage rate measured by composite good A rises, irrespective of the specialization patterns after the opening of free trade; (II) As for each country's real wage rate measured by composite good B, it falls in the case where the foreign country diversifies.*

Proof. See Appendix F.

In the case where the foreign country completely specializes in sector A after the opening of free trade, a change in each country's real wage rate measured by composite good B is ambiguous. As shown in the proposition, the real wage rate measured by composite good B falls if the foreign country diversifies under free trade. In this case,

(38) is not met because the opposite inequality sign holds (see Appendix E). In addition, both countries have positive outputs in sector B, so it follows from (21) that $a_B w^t = a_B^* w^{*t}$. Keeping these in mind and taking a careful look at (26), (27) and (35), it is found that each country's real wage rate measured by composite good B is lowered by moving from semi-autarky to free trade. In contrast, if the foreign country completely specializes under free trade, (38) may or may not hold and $a_B w^t$ does not always equal $a_B^* w^{*t}$. These make it difficult to identify the change in the real wage rate measured by composite good B [12].

A lower real wage rate measured by composite good B leads to the likelihood of welfare losses. Hence, Proposition 4 implies that under $\sigma > 1$, each country may be worse off by the opening of free trade. Indeed, we can show the numerical examples in which one of the two countries faces lower welfare relative to semi-autarky after entering into free trade. Table 1 illustrates such examples in the case where the foreign country diversifies in the free-trade equilibrium. The first and second columns present the wage rates divided by the unit expenditure function that are evaluated at (L_A^{sa}, L_A^{*sa}) and (L_A^t, L_A^{*t}), respectively. The third column presents the ratio of the value in the second column to that in the first column, which shows whether each country is better

[12] Strictly, in the case where the foreign country completely specializes under free trade, a change in the home country's real wage rate measured by composite good B does not depend on whether $a_B w^t = a_B^* w^{*t}$ holds. In this case, it follows that $L_B^{*t} = 0$. This, together with (26), implies that $a_B w^t$ and $a_B^* w^{*t}$ do not affect the home country's real wage rate in the free-trade equilibrium.

off under free trade than under semi-autarky. In Case 1, the home country suffers welfare losses by moving from semi-autarky to free trade. In Case 2, such a shift in the trade regime confers welfare losses on the foreign country.

6. Concluding remarks

This paper studies the implications of the variable share of consumers' expenditure on each sector for international trade in a two-country, two-sector and one-factor (labor) Chamberlinian-Ricardian model. In this paper, we assume a CES upper-tier utility function to incorporate variable expenditure shares into the model. And we also assume a cross-sector adjustment process of labor reallocation induced by the wage gap to check the stability of equilibrium. Under these assumptions, we explore specialization patterns achieved as a result of labor reallocation and the associated welfare changes.

In our model, the stability of equilibrium crucially depends on the sensitivity of expenditure shares to a change in the price ratio of two composite goods, each of which is made up of products in the corresponding sector. If there is no restriction on the elasticity of cross-sector substitution in consumption, we are bothered with instability and/or multiplicity of the equilibrium. The sufficient condition for stability is that the substitution between the composite goods is less elastic than the substitution between every pair of products in each sector. All the results in this paper are premised on this condition.

Under the above condition for stability, each country specializes and hence has positive net exports in the sector where it is relatively productive in terms of the technology index made up of marginal and fixed costs. As in the classical Ricardian model, at least one county completely specializes in the free-trade equilibrium, so that intra-industry trade is less likely to arise in our model than in the Chamberlinian model without cross-country technical heterogeneity. Moreover, each country's relative price under autarky, which is proportional only to its relative marginal cost, is no longer useful for predicting trade patterns.

In the normative analysis of trade, we consider two types of trade regime: free trade, in which both sectors are under free trade; and semi-autarky, in which one sector is under free trade, while the other is closed. Our analysis reveals that both countries become better off under free trade than under autarky, whereas free trade does not always attain higher welfare in both countries relative to semi-autarky.

In our model, a country can lose from the opening of free trade from semi-autarky if the elasticity of cross-sector substitution exceeds unity. The intuition behind the result is as follows. Other things being equal, the opening of free trade lowers the relative price of the composite good made up of products in the sector which is closed under semi-autarky. This fall in the relative price drastically reduces the share of expenditure on the other sector in the presence of sufficiently elastic cross-sector substitution. Hence, each country's labor allocation between sectors under free trade is strongly biased toward the sector with no trade under semi-autarky. This tendency toward fewer varieties in the other sector leads to the negative effects on welfare, which may dominate all the positive effects arising from the interaction between the trade-induced labor reallocation and international cost differences.

The possibility of losses from trade presented in this paper provides an important policy implication. Once we turn to the actual international landscapes, it is readily found that a state of semi-autarky is more plausible as a situation before the opening of free trade, compared to autarky (i.e., a completely self-sufficient state). This suggests that we should be more cautious when considering the welfare effects of trade liberalization.

Appendix A: Proof of Proposition 2

To establish the proposition, we show that the trade-induced production shifts raise each country's real wage rates measured by composite goods. From (25) and (26), we find that w^t / P_k^t exceeds w^a / P_k^a if and only if

$$\frac{L_k^t}{a_k} + \frac{L_k^{*t}}{a_k^*}\left(\frac{a_k w^t}{a_k^* w^{*t}}\right)^{\sigma_k - 1} > \frac{L_k^a}{a_k}. \tag{A1}$$

Similarly, from (25) and (27) it follows that $w^{*t} / P_k^t > w^{*a} / P_k^{*a}$ if and only if

$$\left(\frac{a_k^* w^{*t}}{a_k w^t}\right)^{\sigma_k - 1} \frac{L_k^t}{a_k} + \frac{L_k^{*t}}{a_k^*} > \frac{L_k^{*a}}{a_k^*}. \tag{A2}$$

It can be proved that (A1) and (A2) hold for $k = A, B$, irrespective of the specialization patterns after the opening of trade.

