学 位 論 文 内 容 の 要 旨
Abstract of Doctoral Dissertation

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Mathematical Studies on Dirac Operators with a Variable Mass with Application to the Chiral Quark Soliton Model
変質量をもつディラック作用素の数学的研究とカイラルクォークソリトンモデルへの応用

The Hamiltonian of Chiral Quark Soliton model (CQS) in nuclear physics is described by following Dirac type operator

$$H_{\mathrm{CQS}} := -i\sum_{j=1}^{3}\alpha_j D_j \otimes 1_2 + m\beta \otimes 1_2 e^{i\sum_{j=1}^{3} F\gamma_5\otimes\sigma_j n_j},$$

on the Hilbert space $L^2(\mathbb{R}^3;\mathbb{C}^4)\otimes\mathbb{C}^2$.

Let's compare it to usual free Dirac operator with mass m

$$H_{\mathrm{UD}} := -i\sum_{j=1}^{3}\alpha_j D_j + m\beta,$$

on the Hilbert space $L^2(\mathbb{R}^3;\mathbb{C}^4)$.

Here i is the imaginary unit, $\alpha_1, \alpha_2, \alpha_3$ and $\beta = \alpha_4$ are 4×4 Dirac matrices and 1_n denotes the $n\times n$ unit matrix, D_j $(j = 1, 2, 3)$ is the generalized partial differential operator in the space variable x_j $(\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3)$, $m > 0$ denotes the mass of a quark,

$$\gamma_5 := -i\alpha_1\alpha_2\alpha_3,$$

$F : \mathbb{R}^3 \to \mathbb{R}$ is called a *profile function*, Borel measurable, almost everywhere (a.e.) finite $\mathbf{x} \in \mathbb{R}^{\mathbf{3}}$, σ_1, σ_2 and σ_3 are the Pauli matrices and $n_j : \mathbb{R}^3 \to \mathbb{R}$ is Borel measurable functions such that

$$\sum_{j=1}^{3} n_j(x)^2 = 1$$

for a.e. $\mathbf{x} \in \mathbb{R}^{\mathbf{3}}$.

The main difference in the above operators are the mass term. The mass term of H_{CQS} is spatially variable in general. Hence, the CQS model may be regarded as a model of Dirac particle with a variable mass.

The main propose of this work is to build a model, which can be an abstract d-dimensional extension of CQS model and under suitable conditions to investigate it's Hamiltonian's self-adjoint property, supersymmetric aspects and spectral properties. We will name the Hamiltonian of this extended model by *"d-dimensional Dirac operator with a variable mass"*. The Hamiltonian of *d-dimensional chiral quark soliton model* is defined as follows:

$$H := -i \sum_{j=1}^{d} \alpha_j D_j + \alpha_{d+1} e^{i\Phi} M,$$

here $d \geq 2$ is natural number,

$$N_d := \begin{cases} 2^{d/2} & \text{for } d \text{ even} \\ 2^{(d+1)/2} & \text{for } d \text{ odd,} \end{cases}$$

$\alpha_j, \quad j = 1, \ldots, d+1$ is $N_d \times N_d$ Hermitian matrices satisfying

$$\{\alpha_j, \alpha_k\} = 2\delta_{jk} 1_{N_d}, \quad j, k = 1, \cdots, d+1.$$

Let $\mathcal{K}$ be a finite dimensional Hilbert space. We denote by $\mathcal{F}_{\text{s.a.}}$ the self-adjoint operators on $\mathbb{C}^{N_d} \otimes \mathcal{K}$ such that the mapping :$\mathbb{R}^d \ni \mathbf{x} \to (\Phi(\mathbf{x}) + i)^{-1}$ is measurable. $\Phi(\cdot), \quad M(\cdot) \in \mathcal{F}_{\text{s.a.}}$ and respectively Φ and M be the direct integrals over $\mathbb{R}^d$.

The *"d-dimensional Dirac operator with a variable mass"* acts on

$$\mathcal{H} := L^2(\mathbb{R}^d; \mathbb{C}^{N_d}) \otimes \mathcal{K}.$$

In this work we will give a simple condition for H to be self-adjoint and discuss supersymmetric aspects and spectrum of H. Also we give a condition for H to be a supercharge of a supersymmetric quantum mechanical model. In that case, $\ker H$, the kernel of H, describes the supersymmetric states. Hence it is interesting and important to analyze $\ker H$. We will prove that, under some condition, $\ker H$ is trivial: $\ker H = \{0\}$. In the case where H is a supercharge, this means that there is no supersymmetric states, namely, the supersymmetry is spontaneously broken. We concerned with a unitary equivalence of H to a gauge theoretic Dirac operator. This may be physically interesting. Using this structure, we find another condition for the kernel of H to be trivial. We identify the essential spectrum of H under a suitable condition. In the last, we will discuss the number of eigenvalues of H in the interval $(-m, m)$ with $m > 0$ being a constant.