Dynamic Normative Logic and Information Update

Yasuo Nakayama
Graduate School of Human Sciences, Osaka University
1-2 Yamada-oka, Suita, Osaka, 565-0871 JAPAN
nakayama@hus.osaka-u.ac.jp

In Nakayama (2010), *Logic for Normative Systems* (LNS) was proposed. In this paper, I show how to deal with information update within LNS. I call LNS with information update device *Dynamic Normative Logic* (DNL). Recently, the dynamic epistemic logic (DEL) has been established as a framework for logical description of social interactions.[1] DNL can be considered as an alternative framework for the same purpose. DNL can explicitly express conditions for social behaviors and describe interactions between social actions and normative inference in detail.

1. Logic for Normative Systems and Dynamic Normative Logic

The following is a modification of LNS in Nakayama (2010).[2]

Let T and OB be a set of sentences in *First-Order Logic* (FOL) and q be a sentence of FOL.

(1a) A pair $\langle T, OB\rangle$ consisting of *belief base T* and *obligation base OB* is called a *normative system* ($NS = \langle T, OB\rangle$).

(1b) q belongs to the *belief set* of normative system NS (abbreviated as $\mathbf{B}_{NS}\, q$) $\Leftrightarrow$ q follows from T.

(1c) q belongs to the *obligation set* of NS (abbreviated as $\mathbf{O}_{NS}\, q$) $\Leftrightarrow$ $T\cup OB$ is consistent & q follows from $T\cup OB$ & q does not follow from T.

(1d) q belongs to the *prohibition set* of NS (abbreviated as $\mathbf{F}_{NS}\, q$) $\Leftrightarrow$ $\mathbf{O}_{NS}\, \neg q$.

(1e) *q belongs to the permission set of NS* (abbreviated as $\mathbf{P}_{NS}\, q$) $\Leftrightarrow$ $T\cup OB\cup\{q\}$ is consistent & q does not follow from T.

(1f) A normative system $\langle T, OB\rangle$ is consistent $\Leftrightarrow$ $T\cup OB$ is consistent.

(1g) In this paper, we interpret that NS represents a normative system accepted by a person or by a group at a particular time. Thus, we insert *what a person* (or *a group*) *believes to be true* into the *belief base* and *what he believes that it ought to be done* into the *obligation base*.

Based on the above definition, we can easily prove the following main theorems that characterize

[1] For the development of the dynamic epistemic logic, you nay consult van Benthem (2011). His description is restricted on various kinds of (dynamic) extension of *propositional* modal logics.

[2] We use &, $\Rightarrow$, and $\Leftrightarrow$ as meta-semantic abbreviation for *and*, *if … then*, and *if and only if*.

LNS, where $NS = \langle T, OB \rangle$.

(2a) $(\mathbf{B}_{NS}\, (p \rightarrow q)$ & $\mathbf{B}_{NS}\, p) \Rightarrow \mathbf{B}_{NS}\, q$.

(2b1) $(\mathbf{O}_{NS}\, (p \rightarrow q)$ & $\mathbf{O}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.

(2b2) $\mathbf{O}_{NS}\, p \Rightarrow \mathbf{P}_{NS}\, p$.

(2b3) $\mathbf{F}_{NS}\, p \Rightarrow$ not $\mathbf{P}_{NS}\, p$.

(2c1) $\mathbf{P}_{NS}\, p \Rightarrow$ not $\mathbf{B}_{NS}\, p$.

(2c2) $\mathbf{B}_{NS}\, p \Rightarrow$ (not $\mathbf{O}_{NS}\, p$ & not $\mathbf{F}_{NS}\, p$ & not $\mathbf{P}_{NS}$ p).

(2d1) $(\mathbf{O}_{NS}\, (p \rightarrow q)$ & $\mathbf{B}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.

(2d2) $(\mathbf{O}_{NS}\, (p \wedge q)$ & not $\mathbf{B}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, p$.

(2d3) $(\mathbf{O}_{NS}\, (p \wedge q)$ & $\mathbf{B}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.

(2d4) $(\mathbf{O}_{NS}\, (p \vee q)$ & $\mathbf{B}_{NS}\, \neg p) \Rightarrow \mathbf{O}_{NS}\, q$.

(2d5) $(\mathbf{O}_{NS}\, (p \vee q)$ & $\mathbf{F}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.

(2d6) $(\mathbf{O}_{NS}\, p$ & not $\mathbf{B}_{NS}\, q) \Rightarrow \mathbf{O}_{NS}\, (p \vee q)$.