We prove only (A1) here. Parallel arguments apply to the proof of (A2). Let us begin with the case of $k = B$. It is evident that (A1) holds for $k = B$ if the home country increases the amount of labor allocated to sector B in response to the opening of trade, or equivalently,

$$L_B^t > L_B^a. \tag{A3}$$

Obviously, (A3) holds if the home country completely specializes in sector B. In the case where the home country diversifies, (A3) can be proved in the following way. Evaluating the home country's wage ratio given in (22) at $L_A = L_A^t$ and $L_A^* = L_A^{*t} = L^*$, we obtain

$$\frac{w_A^t}{w_B^t} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L / a_B - L_A^t / a_B)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A^t / a_A + L^* / a_A^*)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} = 1,$$

from which it follows that

$$\Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L / a_B - L_A^t / a_B)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A^t / a_A)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} > 1 . \quad \text{(A4)}$$

The wage ratio in the autarky equilibrium is

$$\frac{w_A^a}{w_B^a} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L / a_B - L_A^a / a_B)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A^a / a_A)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} = 1 , \quad \text{(A5)}$$

which is yielded by evaluating (14) at $L_A = L_A^a$. (A4) and (A5), together with the decreasingness of (14) with respect to L_A, implies that $L_A^t < L_A^a$. Combining this and the full employment condition $L_A + L_B = L$, we obtain (A3). Therefore, (A1) always holds for $k = B$.

Next, we prove that (A1) holds for $k = A$. This is true if the following hold:

$$a_A w^t \geq a_A^* w^{*t} ; \quad \text{(A6)}$$

$$L_A^t / a_A + L_A^{*t} / a_A^* > L_A^a / a_A . \quad \text{(A7)}$$

Let us first verify (A6). Since the home (foreign) country has positive outputs in sector B (sector A) after the opening of trade, the country's wage rate in the free-trade equilibrium is given by $w^t = w_B^t$ ($w^{*t} = w_A^{*t}$), where w_B^t (w_A^{*t}) is the home (foreign) country's wage rate in sector B (sector A) in the free-trade equilibrium. Hence, the proof of (A6) is given by

$$a_A w^t = a_B w_B^t \cdot (a_A / a_B)$$

$$= a_B^* w_B^{*t} \cdot (a_A / a_B) \quad \text{(A8)}$$

$$\geq a_A^* w_A^{*t} \quad \text{(A9)}$$

$$= a_A^* w^{*t} .$$

(A8) immediately follows from (21). To show (A9), we need to use (21) and the fact that the home country's wage ratio does not exceed unity in the free-trade equilibrium, or $w_A^t / w_B^t \leq 1$. From (21), it follows that

$$a_A w_A^t / a_B w_B^t = a_A^* w_A^{*t} / a_B^* w_B^{*t} .$$

In view of this and by the use of $w_A^t / w_B^t \leq 1$, we can show the inequality sign in (A9). Next, let us prove (A7). In the free-trade equilibrium, the following hold:

$$\frac{w_A^t}{w_B^t} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L/a_B + L^*/a_B^* - L_A^t/a_B - L_A^{*t}/a_B^*)^{(\sigma_B-\sigma)/\sigma(\sigma_B-1)}}{(L_A^t/a_A + L_A^{*t}/a_A^*)^{(\sigma_A-\sigma)/\sigma(\sigma_A-1)}} \leq 1, \qquad \text{(A10)}$$

which is obtained by substituting L_A^t and L_A^{*t} for L_A and L_A^* in (22). Combining (A5) with (A10) and taking account of both countries' full employment conditions yield

$$\left(\frac{L_B^t / a_B + L_B^{*t} / a_B^*}{L_B^a / a_B} \right)^{\frac{\sigma_B-\sigma}{\sigma(\sigma_B-1)}} \leq \left(\frac{L_A^t / a_A + L_A^{*t} / a_A^*}{L_A^a / a_A} \right)^{\frac{\sigma_A-\sigma}{\sigma(\sigma_A-1)}} .$$

By (A3), the left-hand side is greater than unity. Hence, (A7) is obtained.

From the above results, (A1) holds for $k = A, B$, irrespective of the specialization patterns after the opening of trade. **Q.E.D.**

Appendix B: Stability of the Long-Run Equilibrium under Semi-Autarky

As shown in Figure 3, the long-run equilibrium under semi-autarky is unique and globally stable (i) if each country's wage ratio is falling in the quantity of its labor forces allocated to sector A and (ii) if the $\dot{L}_A = 0$ locus depicted by hh' is steeper than the $\dot{L}_A^* = 0$ locus depicted by ff' at their intersection. Keeping (29) in mind and taking a careful look at (28) and (31), we find that the first condition for stability is satisfied. On the other hand, some calculations are required to confirm the second condition for stability. This condition implies that the following Jacobean takes a positive value at the intersection of the two loci:

$$J = \begin{vmatrix} \dfrac{\partial(w_A / w_B)}{\partial L_A} & \dfrac{\partial(w_A / w_B)}{\partial L_A^*} \\ \dfrac{\partial(w_A^* / w_B^*)}{\partial L_A} & \dfrac{\partial(w_A^* / w_B^*)}{\partial L_A^*} \end{vmatrix}. \tag{B1}$$

Taking account of (29) and differentiating (28) and (31) with respect to L_A and L_A^*, we obtain the partial derivatives in (B1). Evaluating them at the intersection of the loci yields the following:

$$\frac{\partial(w_A / w_B)}{\partial L_A} = -\left[\frac{\theta_B + (1-\lambda)/\sigma\lambda}{L - L_A + (a_B / a_B^*)(L^* - L_A^*)} + \frac{\theta_A}{L_A} \right], \tag{B2}$$

$$\frac{\partial(w_A / w_B)}{\partial L_A^*} = \frac{1/\sigma - \theta_B}{(a_B^* / a_B)(L - L_A) + L^* - L_A^*}, \tag{B3}$$

$$\frac{\partial(w_A^* / w_B^*)}{\partial L_A^*} = -\left[\frac{\theta_B + \lambda/\sigma(1-\lambda)}{(a_B^* / a_B)(L - L_A) + L^* - L_A^*} + \frac{\theta_A}{L_A^*} \right], \tag{B4}$$

$$\frac{\partial(w_A^* / w_B^*)}{\partial L_A} = \frac{1/\sigma - \theta_B}{L - L_A + (a_B / a_B^*)(L^* - L_A^*)}, \tag{B5}$$

where $\theta_k \equiv (\sigma_k - \sigma)/\sigma(\sigma_k - 1)$ ($k = A, B$). Insertion of (B2), (B3), (B4) and (B5) into (B1) and a little calculation reveal that (B1) takes a positive value at the intersection of the loci. Hence, the second condition for stability is also satisfied.

