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Abstract

This paper explores the long-run impacts of tax policy in a two-country model of endogenous growth with variable labor supply. We focus on international spillover effects of tax reforms under alternative trade structures. It is shown that if the instantaneous utility function of the representative family in each country is additively separable and if international capital mobility is absent, then a change in taxation in one country does not directly affect capital formation in the other country. Such a conclusion is fundamentally modified if international borrowing and lending are allowed. Due to free financial flows, a change in tax policy in one country directly diffuses to the growth performance of the other country, even though preference structures are assumed to be log-additive forms.

keywords: factor-income tax, consumption tax, equilibrium dynamics, two-country model, endogenous growth, variable labor supply

JEL classification: F43, O41

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1 Introduction

It has been well recognized that globalization of an economy significantly alters the effects of tax policy. First of all, in the presence of international trade of goods and services, the tax policy aiming at resource reallocation and income redistribution in the home country may have spillover effects on the economies of trade partners. Additionally, given development of world capital market, factor income taxation in general and capital income tax in particular directly affect investment decisions of foreign firms and households. Since a change in taxation in a country may give rise to relevant international spillover effects, the study on tax policy in a global setting should pay much attention not only to the trade patterns of goods and services but also to the degree of financial integration between the countries.

The central concern of this paper is to explore the relation between trade structure and the outcomes of tax policy in a global economy. We first examine a two-country world where each country engages in free trade of commodities in the absence of capital mobility. We then introduce international borrowing and lending into the base model and consider how financial integration affects impacts of fiscal action undertaken by an individual country. In the model without international capital mobility, each country produces a country-specific good and exchanges it with the other country's product. If the world financial market exists, in addition to free trade of commodities, the households in the home country freely lend to or borrow from the foreign households. We focus on fiscal interactions between the countries under these alternative settings of international trade.

More specifically, the basic setup of our study is a two-country model of endogenous growth with variable labor supply. We use a two-country version of the endogenous growth model with production externalities first presented Benhabib and Farmer (1994).[1] Our analytical basis is simple enough to treat endogenous growth of a two-country world in a highly tractable manner. In addition, the assumption of variable labor-leisure choice provides us with a flexible framework for examining effects of various forms of taxation, which allows to captures distortionary effects of factor income taxes. We introduce factor income and consumption taxes into the baseline model and explore the effects of taxation in the world

[1] Benhabib and Farmer (1994) construct exogenous as well as endogenous growth models with external increasing returns. Our model is based on the endogenous growth version of their base model. Amano et al. (2009) explore tax incidence in the Benhabib-Farmer model to consider the relation between fiscal outcomes and equilibrium indeterminacy.

economy under alternative specifications of trade structure. In particular, we pay our attention to international spillover effects of tax policy in a two-country setting.

This paper presents two main findings. First, both domestic and international impacts of tax policy heavily depend on whether or not there exist the opportunities for international financial transactions. We show that if international borrowing and lending are absent (so that capital is immobile) and if the instantaneous utility function of the representative household in each country takes a log-additive form, then the dynamic behavior of each country is independent of the other country's fiscal policy. Put differently, a change in taxation of one country does not affect the other country's growth performance. In contrast, if international borrowing and lending are allowed but physical capital stocks are still immobile,[2] a tax reform in one country can affect the growth performance of the other country.

Our second finding is that the growth effects of taxation also depend on whether or not the balanced-growth path (BGP) of the world economy is unique. Our model of the world economy has a unique BGP if the degree of external increasing returns in each country is not strong enough. In this case, if there is neither international borrowing nor lending, taxation in one country has a negative growth effect in that country. In contrast, if the opportunities of international lending and borrowing are present, a tax rate change in one country generates complex global effects. The resulting effects on the growth performance of the world economy hinge critically upon the magnitudes of country-specific production externalities. If production external effects are sufficiently strong, then the global economy may have multiple balanced-growth equilibria. In this case, regardless of the presence of international financial market, tax policy generally yields qualitatively different effects depending on which BGP is attained. When the economy is on the BGP with a higher growth rate, the effects of tax policy are similar to those obtained in the case of unique BGP. By contrast, we obtain the opposite policy impacts, if the economy stays on the BGP with a lower growth rate. We examine the relation between the outcomes of tax reforms and the selection of a particular BGP in detail.

The issue of impacts of tax policy in global settings has attracted considerable atten-

[2] This assumption slightly departs from the standard model which allows for international borrowing and lending in dynamic economies. However, in such a model there are only transitional dynamics for the aggregate world economy but no transitional dynamics separately for each country, because international capital flows ensure that each country immediately has the same capital-labor ration. This result may not be consistent with many empirical studies including Feleddstain and Horioka (1980).

tion in the literature. Early contributions such as Ihori (1991), Frenkel and Razin (1989), Frenkel et al. (1991), Nielsen and Sørensen (1991), Bianconi and Turnovsky (1991), Ono and Shibata (1992), and Bianconi (1995) analyze two-country dynamic models with *perfect capital mobility*. Most of these studies have explored various effects of factor income taxes and government spending under the source-based principle of capital income taxation. In a similar vein, Lejour and Verbon (1998) examine a two-country growth model where capital is imperfectly mobile. All the contributions mentioned above employ *exogenous* (neoclassical) growth models where both countries produce homogenous goods. Therefore, in their analyses the opportunities of international borrowing and lending are equivalent to intertemporal trade of goods and services. Additionally, in their models reforms of fiscal policy brings about *level effects* alone in the steady state equilibrium. By contrast, since our model allows for endogenous growth, we can focus on the *growth effects* of various forms of taxation. In addition, we assume that each country produces a country-specific single homogenous good, so that our model with international borrowing and lending treats intertemporal as well as intratemporal trades between the two countries.