(2d7) $(\mathbf{B}_{NS}\, (p \rightarrow q)$ & $\mathbf{O}_{NS}\, p$ & $\mathbf{P}_{NS}\, q) \Rightarrow \mathbf{O}_{NS}\, q$.

(2e1) $(\mathbf{O}_{NS}\, \forall x_{1\ldots}\, \forall x_{\mathrm{n}}\, (P(x_1,\ldots, x_{\mathrm{n}}) \rightarrow Q(x_1,\ldots, x_{\mathrm{n}}))$ & $\mathbf{B}_{NS}\, P(a_1,\ldots, a_{\mathrm{n}})$ & not $\mathbf{B}_{NS}\, Q(a_1,\ldots, a_{\mathrm{n}}))$ $\Rightarrow \mathbf{O}_{NS}\, Q(a_1,\ldots, a_{\mathrm{n}})$. [This means: If $\forall x_{1\ldots}\, \forall x_{\mathrm{n}}\, (P(x_1,\ldots, x_{\mathrm{n}}) \rightarrow Q(x_1,\ldots, x_{\mathrm{n}}))$ is an obligation and you believe $P(a_1,\ldots, a_{\mathrm{n}})$, then $Q(a_1,\ldots, a_{\mathrm{n}})$ is an obligation unless you believe that it was already done.]

(2e2) $(\mathbf{F}_{NS}\, \exists x_{1\ldots}\, \exists x_{\mathrm{n}}\, (P(x_1,\ldots, x_{\mathrm{n}}) \wedge Q(x_1,\ldots, x_{\mathrm{n}}))$ & $\mathbf{B}_{NS}\, P(a_1,\ldots, a_{\mathrm{n}})$ & not $\mathbf{B}_{NS}\, \neg Q(a_1,\ldots, a_{\mathrm{n}}))$ $\Rightarrow \mathbf{F}_{NS}\, Q(a_1,\ldots, a_{\mathrm{n}})$.

We update normative system $\langle T, OB \rangle$ through extending T or OB with new information p (i.e. $T \cup \{p\}$ or $OB \cup \{p\}$). In this paper, we call sometimes a normative system a *normative state*. As we see in the next section, a normative state of a person can be dependent on that of other person. To emphasize aspects of information update, we call LNS with information update device *Dynamic Normative Logic* (DNL).

2. An Application of DNL

To clarify update processes, we divide belief base T into two parts, namely elementary theory ET and a set of facts $FACT$. Thus, it holds, $T = ET \cup FACT$ & $ET \cap FACT = \varnothing$. In the example in this section, only $FACT$ is updated.

As an example, we consider a simple scene in a restaurant described by (van Benthem 2011: 4):

> In a restaurant, your Father has ordered Fish, your Mother ordered Vegetarian, and you have Meat. Out of the kitchen comes some new person carrying the three plates. What will happen?

We assume, here, that the asked person is a boy. The following list describes possible developments of the scene and translations of the described sentences into formula of FOL.

The waiter asks, 'Who has the Meat?	*ask* (*w*, *Family*, *ιj ordered* (*j*, *meat*), 0)
The boy says 'Me'.	*answer* (*b*, *ιj ordered* (*j*, *meat*), *b*, 0)
The waiter serves him with the meat plate.	*serve* (*w*, *b*, *meat*, 0)
The waiter asks, 'Who has the Fish?'	*ask* (*w*, *Family*, *ιj ordered* (*j*, *fish*), 1)
The father says 'Me'.	*answer* (*f*, *ιj ordered* (*j*, *fish*), *f*, 1)
The waiter serves him with the fish plate.	*serve* (*w*, *f*, *fish*, 1)
The waiter serves the mother with the vegetarian plate without asking.	*serve* (*w*, *m*, *v*, 2)

To describe this scene within DNL, we need to make explicit each component of *NS* in this story.

Elementary Theory for the group (i.e. the family members and the waiter): ET^G = {(3a), (3b), (3c), (3d)}.

(3a) [Set Theoretical Principles] $\forall G_1 \forall G_2\, (G_1 = G_2 \leftrightarrow \forall x\, (x \in G_1 \leftrightarrow x \in G_2)) \wedge \forall x\, \forall G_1\, \forall G_2\, (x \in G_1 \cup G_2 \leftrightarrow (x \in G_1 \vee x \in G_2)) \wedge \forall x\, \forall\, G_1\, \forall G_2\, (x \in G_1 - G_2 \leftrightarrow (x \in G_1 \wedge \neg x \in G_2))$.

