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学 位 論 文 内 容 の 要 旨 Abstract of Doctoral Dissertation

博士の専攻分野の名称 博士 (理学) 氏 名 ダグバ ダヤンソロモン Degree requested Doctor of Science Applicant 's name Dagva Dayantsolmon

> 学位論文題名 Title of Doctoral Dissertation

Mathematical Studies on Dirac Operators with a Variable Mass with Application to the Chiral Quark Soliton Model 変質量をもつディラック作用素の数学的研究とカイラルクォーク ソリトンモデルへの応用

The Hamiltonian of Chiral Quark Soliton model (CQS) in nuclear physics is described by following Dirac type operator

$$H_{\text{CQS}} := -i \sum_{j=1}^{3} \alpha_j D_j \otimes \mathbb{1}_2 + m\beta \otimes \mathbb{1}_2 e^{i \sum_{j=1}^{3} F \gamma_5 \otimes \sigma_j n_j},$$

on the Hilbert space $L^2(\mathbb{R}^3; \mathbb{C}^4) \otimes \mathbb{C}^2$.

Let's compare it to usual free Dirac operator with mass m

$$H_{\rm UD} := -i \sum_{j=1}^{3} \alpha_j D_j + m\beta,$$

on the Hilbert space $L^2(\mathbb{R}^3; \mathbb{C}^4)$.

Here *i* is the imaginary unit, $\alpha_1, \alpha_2, \alpha_3$ and $\beta = \alpha_4$ are 4×4 Dirac matrices and 1_n denotes the $n \times n$ unit matrix, D_j (j = 1, 2, 3) is the generalized partial differential operator in the space variable x_j ($\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$), m > 0 denotes the mass of a quark,

$$\gamma_5 := -i\alpha_1\alpha_2\alpha_3,$$

 $F : \mathbb{R}^3 \to \mathbb{R}$ is called a *profile function*, Borel measurable, almost everywhere (a.e.) finite $\mathbf{x} \in \mathbb{R}^3$, σ_1, σ_2 and σ_3 are the Pauli matrices and $n_j : \mathbb{R}^3 \to \mathbb{R}$ is Borel measurable functions such that

$$\sum_{j=1}^{3} n_j(x)^2 = 1$$

for a.e. $\mathbf{x} \in \mathbb{R}^3$.

The main difference in the above operators are the mass term. The mass term of H_{CQS} is spatially variable in general. Hence, the CQS model may be regarded as a model of Dirac particle with a variable mass.

The main propose of this work is to build a model, which can be an abstract d-dimensional extension of CQS model and under suitable conditions to investigate it's Hamiltonian's self-adjoint property, supersymmetric aspects and spectral properties. We will name the Hamiltonian

of this extended model by "d-dimensional Dirac operator with a variable mass". The Hamiltonian of d-dimensional chiral quark soliton model is defined as follows:

$$H := -i\sum_{j=1}^{d} \alpha_j D_j + \alpha_{d+1} e^{i\Phi} M$$

here $d \ge 2$ is natural number,

$$N_d := \begin{cases} 2^{d/2} & \text{for } d \text{ even} \\ 2^{(d+1)/2} & \text{for } d \text{ odd}, \end{cases}$$

 $\alpha_j, \quad j = 1, \dots, d+1$ is $N_d \times N_d$ Hermitian matrices satisfying

$$\{\alpha_j, \alpha_k\} = 2\delta_{jk} \mathbf{1}_{N_d}, \quad j, k = 1, \cdots, d+1.$$

Let \mathcal{K} be a finite dimensional Hilbert space. We denote by $\mathcal{F}_{s.a.}$ the self-adjoint operators on $\mathbb{C}^{N_d} \otimes \mathcal{K}$ such that the mapping $: \mathbb{R}^d \ni \mathbf{x} \to (\Phi(\mathbf{x}) + i)^{-1}$ is measurable. $\Phi(\cdot)$, $M(\cdot) \in \mathcal{F}_{s.a.}$ and respectively Φ and M be the direct integrals over \mathbb{R}^d .

The "d-dimensional Dirac operator with a variable mass" acts on

$$\mathcal{H} := L^2(\mathbb{R}^d; \mathbb{C}^{N_d}) \otimes \mathcal{K}.$$

In this work we will give a simple condition for H to be self-adjoint and discuss supersymmetric aspects and spectrum of H. Also we give a condition for H to be a supercharge of a supersymmetric quantum mechanical model. In that case, ker H, the kernel of H, describes the supersymmetric states. Hence it is interesting and important to analyze ker H. We will prove that, under some condition, ker H is trivial: ker $H = \{0\}$. In the case where H is a supercharge, this means that there is no supersymmetric states, namely, the supersymmetry is spontaneously broken. We concerned with a unitary equivalence of H to a gauge theoretic Dirac operator. This may be physically interesting. Using this structure, we find another condition for the kernel of H to be trivial. We identify the essential spectrum of H under a suitable condition. In the last, we will discuss the number of eigenvalues of H in the interval (-m, m) with m > 0 being a constant.