More recently, Iwamoto and Shibata (2008) and Palomba (2008) have explored the fiscal policy impacts in two-country *endogenous* growth models with perfect capital mobility.[3] Those studies use two-period lived overlapping generations models with AK technologies and fixed labor supply. It is also assumed that both countries produce homogeneous goods. In contrast, our model allows for intratemporal trade between countries and endogenous labor supply, which enables us to provide a more general analysis on the role of tax policy than the foregoing investigations based on endogenous growth models.[4]

From the analytical view point, our modelling strategy is closely related to Turnovsky (1997, Chapter 7), Turnovsky (1999 and 2000) and Bianconi (2003). Turnovsky (1997) and Bianconi (2003) explore impacts of fiscal policy in two-country models where each country specializes in a country-specific product under perfect capital mobility. Hence, the trade

[3] Razin and Yuen (1996) examine a two-country endogenous growth model with fertility choices and perfect mobility of physical capital between countries. Lejour and Verbon (1997) examine a two-country endogenous growth model with imperfect capital mobility. They use a representative-agent framework, but they assume that both countries produce homogeneous goods, so that commodity trade is ignored.

[4] Several authors such as Acemoglu and Ventura (2002), Bond et al. (2003) and Farmer and Lahiri (2005 and 2006) construct two-(or multi-) country, endogenous growth models that are more general than the AK growth model. Those studies, however, do not focus on fiscal policy.

structure of our model is essentially the same as these studies, but they use exogenous growth models so that growth effects of taxation are not discussed in their papers. On the other hand, Turnovsky (1999 and 2000) investigate impacts of various fiscal policies by use of small-country models with endogenous growth and variable labor supply. Our analytical framework could be viewed as a two, large-country extension of the Turnovsky (1999 and 2000).

The reminder of the paper is organized as follows. The next section constructs the base model and examines the effects of tax policy in the absence of capital mobility. Section 3 introduces the opportunities of international borrowing and lending into the base model to highlight how the policy effects are sensitive to the trade structure of the world economy. A brief concluding remark is given in Section 4.

2 The Model without International Borrowing and Lending

2.1 Model Structure

There are two countries, country 1 and country 2. In each country there is a continuum of infinitely lived identical households with a unit mass. Each country has the same form of production function and the identical preference structure. In this paper we use the simplest representation of Armington's (1969) assumption: each country produces a country-specific, single homogenous good.[5] We assume that country 1 specializes in good x and country 2 specializes in good y. Each good can be either consumed or invested for physical capital accumulation. We also assume that the imported goods can be consumed, but they cannot be used for investment.[6] In the baseline model, households in each country cannot access to the international capital market, so that they can neither borrow from nor lend to the foreign households.

Production

The production technology of each country is described by

$$z_i = Ak_i^a l_i^{1-a} \bar{k}_i^{\alpha-a} \bar{l}_i^{\beta-1+a}, \quad 0 < a < 1, \quad \alpha > a, \quad \beta > 1-a, \quad i = 1, 2, \tag{1}$$

[5] See also Lloyd and Zhang (2006) for the Armington's modelling.

[6] The present setting could be viewed as a simplified version of the two-sector model in which one sector produces country-specific tradable consumption goods and the other sector produces non-tradable investment goods.

where z_i, k_i and l_i respectively denote output, capital and labor input of country i. Here, $\bar{k}_i$ and $\bar{l}_i$ express external effects associated with the social levels of capital and labor in country i. The production function (1) means that the private technology under given levels of external effects satisfies constant returns but the social technology exhibits increasing returns with respect to the aggregate levels of capital and labor. We assume that those external effects are country-specific so that there is no international technological spillover of production. In addition, to make sustainable growth possible, we set $\alpha = 1$.[7] Due to symmetry of firms, the social production function derived by setting $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$ is

$$z_i = Ak_i l_i^{\beta}, \qquad i = 1, 2. \tag{2}$$

The commodity and factor markets in both countries are competitive. Firms maximize their instantaneous profits under given levels of external effects generated by production factors. Thus, letting r_i and w_i be the rate of return to capital and the real wage rate in country i, respectively, they are determined by $r_i = az_i/k_i$ and $w_i = (1-a)\, z_i/l_i$. Hence, the equilibrium levels of the rate of return to capital and the real wage are respectively written,

$$r_i = aAl_i^{\beta}, \qquad i = 1, 2, \tag{3}$$

$$w_i = (1-a)\, Ak_i l_i^{\beta-1}, \qquad i = 1, 2. \tag{4}$$

Households

There is a representative household in each country. The households in country i consumes domestic as well as foreign goods and supply l_i units of labor in each moment. The objective functional of the representative household in country i is a discounted sum of utilities over an infinite horizon:

$$U_i = \int_0^{\infty} u\left(x_i, y_i, l_i\right) e^{-\rho t} dt, \quad \rho > 0; \quad i = 1, 2,$$

where x_i and y_i respectively denote country i's consumption of x and y goods. By our

