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<th>Title</th>
<th>Strategic Reasoning in Extensive Games with Short Sight</th>
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Introduction
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A calculation

In a game like Chess, game tree’s size grows exponentially with both its depth and its branching factor.
Time complexity: $O(b^d)$ (b for branching factor, d for depth )

<table>
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<th>b</th>
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Branching factor: $b \approx 35$, depth: $d \approx 100$.
Number of paths in the game tree: $35^{100} \approx 10^{135}$. Much too big for a normal game tree search.

Comparison: Number of particles in the universe $\approx 10^{87}$

• Strong assumption:
Entire structure of a game is common knowledge to all players.

• Solution:
Grossi and Turrini proposed the concept of games with short sight (Grossi and Turrini, 2012), in which players can only see part of the game tree.

• Contribution:
A modal Logic system for reasoning about games with short sight.
**Extensive game: an example**

Figure: Tic-Tac-Toe game

Rule: Two players take turns to mark the spaces in a $3 \times 3$ grid. The player who succeeds in placing three respective marks in a horizontal, vertical, or diagonal row wins the game.

- $\Sigma_i$ is a non-empty set of strategies. $\Sigma_i = \{\sigma_i\}$.
- $\sigma_i$ is a strategy of player $i$, which is a function: $\{v \in V \setminus Z \mid t(v) = i\} \rightarrow V|v$, assigning a successor $v'$ of $v$ to each non-terminal node $v$ when it is $i$'s turn to move. (where $V|v$ is the set of nodes extending $v$.)
- $\sigma = (\sigma_i)_{i \in N}$ represents a strategy profile which is a combination of strategies from all players and $\Sigma$ represents the set of all strategy profiles.
- $\sigma_{-i}$ denotes the collection of strategies in $\sigma$ excluding those for player $i$.
- $O(\sigma)$ is the outcome if the strategy profile $\sigma$ is followed by all players.
- $O(\sigma', \sigma_{-i})$ is the outcome if player $i$ use strategy $\sigma'$ while all other players employ $\sigma$.
- $\succeq_i$ is a preference relation over $V^2$ for each player $i$.

**Definition**

An extensive game is a tuple $G = (N, V, A, t, \Sigma, \preceq)$, where

- $N$ is a non-empty set of the players,
- $V$ a set of nodes or vertices including a root $v_0$
- $A \subseteq V^2$ a set of arcs. If $(v, v') \in A$, we call $v'$ a successor of $v$.
- Leaves are the nodes that have no successors, denoted by $Z$.
- $t$ is turn function assigning a member of $N$ to each non-terminal node. $t(v) = i$;

**Example**

Two players: player 1($\times$) and player 2($\circ$). Solid arrows: the moves of player 1, dotted arrows: moves of player 2. The initial state is $v_0$.

$v_1, v_2, v_3$ are all successors of $v_0$.
$v_{10}, v_{11}, v_{12}$ are the terminal nodes (leaves).
Characterizing solutions concepts for games with short sight

Example

Formally, \( G = (N, V, A, t, \Sigma, \succeq) \)

- \( N = \{1, 2\} \)
- \( V = \{v_0, v_1, v_2, \ldots\} \)
- \( (v_0, v_1), (v_0, v_2), (v_0, v_3) \in A \)
- \( v_{10}, v_{11}, v_{12} \in \mathbb{Z} \)
- \( t(v_0) = 1, t(v_2) = 2, j \ldots \)
- \( a \sigma_1 \) such that \( \sigma_1(v_0) = v_2, \sigma_1(v_5) = v_8, \ldots \)
- \( a \sigma_2 \) such that \( \sigma_2(v_2) = v_5, \ldots \)

Thus, a strategy profile \( \sigma = (\sigma_1, \sigma_2) \) such that \( O(\sigma) = v_{11} \).