Appendix C: Proof of Lemma 1

Proof of (36): We can prove (36) by showing that the following (i) and (ii) hold:

(i) The home country's wage ratio under free trade (i.e., w_A / w_B given in (22)) is unchanged or falls as we move down along the locus of (L_A, L_A^*) such that $L_A / a_A + L_A^* / a_A^* = l$, where l is an arbitrary positive constant.

(ii) $L_A^{sa} < \tilde{L}_A$, where $\tilde{L}_A$ is the value of L_A such that the home country's wage ratio under free trade becomes equal to unity for $L_A^* = 0$.

With the aid of Figure 2, we can demonstrate that the above results are sufficient for (36). As has already been mentioned, schedule HH' in the figure is the locus of (L_A, L_A^*) satisfying $\dot{L}_A = 0$ under free trade, or equivalently, the locus of (L_A, L_A^*) such that the home country's wage ratio under free trade becomes equal to unity. (i) implies that the $L_A / a_A + L_A^* / a_A^* = l$ locus is parallel to or flatter than HH'. Moreover, the long-run equilibrium under free trade, or (L_A^t, L_A^{*t}), is attained at a point on FHH' and $(\tilde{L}_A, 0)$ is represented by H' (see the definition of $\tilde{L}_A$ given in (ii)). Setting l at $L_A^t / a_A + L_A^{*t} / a_A^*$, $(\tilde{L}_A, 0)$ is located in the lower region of the $L_A / a_A + L_A^* / a_A^* = l$ locus. Thus the following holds:

$$L_A^t / a_A + L_A^{*t} / a_A^* > \tilde{L}_A / a_A .$$

From this and (ii), we immediately obtain (36). Hence, (i) and (ii) are sufficient for (34).

Proof of (i): Eliminating L_A^* from the home country's wage ratio under free trade by the use of $L_A / a_A + L_A^* / a_A^* = l$, we obtain

$$\frac{w_A}{w_B} = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{[L / a_B + L^* / a_B^* - \Lambda(l)]^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{l^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}}, \tag{C1}$$

where $\Lambda(l)$ is given by

$$\Lambda(l) = \frac{a_A^*}{a_B^*} \cdot l + \left(\frac{a_A}{a_B} - \frac{a_A^*}{a_B^*} \right) \frac{L_A}{a_A}. \tag{C2}$$

(C2) rises in response to an increase in L_A under (19), so (C1) is decreasing in regard to L_A if $\sigma_B > \sigma$. In the case where $\sigma_B = \sigma$, (C1) is invariable even if L_A changes. These imply that the home country's wage ratio under free trade is unchanged or decreases as we move down along the $L_A / a_A + L_A^* / a_A^* = l$ locus.

Proof of (ii): It is sufficient to show that the following (ii-1), (ii-2) and (ii-3) hold:

(ii-1) $\tilde{L}_A$ is invariable if L^* / L is changed subject to $L / a_B + L^* / a_B^* = \bar{L}$, where $\bar{L}$ is constant.

(ii-2) L_A^{sa} equals $\tilde{L}_A$ if L^* is set at zero, or equivalently, $L = a_B \bar{L}$.

(ii-3) L_A^{sa} decreases if L^* / L is increased subject to $L / a_B + L^* / a_B^* = \bar{L}$.

The sufficiency of these can proved as follows. Suppose that $L_A^{sa} \geq \tilde{L}_A$ holds for some endowment pair denoted by $(\hat{L}, \hat{L}^*)$. Then, by (ii-3), we obtain $L_A^{sa} > \tilde{L}_A$ for any (L, L^*) satisfying $L^* / L < \hat{L}^* / \hat{L}$ and $L / a_B + L^* / a_B^* = \widehat{L}$, where $\widehat{L} = \hat{L} / a_B + \hat{L}^* / a_B^*$. However, this runs in contradiction with the implication of (ii-1), (ii-2) and (ii-3) that L_A^{sa} does not exceed $\tilde{L}_A$ for any (L, L^*) satisfying $L / a_B + L^* / a_B^* = \widehat{L}$. Therefore, $L_A^{sa} < \tilde{L}_A$ always holds.

Let us turn to the proof of (ii-1), (ii-2) and (ii-3). (ii-1) and (ii-2) are obvious from (22) and (30), respectively. (ii-3) can be proved with the aid of Figure 3, where hh' (ff') represents the locus of (L_A, L_A^*) for which $w_A / w_B = 1$ ($w_A^* / w_A^* = 1$) under semi-autarky. By the decreasingness of (30) and (32) in regard to L_A and L_A^*, respectively, hh' shifts inward in response to an decrease in L, while ff' shifts outward in response to an increase in L^*. Since hh' is steeper than ff', as shown in Appendix B, such shifts of the two loci move their intersection to the upper left in the figure. Hence, (ii-3) holds.

Proof of (37): It is evident that (37) holds when the foreign country completely specializes in sector A after the opening of free trade, or $L_A^{*t} = L^*$. In the case where the foreign country diversifies, the proof is given as follows. Let $\tilde{L}_A^*$ denote the value of L_A^* such that the foreign country's wage ratio under free trade becomes equal to unity for $L_A = 0$. Then, in a similar way as the case of (36), it can be shown that $L_A^{*sa} < \tilde{L}_A^*$. From Figure 2, we find that the free-trade equilibrium is attained at H, or $L_A^{*t} = \tilde{L}_A^*$, in the current case. Hence, (37) also applies to the case in which the foreign country diversifies. **Q.E.D.**

Appendix D: Proof of Lemma 2

Let $\Omega(L_A, L_A^*)$ be the wage ratio of the home country under free trade (i.e., w_A / w_B given in (22)) and let $\Omega^*(L_A, L_A^*)$ be that of the foreign country under free trade (i.e., w_A^* / w_B^* given in (23)). Then, we can prove the lemma by showing that the following (i), (ii), (iii) and (iv) hold:

(i) $\Omega(L_A, L_A^*)$ increases as we move down along the locus of (L_A, L_A^*) such that $L_A / a_B + L_A^* / a_B^* = l$, where l is an arbitrary positive constant.