[7] Our formulation of production technology is first presented by Benhabib and Farmer (1994).

assumption, y_1 is imported by country 1 and x_2 is imported by country 2. The instantaneous utility is assumed to be increasing in x_i and y_i, and decreasing in labor l_i. The standard concavity assumption is imposed on $u(.)$. For simplicity, we also assume that the households in both countries have the same form of utility function and an identical time discount rate, ρ. In this paper we specify the instantaneous felicity function in the following manner:

$$u(x_i, y_i, l_i) = \theta \log x_i + (1-\theta) \log y_i - \frac{l_i^{1+\gamma}}{1+\gamma}, \quad 0 < \theta < 1, \quad \gamma > 0. \tag{5}$$

The flow budget constraint for the representative household in each country is given by

$$\dot{\omega}_i = (1-\tau_i^r) r_i \omega_i + (1-\tau_i^w) w_i l_i - (1+\tau_i^c) m_i + T_i, \quad i = 1, 2, \tag{6}$$

where ω_i is the asset holding, m_i is real consumption expenditure and T_i denotes the real transfer from the domestic government. In addition, $\tau_i^c \in [0,1)$, $\tau_i^w \in [0,1)$ and $\tau_i^r \in [0,1)$ denote the rates of consumption, capital income and wage income, respectively. For notational convenience, ω_i, w_i, T_i and m_i are expressed in terms of the good country i produces. Hence, if p denotes the price of good y in terms of good x, then the after-tax consumption spendings in both countries are respectively determined by

$$m_1 = x_1 + p y_1, \quad m_2 = \frac{x_2}{p} + y_2.$$

The household's budget should satisfy the non-Ponzi-game scheme, so that it holds that

$$\lim_{t \to \infty} \omega_i(t) \exp\left[-\int_0^t (1-\tau_i^r) r_i(s)\, ds\right] \geq 0, \quad i = 1, 2.$$

As a result, the following intertemporal budget constraint holds as well:

$$\begin{aligned} & \omega_i(0) + \int_0^\infty \exp\left[-\int_0^t (1-\tau_i^r) r_i(s)\, ds\right] [(1-\tau_i^w) w_i(t) l_i(t) + T_i(t)] dt \\ \geq \; & \int_0^\infty \exp\left[-\int_0^t (1-\tau_i^r) r_i(s)\, ds\right] (1+\tau_i^c) m_i(t)\, dt, \quad i = 1, 2, \end{aligned} \tag{7}$$

where $\omega_i(0)$ is the initial wealth holding of country i's households.

The Government

The government distributes back its total tax revenue to the domestic households. Therefore, the balanced budget constraint for the government in country i is given by

$$T_1 = \tau_1^r r_1 \omega_1 + \tau_1^w w_1 l_1 + \tau_1^c (x_1 + p y_1), \tag{8}$$

$$T_2 = \tau_2^r r_2 \omega_2 + \tau_2^w w_2 l_2 + \tau_2^c \left(\frac{x_2}{p} + y_2 \right). \tag{9}$$

Market Equilibrium Conditions

Since *physical capital stocks are not traded*, the market equilibrium conditions for goods x and y are:

$$z_1 = x_1 + x_2 + \dot{k}_1, \tag{10}$$

$$z_2 = y_1 + y_2 + \dot{k}_2. \tag{11}$$

For simplicity, we assume that physical capital in each country does not depreciate.

Since we have assumed not only that international borrowing and lending are not allowed, but also that the government of each country runs a balanced budget, the only asset households own is the capital stock in each country. Thus the asset market equilibrium condition in each country is given by

$$\omega_i = k_i, \quad i = 1, 2. \tag{12}$$

Finally, in the absence of international capital markets, the trade balance condition in the world market should hold at each moment in time:

$$p y_1 = x_2. \tag{13}$$

Perfect-Foresight Competitive Equilibrium

To sum up, the perfect-foresight competitive equilibrium (PFCE) of the world economy is defined in the following manner:

Definition: *The PFCE of the world economy composed of two countries* $(i = 1, 2)$ *holds if the following conditions are satisfied at each moment in time* $t \geq 0$*:*

(i) The firms maximize instantaneous profits under given levels of external effects, $\bar{k}_i$ *and*

$\bar{l}_i$.