- \( v_{12} \succeq v_{11} \succeq v_{10} \) (since player 1 wins the game in \( v_{12} \), loses it in \( v_{10} \), and gains a draw in \( v_{11} \)).

Definition

(sight function). Let \( G = (N, V, A, t, \Sigma, \succeq) \) be an extensive game. A short sight function for \( G \) is a function \( s : V \setminus Z \rightarrow 2^{V \setminus \emptyset} \), associating to each non-terminal node \( v \) a finite subset of all the available nodes at \( v \).

Definition

(Extensive game with short sight). An extensive game with short sight (Egss) is a tuple \( S = (G, s) \) where \( G \) is a finite extensive game and \( s \) a sight function.

- Endowing an extensive game with a sight function.
Syntax and Semantics

$$\varphi := p | \neg \varphi_0 \wedge \varphi_1 | (\leq) \varphi | (\hat{\sigma}) \varphi | (\hat{\sigma}) \varphi | (\hat{\sigma}) \varphi | (\hat{\sigma}) \varphi | (\hat{\sigma}) \varphi$$

- The label $\leq_i$ denotes player $i$'s preference relation.
- The label $\hat{\sigma}$ stands for the outcomes of strategy profiles. $(v, v') \in R_{\sigma}$ if $v'$ is the terminal node reached from $v$ by following $\sigma$.

(v₀, v₁₁) ∈ R_{v₀}

Example

- $N_{v₀} = N$;
- $V_{v₀} = \{v₀, v₁, v₂, v₃, v₅\}$;
- $Z_{v₀} = \{v₁, v₃, v₄, v₅\}$;
- $A_{v₀} = \{(v₀, v₁), (v₂, v₄), (v₂, v₄), \cdots\}$;
- $σ_{v₀} = (σ₁_{v₀}, σ₂_{v₀})$ such that $O_{v₀}(σ_{v₀}) = v₅$, with $σ₁_{v₀}(v₀) = v₂$ and $σ₂_{v₀}(v₂) = v₅$

Figure: Sight-filtarated extensive game $S_{v₀}$
Characterizing solutions concepts for games with short sight

**Syntax and Semantics**

\[ \varphi ::= p | \neg \varphi | \varphi_0 \land \varphi_1 | \langle \leq \rangle \varphi | (\langle \delta \rangle \varphi) | (\langle \delta^\# \rangle \varphi) | (\langle \delta_{\sigma} \rangle \varphi) \]

- The label \( < \) is sight function for the current node, and 
- \((v, v') \in R_i\) means 'node \( v \)' is within the sight at the present node \( v \).

**Frame** \( F \): \((V, R_{\leq}, R_{\circ}, R_{\circ}, R_{\circ}, R_{\circ}, R_{\circ})\), where

- \( vR_{\leq}v' \iff v' \geq_i v \)
- \( vR_{\circ}v' \iff v' = O[v(\sigma|_V)] \)
- \( vR_{\circ}v' \iff v' \in O[V\sigma|_V] \)
- \( vR_{\circ}v' \iff v' \in s(v) \)
- \( vR_{\circ}v' \iff v' = O[v(\sigma|_V)] \)
- \( vR_{\circ}v' \iff v' \in O[V\sigma|_V] \)

**Model:** \( M = (V, R, I) \)

- \( M, v \models (\leq) \varphi \iff M, u \models \varphi \) for some \( u \in V \) with \( vR_{\leq}u \).
- \( M, v \models (\delta) \varphi \iff M, u \models \varphi \) for some \( u \in V \) with \( vR_{\circ}u \).
- \( M, v \models (\delta_{\sigma}) \varphi \iff M, u \models \varphi \) for some \( u \in V \) with \( vR_{\circ}u \).
- \( M, v \models (\delta^\#) \varphi \iff M, u \models \varphi \) for some \( u \in V \) with \( vR_{\circ}u \).
- \( M, v \models (\delta^\#) \varphi \iff M, u \models \varphi \) for some \( u \in V \) with \( vR_{\circ}u \).
Validities