(3b) $\forall i\, \exists^{=1} x\, ordered\, (i, x) \wedge \forall i\, \forall j\, \forall x\, \forall y\, (ordered\, (i, x) \wedge ordered\, (j, y) \wedge i \neq j \rightarrow x \neq y)$.

(Each family member ordered exactly one plate.)

(3c) $\forall i\, \forall x\, (\exists n\, answer\, (i, \iota j\, ordered\, (j, x), i, n) \rightarrow i = \iota j\, ordered\, (j, x))$, where $\iota j\, ordered\, (j, x)$ refers to the person who ordered *x*. This use of *ι*-operator is justified by (3b).

(If someone answers that he ordered *x*, then he is the person who ordered *x*.)

(3d) $\forall i\, \forall G_1\, \forall n\, (served\, (G_1, n) \wedge \exists x\, serve\, (w, i, x, n) \rightarrow served\, (G_1 \cup \{i\}, n+1))$.

(At stage *n* where G_1 is already served, if the waiter serves person *i* with a plate, then $G_1 \cup \{i\}$ is served at stage *n*+1.)

Elementary Theory for the waiter: ET^w = {(3e)}

(3e) $\forall x\, \forall D\, \forall n\, (have\text{-}plate\, (*, D, n) \wedge x \in D \wedge \exists i\, serve\, (*, i, x, n) \rightarrow have\text{-}plate\, (*, D - \{x\}, n+1))$, where sign '*' indicates that this belief is a *de se belief* (i.e. belief about himself).

(The waiter believes: At stage *n* where he has plates *D*, if he serves someone with plate *x*, then he has plates *D* – {*x*} at stage *n*+1.)

Obligation Base for the group: OB^G = {(4a)}

(4a) $\forall i\, \forall x\, (ordered\, (i, x) \rightarrow \forall n\, (ask\, (w, Family, \iota j\, ordered\, (j, x), n) \rightarrow answer\, (i, \iota j\, ordered\, (j, x), i, n)))$.

(If the waiter asks the family 'Who ordered *x?*', then the person who ordered *x* should answer that he (or she) did. This rule expresses a social norm for guests in a restaurant.)[3]

[3] Here, the speech act of asking is interpreted as a request for an answer from a person who has sufficient

Obligation Base for the waiter: $OB^w = \{(4b)\}$

(4b) $\forall i\ \forall x\ (ordered\ (i, x) \rightarrow \forall D\ \forall n\ (have\text{-}plate\ (*, D, n) \wedge x \in D \rightarrow serve\ (*, i, x, n)))$, where sign '*' indicates that this obligation is a *de se norm* (i.e. norm about himself).

(The waiter should serve a guest with the meal that he (or she) ordered.)

(5a) Initial State:

$FACT^G_0 = \{Family = \{b, f, m\}, Plate = \{meat, fish, v\}, served\ (\varnothing, 0)\}$.

$FACT^b_0 = \{ordered\ (*, meat), * \in Family\}$. The content of $FACT^b_0$ means 'I ordered *meat* and I belong to the *Family*', where 'I' refers to the boy.

$FACT^f_0 = \{ordered\ (*, fish), * \in Family\}$.

$FACT^m_0 = \{ordered\ (*, v), * \in Family\}$.

$FACT^w_0 = \{have\text{-}plate\ (*, Plate, 0)\}$.

(5b) Normative systems on state *n* (In this story, $FACT^G_n$ is updated along the development of the situation.)

$G(n) = \langle T^G_n, OB^G\rangle$, where $T^G_n = ET^G \cup FACT^G_n$.

$boy(n) = \langle T^G_n \cup FACT^b_0, OB^G\rangle$,

$father(n) = \langle T^G_n \cup FACT^f_0, OB^G\rangle$,

$mother(n) = \langle T^G_n \cup FACT^m_0, OB^G\rangle$,

$waiter(n) = \langle T^G_n \cup ET^w \cup FACT^w_0, OB^G \cup OB^w\rangle$.

Based on (5b), we can easily show that $\mathbf{B}_{G(n)}$ expresses a shared belief among four people in the story, namely it holds: $\mathbf{B}_{G(n)}\, p \Rightarrow (\mathbf{B}_{waiter(n)}\, p\ \&\ \mathbf{B}_{boy(n)}\, p\ \&\ \mathbf{B}_{father(n)}\, p\ \&\ \mathbf{B}_{mother(n)}\, p)$.

I propose to interpret the restaurant story as a game played by the waiter and three guests who are cooperative with the waiter. We assume here that each of players obeys and performs any obligation that is required in each situation. It is the goal of this game that the waiter correctly distributes all plates he had at the initial state. Now, we can describe the development with help of DNL as follows.