(ii) Given rates of τ_i^c, τ_i^r *and* τ_i^w, *the households maximize their discounted sum of utilities under given the paths of prices,* $\{r_i(t), w_i(t), p(t)\}_{t=0}^{\infty}$.

(iii) The commodity and asset markets clear in each country and the trade balance condition (13) *holds.*

(iv) The government budget constraints (8) *and* (9) *are fulfilled.*

(v) External effects satisfy the consistency conditions such that $\bar{k}_i(t) = k_i(t)$ *and* $\bar{l}_i(t) = l_i(t)$.

2.2 Optimization Conditions

First, consider the following static problem for the representative household in country 1:

$$\max \ \theta \log x_1 + (1-\theta) \log y_1$$

subject to $x_1 + py_1 = m_1$. The resulting optimal choices of x_1 and y_1 are:

$$x_1 = \theta m_1, \quad y_1 = (1-\theta) m_1/p. \tag{14}$$

Using (14), we drive the instantaneous indirect subutility in such a way that

$$\hat{u}^1(m_1, p) = \log m_1 - (1-\theta)\log p + \theta \log \theta + (1-\theta)\log(1-\theta).$$

Similarly, we find that the static demand functions of the country 2's household are:

$$x_2 = \theta m_2 p, \quad y_2 = (1-\theta) m_2, \tag{15}$$

implying that the instantaneous indirect subutility of the representative household in country 2 is

$$\hat{u}^2(m_2, p) = \log m_2 + \theta \log p + \theta \log \theta + (1-\theta)\log(1-\theta).$$

To derive the optimization conditions for the household in each country, we set up the

Hamiltonian function in which m_i is the control variable:

$$H_i = \hat{u}^i (m_i, p) - \frac{l_i^{1+\gamma}}{1+\gamma} + q_i \left[(1-\tau_i^r) r_i k_i + (1-\tau_i^w) w_i l_i - (1+\tau_i^c) m_i + T_i \right], \quad i = 1, 2,$$

where q_i represents the shadow value of net asset evaluated in terms of utility. The necessary conditions for an optimum include the following:

$$1/m_i = q_i (1+\tau_i^c), \quad i = 1, 2, \tag{16}$$

$$l_i^\gamma = q_i (1-\tau_i^w) w_i, \quad i = 1, 2, \tag{17}$$

$$\dot{q}_i = q_i \left[\rho - (1-\tau_i^r) r_i \right], \quad i = 1, 2, \tag{18}$$

$$\lim_{t\to\infty} q_i e^{-\rho t} k_i = 0, \quad i = 1, 2. \tag{19}$$

In the above, (19) is the transversality condition.

Equations (4) and (17) give

$$l_i = \left[(1-\tau_i^w)(1-a) A k_i q_i \right]^{\frac{1}{\gamma+1-\beta}}, \quad i = 1, 2. \tag{20}$$

The above relations show that the labor supply in country i is positively (negatively) related to the *implicit (utility) value of capital*, $k_i q_i$, if $\gamma + 1 > \beta$ ($\gamma + 1 < \beta$). Substituting (17) for l_i into (2) yields

$$z_i = A k_i \left[(1-\tau_i^w)(1-a) A v_i \right]^{\frac{\beta}{\gamma+1-\beta}}, \quad i = 1, 2, \tag{21}$$

and the rate of return to capital is

$$r_i = aA \left[(1-\tau_i^w)(1-a) A v_i \right]^{\frac{\beta}{\gamma+1-\beta}} \equiv r_i (v_i), \quad i = 1, 2, \tag{22}$$

where we set $k_i q_i \equiv v_i$ which represents the utility value of capital in each country.

2.3 Dynamic System

From (13), (14), (15) and (16), the equilibrium price of good y in terms of good x is written as

$$p = \frac{(1-\theta)(1+\tau_2^c)\, q_2}{\theta\,(1+\tau_1^c)\, q_1}. \tag{23}$$

Namely, the relative price of good y in terms of good x is proportional to the relative shadow value of capital, q_2/q_1.

Using (10), (11), (14), (15), (16), (21) and (23), we can write the capital accumulation equations of countries 1 and 2, respectively:

$$\frac{\dot{k}_i}{k_i} = A^{\frac{\gamma+1}{\gamma+1-\beta}}\left[(1-\tau_i^w)(1-a)\, v_i\right]^{\frac{\beta}{\gamma+1-\beta}} - \frac{1}{v_i}\frac{1}{1+\tau_i^c}, \qquad i = 1, 2. \tag{24}$$

Combining (18), (22) and (24) with $(\dot{v}_i/v_i) = (\dot{k}_i/k_i) + (\dot{q}_i/q_i)$, we obtain the differential equations which constitute a complete dynamic system with respect to v_1 and v_2:

$$(\dot{v}_i/v_i) = [1-(1-\tau_i^r)\, a]A^{\frac{\gamma+1}{\gamma+1-\beta}}\left[(1-\tau_i^w)(1-a)\, v_i\right]^{\frac{\beta}{\gamma+1-\beta}} + \rho - \frac{1}{v_i}\frac{1}{1+\tau_i^c}, \quad i = 1, 2. \tag{25}$$