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<td>$[\cdot]_i \varphi \rightarrow \varphi$</td>
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<td>transitivity</td>
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<tr>
<td>D</td>
<td>$[\bar{v}]$</td>
<td>$[\bar{v}] \varphi \leftrightarrow (\bar{v}) \varphi$</td>
<td>determinism</td>
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<tr>
<td>I</td>
<td>$(\bar{v}_1, [\bar{v}_2 \ldots])$</td>
<td>$([\bar{v}_1] [\bar{v}_2 \ldots]) \varphi \rightarrow ([\bar{v}_1]) \varphi$</td>
<td>inclusions</td>
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<td>$(\bar{v}_1), [\bar{v}_2 \ldots]$</td>
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<tr>
<td>M</td>
<td>$[\bar{v}]$</td>
<td>$([\bar{v}] \varphi \leftrightarrow \varphi)$</td>
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<td>$([\bar{v}_1]) ([\bar{v}_2 \ldots]) \varphi \leftrightarrow (\bar{v}_1) \varphi$</td>
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<td>$[\bar{v}_1 \ldots] \varphi \rightarrow [\bar{v}_1 \ldots] \varphi$</td>
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Soundness and Completeness

LEGS is sound and complete w.r.t. the class of all games with short sight.

Solution concepts for traditional extensive games

Definition

(Best response and Nash equilibrium)
A best response for player $i$ of an extensive game is a strategy profile $\sigma^*$ such that $O(\sigma^*_i, \sigma^*_{-i}) \succeq_i O(\sigma_i, \sigma^*_{-i})$ for every strategy $\sigma_i$ of player $i$. A strategy profile $\sigma^*$ is a Nash equilibrium of an extensive game if it is a best response for every player $i$.

Solution concepts for extensive games with short sight

Definition

(Sight-compatible best response and Nash equilibrium).
Let $S = (G, s)$ be an Egs and $S|_v$ be the sight-filtrated extensive game at $v$. A strategy profile $\sigma^*$ is a sight-compatible best response for $i$ if for every nonterminal node $v$, it holds that $O|_v(\sigma^*_i|_v, \sigma^*_{-i}|_v) \succeq_i O|_v(\sigma_i|_v, \sigma^*_{-i}|_v)$ for any strategy $\sigma_i|_v$ available to $i$. $\sigma^*$ is a sight-compatible Nash equilibrium(SCNE) of $S$ if it is a sight-compatible best response for every player $i \in N$. 
Characterize solution concepts

**Theorem**

Let $S$ be an EgsS given by $(N, V, A, t, \Sigma, \succeq, s)$. Then for any player $i$, any strategy profiles $\sigma$ in $S$ and any formulas $\varphi$ of LS:

(a) $\sigma$ is a sight-compatible best response (SCBR) of $S$ for $i$ iff $F_S \models [\hat{\sigma}^S]_i \varphi \rightarrow [\hat{\sigma}^S]_{-i}(\leq_i)\varphi$.

(b) $\sigma$ is a sight-compatible Nash equilibrium (SCNE) of $S$ iff $F_S \models \bigwedge_{i \in N}([\hat{\sigma}]_i \varphi \rightarrow [\hat{\sigma}]_{-i}(\leq_i)\varphi)$.

(c) $\sigma$ is a subgame perfect equilibrium (SPE) of $S$ iff for any $u \in V \setminus Z$, $F_{S | v} \models [\hat{\sigma}]_{-i} \varphi \rightarrow [\hat{\sigma}]_{-i}(\leq_i)\varphi$.

(d) A strategy profile $\sigma$ is a sight-compatible SPE of $S$ iff for all $v \in V \setminus Z$, $F_{S | v} \models [\hat{\sigma}]_{-i} \varphi \rightarrow [\hat{\sigma}]_{-i}(\leq_i)\varphi)$.

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Thank you!