(ii) $\Omega((a_B / a_B^*)\tilde{L}_A^*, 0) > 1$, where $\tilde{L}_A^*$ is the value of L_A^* that solves $\Omega^*(0, L_A^*) = 1$.

(iii) $\dfrac{L_A^{sa}}{a_B} + \dfrac{L_A^{*sa}}{a_B^*} > \dfrac{\tilde{L}_A^*}{a_B^*}$.

(iv) $\Omega(L_A^{sa}, L_A^{*as}) < 1$.

With the aid of Figure 2, we can demonstrate that the above results are sufficient for the lemma. HH' in the figure is the locus of (L_A, L_A^*) satisfying $\dot{L}_A = 0$ under free trade, or equivalently, the locus of (L_A, L_A^*) for which $\Omega(L_A, L_A^*) = 1$. FJ represents the locus of (L_A, L_A^*) satisfying the following equality:

$$L_A / a_B + L_A^* / a_B^* = \tilde{L}_A^* / a_B^* . \tag{D1}$$

If (i) and (ii) are satisfied, FJ intersects HH' such that the former cuts the latter from above. Moreover, (iii) and (iv) imply that the long-run equilibrium under semi-autarky, (L_A^{sa}, L_A^{*sa}), is located in the upper region of FGH'. Thus if the long-run equilibrium under free trade, (L_A^t, L_A^{*t}), is attained at a point on FHG, it immediately follows that

$$\frac{L_A^{sa}}{a_B} + \frac{L_A^{*sa}}{a_B^*} > \frac{L_A^t}{a_B} + \frac{L_A^{*t}}{a_B^*} . \tag{D2}$$

This also applies to the case in which (L_A^t, L_A^{*t}) is on GH'. In this case, setting l at $L_A^t / a_B + L_A^{*t} / a_B^*$, it follows from (i) that the $L_A / a_B + L_A^* / a_B^* = l$ locus passes through (L_A^t, L_A^{*t}) such that it cuts GH' from above. Then, noticing that $L_A^{*t} = L^* > L_A^{*sa}$, we find that (D2) is true of the current case. From the above, it can be concluded that (D2) holds even if the free-trade equilibrium is attained at any point on FHH'. In view of each country's full employment condition, it is easy to confirm that (D2) is equivalent to (38). Hence, (i), (ii), (iii) and (iv) are sufficient for (38).

Proof of (i): Eliminating L_A^* from $\Omega(L_A, L_A^*)$ by the use of $L_A / a_B + L_A^* / a_B^* = l$, we obtain

$$\Omega(L_A, L_A^*) = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L / a_B + L^* / a_B^* - l)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{\Delta(l)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} , \tag{D3}$$

where $\Delta(l)$ is given by

$$\Delta(l) = \frac{a_B^*}{a_A^*} \cdot l - \left(\frac{a_B^*}{a_A^*} - \frac{a_B}{a_A} \right) \frac{L_A}{a_B} . \tag{D4}$$

(D3) represents the wage ratio of the home country under free trade that is evaluated at an arbitrary point on the $L_A / a_B + L_A^* / a_B^* = l$ locus. (D4) falls in response to an increase in L_A under (19), so (D3) is increasing in regard to L_A. This implies that $\Omega(L_A, L_A^*)$ increases as we move down along the $L_A / a_B + L_A^* / a_B^* = l$ locus.

Proof of (ii): Calculating $\Omega((a_B / a_B^*)\tilde{L}_A^*, 0)$ yields the following:

$$\Omega((a_B / a_B^*)\tilde{L}_A^*, 0) = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L / a_B + L^* / a_B^* - \tilde{L}_A^* / a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{[(a_B / a_B^*)\tilde{L}_A^* / a_A]^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} \tag{D5}$$

$$= \Theta \cdot \frac{a_A^*}{a_B^*} \cdot \frac{(L / a_B + L^* / a_B^* - \tilde{L}_A^* / a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(\tilde{L}_A^* / a_A^*)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} \cdot \phi^{(1-\sigma)\sigma_A/\sigma(\sigma_A - 1)}$$

$$= \Omega^*(0, \tilde{L}_A^*) \cdot \phi^{(1-\sigma)\sigma_A/\sigma(\sigma_A - 1)}$$

$$= \phi^{(1-\sigma)\sigma_A/\sigma(\sigma_A - 1)}, \tag{D6}$$

where ϕ is given by

$$\phi = (a_A / a_B) \cdot (a_B^* / a_A^*) . \tag{D7}$$

From (19), it follows that $\phi > 1$. Moreover, by assumption the elasticity of cross-sector substitution is now smaller than unity, or $\sigma < 1$, (D6) exceeds unity. Therefore, we have $\Omega((a_B / a_B^*)\tilde{L}_A^*, 0) > 1$.

Proof of (iii): Let $\omega(L_A, L_A^*)$ be the wage ratio of the home country under semi-autarky (i.e., w_A / w_B given in (28)) and let $\omega^*(L_A, L_A^*)$ be the wage ratio of the foreign country under semi-autarky (i.e., w_A^* / w_B^* given in (33)). Then, (iii) can be proved by establishing that the following (iii-1) and (iii-2) hold:

(iii-1) $\omega(L_A, L_A^*)$ ($\omega^*(L_A, L_A^*)$) decreases (increases) as we move down along the locus of (L_A, L_A^*) satisfying (D1).

(iii-2) $\omega(sL, sL^*) > 1$ and $\omega^*(sL, sL^*) > 1$, where

$$s = \frac{\tilde{L}_A^* / a_B^*}{L / a_B + L^* / a_B^*}. \tag{D8}$$

With the aid of Figure 3, we can demonstrate that (iii-1) and (iii-2) are sufficient for (iii). As has already been mentioned, schedule hh' (ff') is the locus of (L_A, L_A^*) satisfying $\dot{L}_A = 0$ ($\dot{L}_A^* = 0$) under semi-autarky, or equivalently, the locus of (L_A, L_A^*) for which $\omega(L_A, L_A^*) = 1$ ($\omega^*(L_A, L_A^*) = 1$). FJ is the same as that of Figure 2, or the locus of (L_A, L_A^*) satisfying (D1), so that the intersection of FJ and OE gives (sL, sL^*). If (iii-1) is satisfied, hh' (FJ) intersects FJ (ff') such that the former cuts the latter from above. (iii-2) implies that (sL, sL^*) is located in the lower region of hh' and ff'. Pulling these together, we find that the intersection of hh' and ff', or (L_A^{sa}, L_A^{*sa}), is located in the upper region of FJ. Therefore, if (iii-1) and (iii-2) are satisfied, (iii) holds.