2.4 Balanced-Growth Equilibrium

The balanced growth path (BGP) of the world economy is established when v_1 and v_2 stay constant over time, that is, $\dot{v}_i = 0$, $i = 1, 2$ in (25). Due to the assumption of log-additive utility functions, the dynamic behaviors of v_1 and v_2 are independent of each other [see (25)] and the steady-state (or BGP) condition in country i is given by

$$[1-(1-\tau_i^r)\, a](Av_i)^{\frac{\gamma+1}{\gamma+1-\beta}}\left[(1-\tau_i^w)(1-a)\right]^{\frac{\beta}{\gamma+1-\beta}} = \frac{1}{1+\tau_i^c} - \rho v_i, \quad i = 1, 2. \tag{26}$$

It is easy to see that the steady-state value of v_i $(i = 1, 2)$ is uniquely given if $\gamma + 1 > \beta$. We also find that if $\gamma + 1 < \beta$, then either there is no steady state or there are dual steady states. Without loss of generality, we assume throughout the paper that both countries have dual BGPs when $\gamma + 1 < \beta$. To sum up, we may state:

Proposition 1 *Suppose that the instantaneous utility function of the representative family in each country is given by* (**??**) *which is log-additively separable between each commodity and*

labor. If $1+\gamma > \beta$, the world economy has a unique BGP that satisfies global determinacy. If $1+\gamma < \beta$, then there may exist four BGPs: one in which both countries grow at a lower rate is locally determinate, while the other three are locally indeterminate.

Figure 1 depicts the phase diagram of (25) for the case of $1+\gamma > \beta$ (so that the world economy is globally determinate). If this is the case, the world economy always stays on the BGP and so it has no transitional dynamics. In contrast, if $1+\gamma < \beta$, then the world economy involves *four* steady states. As Figure 2 shows, the steady state where both countries attain higher growth rates (lower values of v_1 and v_2) is a sink. The steady state where both countries attain lower growth rates (higher values of v_1 and v_2) is totally unstable and, hence, it exhibits local determinacy. The other two steady states are saddlepoints. The last two steady states exhibit local indeterminacy, because the initial values of v_i $(= q_i k_i)$, $i = 1, 2$, are not specified in the perfect-foresight competitive equilibrium. Note that in the case of saddlepoint, the stable saddle path restricts the relation between v_1 and v_2 to the one-dimensional stable manifold, while the levels of v_1 and v_2 are completely indeterminate around the first high-growth steady state because it is a sink.

To explain how indeterminacy emerges on the BGP, following Bennett and Farmer (2000), we focus on the labor market equilibrium condition. Substituting (4) for w_i into (17) yields

$$l_i^{\gamma}/q_i = (1-\tau_i^w)(1-a)\,Ak_i l_i^{\beta-1}. \tag{27}$$

Unlike Bennett and Farmer (2000), in our two-country model, we take the utility value of capital $q_i k_i$ rather than the marginal utility of consumption expenditure, $1/m_i$, as given since the capital value $v_i = q_i k_i$ is constant along the BGP. After substitution of $(1-a)Ak_i l_i^{\beta-1} = w_i$, we take the logarithm of (27) to get

$$\log w_i = \gamma \log l_i - \log q_i - \log(1-\tau_i^w), \tag{28}$$

which can be viewed as the Frisch labor supply curve in country i with respect to a given price of capital q_i. Notice that the elasticity of this labor supply curve in country i, evaluated on the BGP, is given by

$$\left(\frac{d\log w_i}{d\log l_i}\right)^S = \gamma > 0. \tag{29}$$

Thus the Frisch labor supply curve has a positive slope.

On the other hand, equation (4) delivers the labor demand curve such as

$$\log w_i = \log (1-a) A + \log k_i + (\beta - 1) \log l_i, \tag{30}$$

Under a given level of k_i, the slope of the labor demand curve evaluated on the BGP is

$$\left(\frac{d \log w_i}{d \log l_i} \right)^D = \beta - 1. \tag{31}$$

Accordingly, the difference between (29) and (31) is:

$$\left(\frac{d \log w_i}{d \log l_i} \right)^S - \left(\frac{d \log w_i}{d \log l_i} \right)^D = 1 + \gamma - \beta.$$

This implies that if $\gamma + 1 - \beta > 0$, the labor demand curve is less steeper than the Frisch labor supply curve; see Figure 3. In this case the labor supply and demand curves cross with 'normal' slopes. The opposite holds if $\gamma + 1 - \beta < 0$; see Figure 4 where the labor demand curve has a positive slope and it is steeper than the Frisch labor supply curve (i.e., the labor supply and demand curves cross with 'wrong' slopes). Thus Proposition 1 states that indeterminacy of equilibria emerges in the latter case.[8]