(6a) By constructing a finite model, we can prove: $\mathbf{P}_{waiter(0)}\ ask\ (*, Family, \iota j\ ordered\ (j, meat), 0)$. Thus, the waiter asks, 'Who has the Meat?': $FACT^G_1 = FACT^G_0 \cup \{ask\ (w, Family, \iota j\ ordered\ (j, meat), 0)\}$.

Then, because of (4a) and (5a): $\mathbf{O}_{boy(1)}\ answer\ (*, \iota j\ ordered\ (j, meat), *, 0)$. Following this obligation, the boy says 'Me': $FACT^G_2 = FACT^G_1 \cup \{answer\ (b, \iota j\ ordered\ (j, meat), b, 0)\}$.

Now, because of (3c) and (5b): $\mathbf{B}_{waiter(2)}\ ordered\ (b, meat)$, which means that the waiter realizes that the boy ordered meat. Then, from (4b) follows: $\mathbf{O}_{waiter(2)}\ serve\ (*, b, meat, 0)$.

information.

Following this obligation, the waiter serves the boy with the meat plate: $FACT^G{}_3 = FACT^G{}_2 \cup \{$*serve* (*w*, *b*, *meat*, 0)$\}$. Now, from (3d) and (3e) follows: $\mathbf{B}_{G(3)}$ *served* ({*b*}, 1) & $\mathbf{B}_{waiter(3)}$ *have-plate* (*, {*fish*, *v*}, 1).

(6 b) Similarly as (6a), we obtain the following updates and attitude changes:

$\mathbf{P}_{waiter(3)}$ *ask* (*, *Family*, *ιj ordered* (*j*, *fish*), 1).

$FACT^G{}_4 = FACT^G{}_3 \cup \{$*ask* (*w*, *Family*, *ιj ordered* (*j*, *fish*), 1)$\}$.

$\mathbf{O}_{father(4)}$ *answer* (*, *ιj ordered* (*j*, *fish*), *, 1).

$FACT^G{}_5 = FACT^G{}_4 \cup \{$*answer* (*f*, *ιj ordered* (*j*, *fish*), *f*, 1)$\}$.

$\mathbf{B}_{waiter(5)}$ *ordered* (*f*, *fish*) & $\mathbf{O}_{\mathrm{waiter}(5)}$ *serve* (*, *f*, *fish*, 1).

$FACT^G{}_6 = FACT^G{}_5 \cup \{$*serve* (*w*, *f*, *fish*, 1)$\}$.

$\mathbf{B}_{G(6)}$ *served* ({*b*, *f*}, 2) & $\mathbf{B}_{waiter(6)}$ *have-plate* (*, {*v*}, 2).

(6c) In the third stage, the waiter infers who ordered the vegetarian plate without asking. Because of (3b): $\mathbf{B}_{waiter(6)}$ *ordered* (*m*, *v*). Thus, $\mathbf{O}_{waiter(6)}$ *serve* (*, *m*, *v*, 2). Following this obligation, the waiter serves the mother with the vegetarian: $FACT^G{}_7 = FACT^G{}_6 \cup \{$*serve* (w, *m*, *v*, 2)$\}$. Then, we obtain: $\mathbf{B}_{G(7)}$ *served* ({*b*, *f*, *m*}, 3) & $\mathbf{B}_{waiter(7)}$ *have-plate* (*, $\emptyset$, 3). This shows that the waiter realized that he had accomplished his current task.

Now, you may recognize that this interaction in the restaurant is similar to many language games described in Wittgenstein (1953). Actually, simple language games can be described within DNL. Furthermore, other puzzles like *The Cards* and *The Muddy Children* (cf. van Benthem 2011: 8, 12) can be solved within DNL.

3. Concluding Remarks

In this paper, we extended LNS and defined DNL. Then, we have shown how to describe information update within DNL and applied DNL to a logical elucidation of social interactions in a restaurant scene. The method used in this paper is applicable to descriptions of social interactions among multiple agents, especially when these interactions involve belief update that affects normative attitudes.

References

van Benthem, J. (2011) *Logical Dynamics of Information and Interaction,* Cambridge University Press.

Nakayama, Y. (2010) "Logical Framework for Normative Systems," *SOCREAL 2010: Proceedings of the 2nd International Workshop On Philosophy and Ethics of Social Reality,* 27–28 March 2010, Sapporo: 19-24.

Wittgenstein, L. (1953) *Philosophical Investigations*.