Let us turn to the proof of (iii-1) and (iii-2). We prove only the properties regarding $\omega(L_A, L_A^*)$ here. As for the properties of $\omega^*(L_A, L_A^*)$, they can be shown by the same logic.

Proof of (iii-1): Eliminating L_A^* from $\omega(L_A, L_A^*)$ by the use of (D1), we obtain

$$\omega(L_A, L_A^*) = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{(L/a_B + L^*/a_B^* - \tilde{L}_A^*/a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A/a_A)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} \cdot \tilde{\lambda}^{1/\sigma}, \tag{D9}$$

where $\tilde{\lambda}$ is given by

$$\tilde{\lambda} = \frac{L/a_B - L_A/a_B}{L/a_B + L^*/a_B^* - \tilde{L}_A^*/a_B^*}. \tag{D10}$$

(D9) represents the wage ratio of the home country under semi-autarky that is evaluated at an arbitrary point on the locus of (L_A, L_A^*) satisfying (D1). Taking account of (D10), we find that (D9) is decreasing in L_A. This implies that $\omega(L_A, L_A^*)$ falls as (L_A, L_A^*) is moved downward along the (D1) locus.

Proof of (iii-2): We can prove $\omega(sL, sL^*) > 1$ by establishing that the following (a) and (b) hold:

(a) $\omega(sL, sL^*)$ increases as we raise L^*/L while keeping $L/a_B + L^*/a_B^*$ constant at $\bar{L}$ (i.e., we move up along WW' in Figure 2).

(b) $\omega(sL, sL^*)$ is greater than unity if each country's labor endowment is given by $(L, L^*) = (a_B\bar{L}, 0)$.

It is obvious that these are sufficient for $\omega(sL, sL^*) > 1$. Let us begin with the proof of (a). Now, calculating $\omega(sL, sL^*)$ yields the following:

$$\omega(sL, sL^*) = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{[(1-s)(L/a_B + L^*/a_B^*)]^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(sL/a_A)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} \cdot \bar{\lambda}^{1/\sigma}, \tag{D11}$$

where $\bar{\lambda}$ is given by

$$\bar{\lambda} = \frac{L/a_B}{L/a_B + L^*/a_B^*}. \tag{D12}$$

Taking account of (D12), it is found that for a given s, (D11) increases by raising L^* / L while keeping $L / a_B + L^* / a_B^*$ constant. (D8) shows that s is unchanged if the above change in L^* / L has no influence on $\tilde{L}_A^*$. This is evident from the definition of $\tilde{L}_A^*$ and (23). Hence, (a) holds. Next, let us prove (b). Inserting $L = a_B \bar{L}$ and $L^* = 0$ into (D11), the following is obtained:

$$\omega(s\bar{L}, 0) = \Theta \cdot \frac{a_B}{a_A} \cdot \frac{[(1-s)\bar{L}]^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(s a_B \bar{L} / a_A)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}}.$$

Rewriting this by the use of $L / a_B + L^* / a_B^* = \bar{L}$ and the definition of s given in (D8) yields (D5). This, together with (ii), implies that (b) holds. From the above, it can be concluded that $\omega(sL, sL^*) > 1$ is satisfied.

Proof of (iv): The outline of proof is as follows. Evaluating $\Omega(L_A, L_A^*)$ (i.e., w_A / w_B under free trade) at the semi-autarky equilibrium, (L_A^{sa}, L_A^{*as}), yields the following:

$$\Omega(L_A^{sa}, L_A^{*sa}) = \left[\frac{1 + \phi^{-\sigma} q^{\rho}}{(1+q)^{\rho}} \right]^{1/\sigma}, \tag{D13}$$

where ϕ is the technology parameter given in (D7). Moreover, q and ρ are given by $q = (L_A^{*as} / a_A^*)/(L_A^{sa} / a_A)$ and $\rho = (\sigma_A - \sigma)/(\sigma_A - 1)$ respectively. From $\sigma < 1$, it follows that $\rho > 1$. Then, if we let $\varsigma(q)$ denote the bracket term in the right-hand side of (D13), it satisfies the following:

$$\forall q > 0, \quad \varsigma(q) < 1. \tag{D14}$$

From this, $\Omega(L_A^{sa}, L_A^{*sa}) < 1$ is immediately obtained. Hence, (iv) is proved if we derive (D13) and show that (D14) is satisfied.

Let us begin with derivation of (D13). By $\omega(L_A^{sa}, L_A^{*sa}) = \omega^*(L_A^{sa}, L_A^{*sa}) = 1$, (30), (31) and (32), the following hold in the semi-autarky equilibrium:

$$\Theta \frac{a_B}{a_A} \cdot \frac{(L/a_B + L^*/a_B^* - L_A^{sa}/a_B - L_A^{*sa}/a_B^*)^{(\sigma_B - \sigma)/\sigma(\sigma_B - 1)}}{(L_A^{sa}/a_A)^{(\sigma_A - \sigma)/\sigma(\sigma_A - 1)}} \cdot (\lambda^{sa})^{1/\sigma} = 1, \qquad \text{(D15)}$$

$$\frac{(L - L_A^{*as})/a_B^*}{(L - L_A^{sa})/a_B} = \phi^{-\sigma} \cdot \left(\frac{L_A^{*as}/a_A^*}{L_A^{sa}/a_A} \right)^{\frac{\sigma_A - \sigma}{\sigma_A - 1}}, \qquad \text{(D16)}$$

where λ^{sa} represents the value of λ evaluated at (L_A^{sa}, L_A^{*as}). Insertion of (L_A^{sa}, L_A^{*as}) into (22), together with the use of (D15) and (D16), yields (D13).