Logarithmic-time differentiation of (23), coupled with (18) and (22), delivers the dynamic evolving equation for the terms of trade p:

$$\begin{aligned} \frac{\dot{p}}{p} &= (1-\tau_1^r) r_1 - (1-\tau_2^r) r_2 \\ &= (1-\tau_1^r) aA[(1-\tau_1^w)(1-a) Av_1]^{\frac{\beta}{1+\gamma-\beta}} - (1-\tau_2^r) aA[(1-\tau_2^w)(1-a) Av_2]^{\frac{\beta}{1+\gamma-\beta}} \end{aligned} \tag{32}$$

Since $\dot{k}_i / k_i = -\dot{q}_i / q_i$ holds on the BGP (recall $\dot{v}_i = 0$), the steady-state change in the term of trade can be expressed by the gap between the balanced growth rates of the two countries:

$$\frac{\dot{p}}{p} = \frac{\dot{q}_2}{q_2} - \frac{\dot{q}_1}{q_1} = g_1^k - g_2^k, \tag{33}$$

[8] It has been intensively discussed that equilibrium indeterminacy of the Benhabib-Farmer model in small-open economy settings: see, for example, Weder (2000) and Meng and Velasco (2004). Our implication of indeterminacy conditions is similar to that obtained in their small-country models.

where g_i^k denotes the growth rate of capital of country i along the BGP.

2.5 Growth Effects of Taxation

We are particularly concerned with the growth effects of taxation along the BGP. To do this, using (17) and (22) we rewrite the BGP condition, $g_i^k(\equiv \dot{k}_i/k_i) = -\dot{q}_i/q_i = (1-\tau_i^r)\, r_i - \rho$, as follows:

$$g_i^k = (1-\tau_i^r)\, aA\, [(1-\tau_i^w)\,(1-a)\, Av_i]^{\frac{\beta}{1+\gamma-\beta}} - \rho, \quad i = 1, 2. \tag{34}$$

Solving (34) for v_i gives us

$$v_i = \frac{1}{A\,(1-a)\,(1-\tau_i^w)} \left[\frac{g_i^k + \rho}{(1-\tau_i^r)\, aA}\right]^{\frac{1+\gamma-\beta}{\beta}}, \quad i = 1, 2. \tag{35}$$

We see that the implicit value of capital, v_i, in the steady state is positively (negatively) related to the growth rate of capital, g_i^k, if $\gamma + 1 > \beta$ $(\gamma + 1 < \beta)$. Substituting (35) for v_i into (26) and rearranging terms, we obtain the following:

$$[1-(1-\tau_i^r)\, a]\, \frac{g_i^k + \rho}{(1-\tau_i^r)\, aA} = \frac{(1-\tau_i^w)\,(1-a)}{1+\tau_i^c} \left[\frac{g_i^k + \rho}{(1-\tau_i^r)\, aA}\right]^{\frac{\beta-(1+\gamma)}{\beta}} - \frac{\rho}{A}, \quad i = 1, 2. \tag{36}$$

Figures 5-8 show the graphs of the left-hand (henceforth LHS) and right-hand sides (henceforth RHS) of (36). As shown in Figures 5 and 6, if $1+\gamma > \beta$, then the graphs have a unique intersection. It is easy to see that a rise in every tax rate entails a downward shift of the locus of the RHS and thus the balanced-growth rate of capital, g_i^k, will decline. Figures 7 and 8 depict the case of $\gamma + 1 < \beta$. In this case, the growth effect of a change in fiscal action undertaken by an individual country depends on which BGP the economy stays. Since a rise in τ_i^w or τ_i^c causes a downward shift of the locus of the RHS in Figure 7, it reduces the balanced-growth rate if the economy is on the high-growth BGP. In contrast, if the economy stays on the low-growth BGP, then a rise in every tax rate increases the balanced-growth rate.

To sum up, we have shown:

Proposition 2 *In the case of log-additively separable utility, if $1+\gamma > \beta$, then we obtain: $dg_i^k/d\tau_i^c < 0$, $dg_i^k/d\tau_i^w < 0$, and $dg_i^k/d\tau_i^r < 0$. If $1+\gamma < \beta$, then it holds that $dg_i^k/d\tau_i^c < 0$,*

$dg_i^k/d\tau_i^w < 0$ and $dg_i^k/d\tau_i^r \gtrless 0$ if the economy stays on the BGP with a higher growth rate, while $dg_i^k/d\tau_i^c > 0$, $dg_i^k/d\tau_i^w > 0$ and $dg_i^k/d\tau_i^r \gtrless 0$ on the BGP with a lower growth rate.

The economic intuition for Proposition 2 is as follows.

(i) The case of $1+\gamma > \beta$

Consider first the case where $1+\gamma > \beta$. A rise in the after-tax price of consumption goods in the home country caused by an increase in τ_1^c makes the consumption goods, $x_1 + py_1$, relatively more expensive than leisure.[9] This induces the households to reduce the demand for consumption goods, which makes the locus of the RHS of (36) shift downwards as illustrated in Figure 5. As a result, the value of capital, v_1, has to fall immediately not only because the steady state is determinate (unstable), but also because the initial price of capital, q_1, can freely jump to its new steady state level (recall that the stock of capital is instantaneously fixed). It turns out from (20) that this instantaneous reduction of v_1 depresses labor supply and thus the level of employment since the Frisch labor supply curve is steeper than the labor demand curve (i.e., the normal slope's case); see Figure 3. It follows from (24) that the growth rate of capital, g_1^k, also falls as a result of a lower output level of the home country caused by the decreased employment.

If the government of the home country raises the tax rate on wage income, τ_1^w, then the after-tax real wage in the home country decreases. Since this reduction lowers the opportunity costs of leisure, the resulting substitution effect raises leisure and thus depresses labor supply. Since under the normal slope's case the equilibrium level of employment falls, so does the capital growth rate of the home country due to (24). This situation is also depicted by a downward shift of the locus of the RHS of (36) in Figure 5. Although the increase in τ_1^w also reduces the rate of change in the terms of trade, p, in (32), the resulting intertemporal substitution effect does not affect real decisions of the households because of the assumed log-additive utility function.

If the tax rate on capital income in the home country, τ_1^r, rises, then the after-tax rate of return to capital decreases in the home country. Accordingly, the reduction in the after-tax return to capital discourages the incentive of savings of the households, thereby reducing the growth rate of capital. This causes a counter-clockwise rotation of the locus of the LHS as

[9] In what follows, we call country 1 (country 2) the home country (the foreign country).

well as a downward shift of the locus of the RHS of (36). Hence, the long-run growth rate in the home country rises, as illustrated in Figure 6.

We should notice that, under the assumption that the instantaneous utility function of the representative household is log-additively separable between consumption and labor supply, an increase in τ_i^c, τ_i^wor τ_i^r only affects the own country's capital value v_i and l_i. This arises because their impacts on the other country's growth rate are completely offset by the changes in the relative price p in (33). Such dynamic interdependence between two countries has been also found in Turnovsky (1997) using the two-country, two-good model of *exogenous* growth with additive separable preferences in different consumption goods.

(ii) The case of $1 + \gamma < \beta$

As before, the increase in τ_1^c (or τ_1^w) leads to a downward shift of the locus of the RHS in (36), as illustrated in Figure 7. This changes the growth rate, g_1^k, in both BGPs. In the low-growth BGP, g_1^k rises as a result of the decrease in v_1, while g_1^k falls due to the increase in v_1 on the high-growth BGP. The reason is given as follows. Since whether labor supply increases or not depends on whether the capital value v_1 (or $(1 - \tau_1^w)v_1$) rises or not. As a result, since the Frisch labor supply and labor demand curves cross with 'wrong' slopes under $1 + \gamma < \beta$, the leftward (rightward) shift of the Frisch labor supply curve raises (lowers) the equilibrium level of employment l_1; see Figure 4, which in turn leads to a decline (rise) in the long-run growth rate.

On the other hand, we see that the sign of $dg_i^k/d\tau_i^k$ is ambiguous on both BGPs. Since an increase in τ_1^r reduces the after-tax rate of return to capital in the home country thereby discouraging the incentive of saving. This impact makes the locus of the LHS of (36) rotate counter-clock wise and shift the locus of the RHS downward for the same reason outlined as in the case of $1 + \gamma > \beta$. Accordingly, under the case of $1 + \gamma < \beta$, whether g_1^k increases or decreases depends on the relative size of the movements of both loci, as illustrated in Figure 8. The reason for ambiguity is that the impacts of changes in the capital value v_1 on employment levels are opposed to those of the demand for consumption goods, $x_1 + py_1$, which may also counteract the disincentive effect on saving.

In view of Propositions 1 and 2, we have found the following facts. First, since a change in any tax in one country does not affect the other country's growth performance, the divergence in growth rates between the two countries is completely absorbed by a change in $\dot{p}/p$ through

(33). As stated before, this is so because the resulting intertemporal substitution effect caused by changes in the terms of trade, p, is completely canceled out by its income effect because of the log-additive utility function.

Second, it is worth emphasizing that the presence of indeterminacy does not alter the policy impacts in our model. As shown in Propositions 1 and 2, we obtain a negative relation between taxation and long-term growth on the high-growth BGP that exhibits local indeterminacy. The unconventional policy effect, i.e. a rise in the tax increases the long-run growth rate, is established on the low-growth BGP that also exhibits local determinacy. This result (except for the effects of τ_i^k, $i = 1, 2$) stems from the fact that the local indeterminacy emerges only when there exist dual BGPs (i.e., production externalities are sufficiently large). This prevents us from obtaining a one-to-one correspondence between the comparative statics properties of tax changes and the stability conditions in the BGP.