Let us turn to the proof of (D14). Differentiating $\varsigma(q)$ with respect to q, the following is obtained:

$$\frac{d\varsigma(q)}{dq} = \frac{\rho(\phi^{-\sigma} q^{\rho-1} - 1)}{(1+q)^{\rho+1}}.$$

Thus $\varsigma(q)$ has a negative (positive) slope if q falls short of (exceeds) $\phi^{\sigma/(\rho-1)}$. Moreover, it is shown that $\varsigma(0) = 1$ and that $\varsigma(q) \to \phi^{-\sigma} < 1$ as $q \to \infty$. Pulling these together, we find that (D14) is satisfied.

As a consequence, Lemma 1 holds. **Q.E.D.**

Appendix E: Proof of Proposition 3

To establish the proposition, we show that under $\sigma < 1$, each country's real wage rates are improved by moving from semi-autarky to free trade. (36) and (37) in Lemma 1 are used to demonstrate that each country's real wage rate measured by composite good A rises as a result of labor reallocation induced by the above shift in the trade regime. (38) in Lemma 2 plays an important role in proving that a higher real wage rate measured by composite good B are attained in each country.

Frist, we consider the case of the home country. From (26) and (34), it is easy to verify that w^t / P_A^t exceeds w^{sa} / P_A^{sa} if and only if

$$\frac{L_A^t}{a_A} + \frac{L_A^{*t}}{a_A^*}\left(\frac{a_A w^t}{a_A^* w^{*t}}\right)^{\sigma_A - 1} > \frac{L_A^{sa}}{a_A}. \tag{E1}$$

Similarly, from (26) and (35) it follows that $w^t / P_B^t > w^{sa} / P_B^{sa}$ if and only if

$$\frac{L_B^t}{a_B} + \frac{L_B^{*t}}{a_B^*}\left(\frac{a_B w^t}{a_B^* w^{*t}}\right)^{\sigma_B - 1} > \frac{L_B^{as}}{a_B} + \frac{L_B^{*as}}{a_B^*}. \tag{E2}$$

We begin by proving that (E1) holds. As shown in Lemma 1, (36) is satisfied, so that (E1) follows if $a_A w^t \geq a_A^* w^{*t}$. This inequality has already been proved in (A6). Thus we obtain (E1). The proof of (E2) is as follows. If the foreign country completely specializes in sector A after the opening of free trade, or $L_B^{*t} = 0$, (E2) follows from (38) immediately. In the case where the foreign country diversifies, (21) is necessary for the proof. In this case, both countries have positive outputs in sector B, so each country's wage rate in the free-trade equilibrium is given by its wage rate in sector B. This, together with (21), implies that $a_B w^t / a_B^* w^{*t} = 1$. Hence, (38) is equivalent to (E2) if the foreign country diversifies.

Let us turn to the case of the foreign country. From (27) and (34), it is easily shown that w^{*t} / P_A^t exceeds w^{*sa} / P_A^{*sa} if and only if

$$\left(\frac{a_A^* w^{*t}}{a_A w^t}\right)^{\sigma_A - 1} \frac{L_A^t}{a_A} + \frac{L_A^{*t}}{a_A^*} > \frac{L_A^{*sa}}{a_A^*}. \tag{E3}$$

Similarly, from (27) and (35) it follows that $w^{*t} / P_B^t > w^{*sa} / P_B^{sa}$ if and only if

$$\left(\frac{a_B^* w^{*t}}{a_B w^t}\right)^{\sigma_B - 1} \frac{L_B^t}{a_B} + \frac{L_B^{*t}}{a_B^*} > \frac{L_B^{as}}{a_B} + \frac{L_B^{*as}}{a_B^*}. \tag{E4}$$

(E3) follows from (37) immediately. The proof of (E4) is as follows. In the free-trade equilibrium, we have $a_B w^t \le a_B^* w^{*t}$, which can be shown in the same way as (A6). Combining this with (38) yields (E4).

From the above results, it is found that each country's real wage rates are improved by moving from semi-autarky to free trade. **Q.E.D.**

Appendix F: Proof of Proposition 4

Proof of (I): In the proof of Proposition 3, condition $\sigma < 1$ was not used to prove that the real wage rate measured by composite good A rises. This implies that the real wage rate measured by composite good A is always improved by moving from semi-autarky to free trade, regardless of whether σ is less than unity. Therefore, (I) holds.

Proof of (II): This is true if the following holds:

$$L_A^{sa} / a_B + L_A^{*sa} / a_B^* < \tilde{L}_A^* / a_B^*, \tag{F1}$$

where $\tilde{L}_A^*$ is the value of L_A^* such that the foreign country's wage ratio becomes equal to unity for $L_A = 0$. The sufficiency of (F1) can be shown in the following way. From (26), (27) and (35), it is found that each country's real wage rate measured by composite good B falls if and only if the opposite inequality signs hold in (E2) and (E4). Moreover, in the case where the foreign country diversifies, both countries have positive outputs in sector B, so each country's wage rate in the free-trade equilibrium is given by its wage rate in sector B. This, together with (21), implies that $a_B w^t / a_B^* w^{*t} = 1$.

Hence, the necessary and sufficient condition for lower real wage rates measured by composite good B is given by

$$L_B^t / a_B + L_B^{*t} / a_B^* < L_B^{as} / a_B + L_B^{*as} / a_B^* . \quad \text{(F2)}$$

Note also that $L_A^t = 0$ and $L_A^{*t} = \tilde{L}_A^*$ in the case where the foreign country diversifies. Keeping this in mind and using each country's full employment condition, it is shown that (F2) is equivalent to (F1).

To prove (F1), we show that (iii-2) in Appendix D applies to the case where $\sigma > 1$. With the aid of Figure 4, it can be demonstrated that (iii-2) in Appendix D is sufficient for (F1). Schedule hh' (ff') represents the locus of (L_A, L_A^*) satisfying $\dot{L}_A = 0$ ($\dot{L}_A^* = 0$) under semi-autarky, or equivalently, the locus of (L_A, L_A^*) for which $\omega(L_A, L_A^*) = 1$ ($\omega^*(L_A, L_A^*) = 1$). (As shown in the figure, schedules hh' and ff' slope upward under $\sigma > 1$.) FJ is the locus of (L_A, L_A^*) satisfying (D1), so that the intersection of FJ and OE gives (sL, sL^*). (iii-2) in Appendix D implies that (sL, sL^*) is located in the lower region of hh' and in the upper region of ff'. From this, we find that the intersection of hh' and ff', or (L_A^{sa}, L_A^{*sa}), is located in the lower region of FJ. Therefore, if (iii-2) in Appendix D are satisfied, (F1) holds.

Let us turn to the proof of (iii-2) in Appendix D. We prove only the properties regarding $\omega(L_A, L_A^*)$ here. As for the properties of $\omega^*(L_A, L_A^*)$, they can be shown by the same logic. To establish that $\omega(sL, sL^*) > 1$ applies to the case where $\sigma > 1$, we show that the following (a) and (b) are satisfied:

(a) $\omega(sL, sL^*)$ decreases as we raise L^* / L while keeping $L / a_B + L^* / a_B^*$ constant at $\bar{L}$ (i.e., we move up along WW' in Figure 2).

(b) $\omega(sL, sL^*)$ is less than unity if each country's labor endowment is given by $(L, L^*) = (a_B\bar{L}, 0)$.

It is obvious that these are sufficient for $\omega(sL, sL^*) > 1$. Let us begin with the proof of (a). Calculating $\omega(sL, sL^*)$ yields (D11). Taking account of (D12), we find that for a given s, (D11) increases by raising L^*/L while keeping $L/a_B + L^*/a_B^*$ constant, given that σ is greater than unity. As has been shown in the proof of (iii-2) in Appendix D, the above change in L^*/L has no influence on s. Hence, (a) holds. Next, let us prove (b). Calculating $\omega(s\bar{L}, 0)$ reveals that it equals (D5), which has also been shown in the proof of (iii-2) in Appendix D. By (D6), (D5) is less than unity under $\sigma > 1$. Hence, (b) holds. From the above, it can be concluded that $\omega(sL, sL^*) > 1$ is satisfied. **Q.E.D.**

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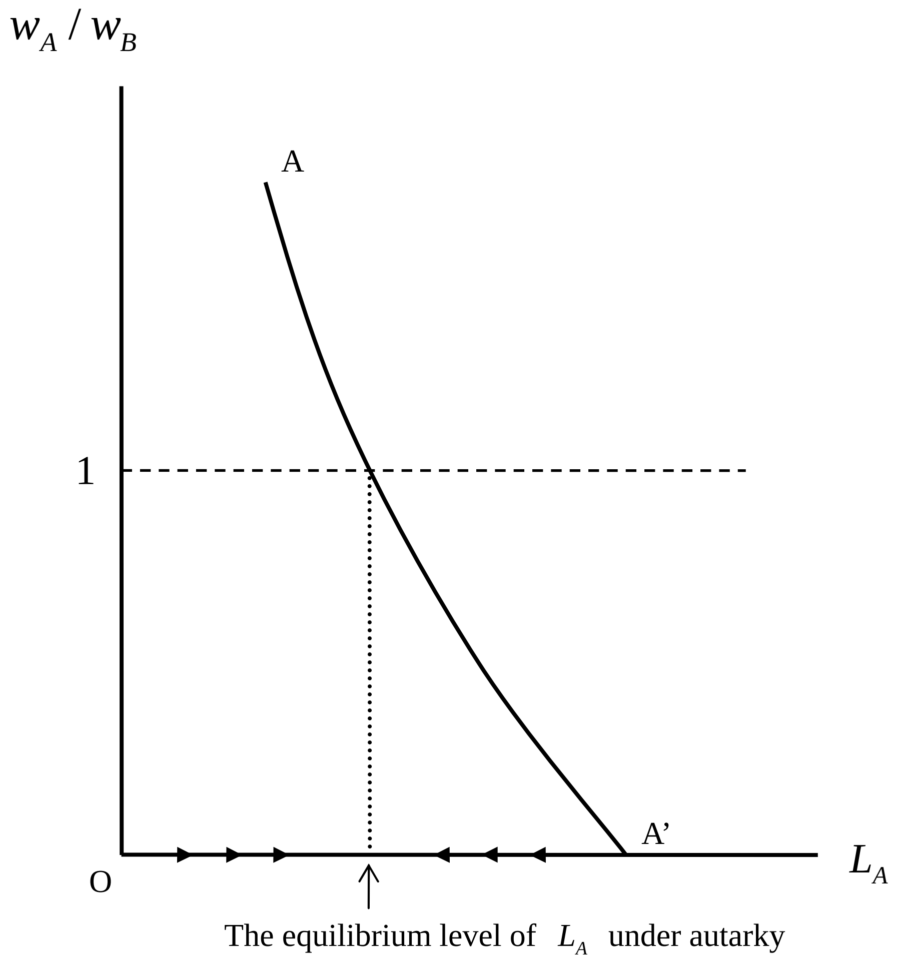

Figure 1

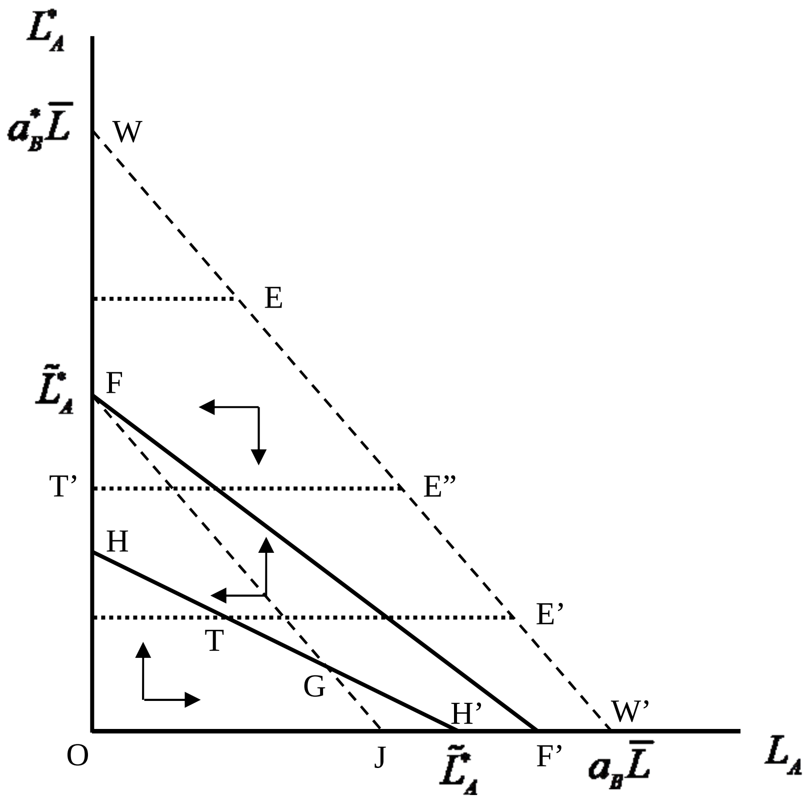

Figure 2

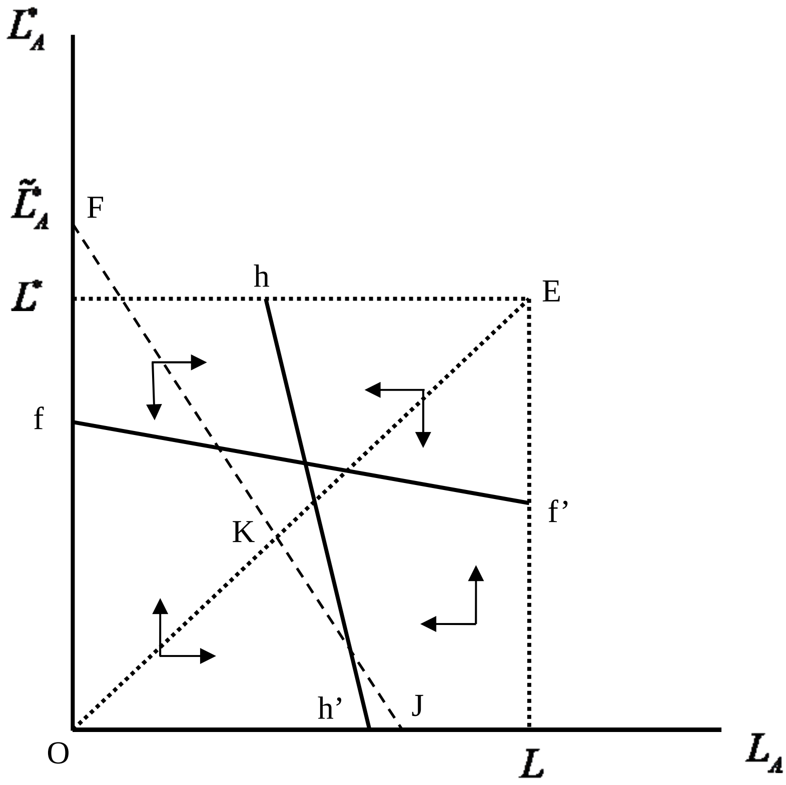

Figure 3

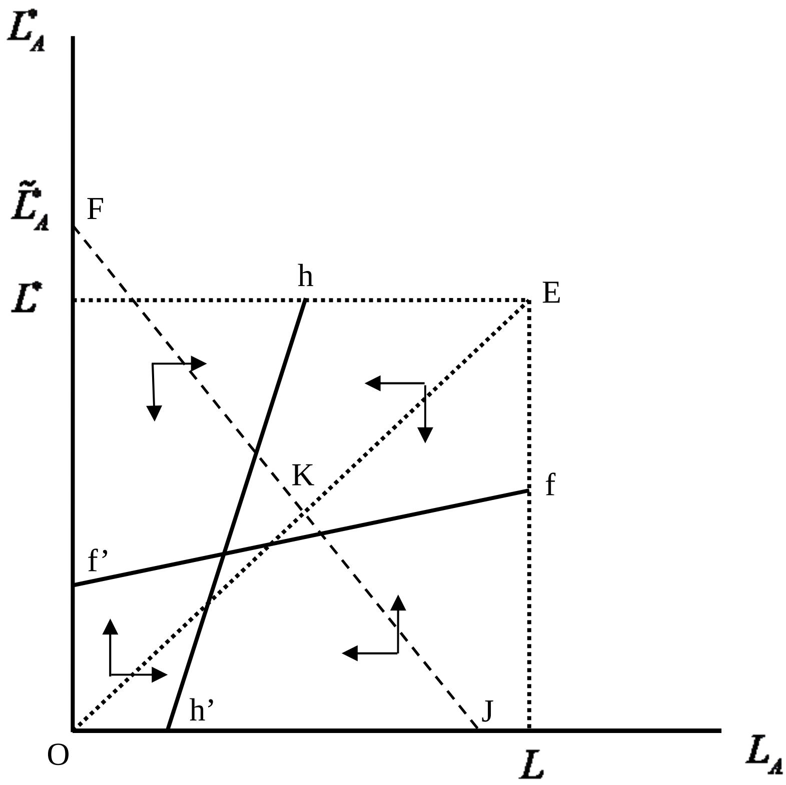

Figure 4

Table 1

Welfare changes induced by moving from semi-autarky to free trade: The case where the foreign country diversifies in the free-trade equilibrium

Case 1: $a_B^* = 1.2$, $L = 0.85$, $L^* = 0.18$			
	Wage rate / Unit expenditure function		(2) / (1)
	(1) Semi-autarky	(2) Free trade	
Home	0.16	0.160304	0.998102
Foreign	0.133333	0.131915	1.010755

Case 2: $a_B^* = 2$, $L = 0.7$, $L^* = 0.6$			
	Wage rate / Unit expenditure function		(2) / (1)
	(1) Semi-autarky	(2) Free trade	
Home	0.16	0.155436	1.029364
Foreign	0.08	0.085885	0.931484

Notes: The other parameter values are set as $a_A = a_B = a_A^* = 1$, $\sigma = \sigma_B = 2$, $\sigma_A = 6$ and $\alpha = 0.2$ both in Case 1 and in Case 2. Precisely, a_k is the technology index composed of parameters b_k, f_k and σ_k. However, this index can be treated as a parameter if the values of b_k and f_k are adjusted such that they are compatible with the values of a_k and σ_k. The same is true of a_k^*.