3 The Model with International Borrowing and Lending

Thus far we have assumed that the households in each country neither borrow from nor lend to the foreign households. In this section we introduce the *opportunities of international borrowing and lending* into the base model examined in the previous section. We will show that this modification leads to substantial differences in the effects of taxation.[10]

3.1 Model Structure

The basic model structure in the presence of international borrowing and lending is the same as the previous model without financial integration (i.e., no capital mobility). Denote b_i as the stock of traded bonds (international IOUs) held by the households in country i. Here, b_1 is evaluated in terms of good x and b_2 is evaluated by good y. The constraints facing the households in each country are now given by the following:

$$\dot{b}_i = (1 - \tau_i^r)(R_i b_i + r_i k_i) + (1 - \tau_i^w) w_i l_i - (1 + \tau_i^c) m_i - I_i + T_i, \quad i = 1, 2, \tag{37}$$

[10] Hu and Mino (2009) study the effect of international financial integration in a two-county, *exogenous* growth model with production externalities. Hu and Mino (2013) investigate the same issue in the context of two-country Heckscher-Ohlin model with social constant returns. Our analytical framework used below could be viewed as a endogenous-growth counterpart of Hu and Mino (2009).

$$\dot{k}_i = I_i, \quad i = 1, 2,$$

where R_i is the real interest rate on b_i and I_i is investment on physical capital. The representative household in country i maximizes U_i subject to the above constraints and the initial holdings of physical capital, $k_i(0)$, and financial asset, $b_i(0)$.

The Hamiltonian function of the optimization problem for the households in country i is set as

$$H_i = \hat{u}^i(m_i, p) - \frac{l_i^{1+\gamma}}{1+\gamma} + \lambda_i \left[(1-\tau_i^r)(R_i b_i + r_i k_i) + (1-\tau_i^w) w_i l_i - (1+\tau_i^c) m_i - I_i + T_i\right] + q_i I_i,$$
$$i = 1, 2.$$

The necessary conditions for an optimum include the following:

$$1/m_i = q_i(1+\tau_i^c) = \lambda_i(1+\tau_i^c), \quad i = 1, 2, \tag{38}$$

$$l_i^\gamma = q_i(1-\tau_i^w) w_i, \quad i = 1, 2, \tag{39}$$

$$\dot{\lambda}_i = \lambda_i \left[\rho - (1-\tau_i^r) R_i\right], \quad i = 1, 2, \tag{40}$$

$$\dot{q}_i = q_i \left[\rho - (1-\tau_i^r) r_i\right], \quad i = 1, 2, \tag{41}$$

together with the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} \lambda_i b_i = 0, \quad \lim_{t\to\infty} e^{-\rho t} q_i k_i = 0, \quad i = 1, 2,$$

and the non-Ponzi-game scheme:

$$\lim_{t\to\infty} \exp\left[-\int_0^t R_i(s)\, ds\right] b_i(t) \geq 0, \quad i = 1, 2.$$

From (38), (40) and (41), it holds that

$$R_i = r_i, \quad i = 1, 2. \tag{42}$$

The financial integration means that the real rate or return to holding bonds is the same in

both countries; consequently, it always holds that[11]

$$R_1 = R_2 + \frac{\dot{p}}{p}, \tag{43}$$

which, coupled with (42), leads to

$$\frac{\dot{p}}{p} = r_1 - r_2. \tag{44}$$

Namely, the terms of trade between goods x and y varies according to the discrepancy between the real rates of return to capital in both countries.

The equilibrium condition in the bond (or international financial) market is

$$b_1 + pb_2 = 0. \tag{45}$$

Note that the homogeneity of private technologies gives $z_i = r_i k_i + w_i l_i$ $(i = 1, 2)$. Hence, from the flow budget constraints for the households and the government in each country; i.e., (8), (9) and (37) the dynamic equations of b_1 and b_2 are:

$$\dot{b}_1 = r_1 b_1 + x_2 - py_1, \tag{46}$$

$$\dot{b}_2 = \left(r_1 - \frac{\dot{p}}{p}\right) b_2 + y_1 - \frac{x_2}{p}. \tag{47}$$

Equations (46) and (47) respectively describe the current accounts of country 1 and 2.[12]

3.2 Dynamic System

Conditions (22) and (44) yield:

$$\frac{\dot{p}}{p} = aA\left[\left(1-\tau_1^w\right)(1-a)\,Av_1\right]^{\frac{\beta}{\gamma+1-\beta}} - aA\left[\left(1-\tau_2^w\right)(1-a)\,Av_2\right]^{\frac{\beta}{\gamma+1-\beta}}. \tag{48}$$

[11] This formulation implies that we have eventually assumed that the tax on the return on bonds is subject to the residence-based tax principle. The reason for this is that b_i may have a negative when households in country i owe debt. If this is the case, it is impossible to levy the return on bonds. To eliminate this situation, we have assumed the residence-based tax principle.

[12] Using conditions (46)and (47), we obtain $\dot{b}_1 + p\dot{b}_2 = r_1(b_1 + pb_2) - \dot{p}b_2$. Thus (45) gives $\dot{b}_1 + p\dot{b}_2 + \dot{p}b_2 = 0$, which is consistent with the bond-market equilibrium condition given by (45).

The capital stock in each country evolves in the following manner:

$$\frac{\dot{k}_1}{k_1} = A^{\frac{\gamma+1}{\gamma+1-\beta}} \left[\left(1-\tau_1^w\right)(1-a)\, v_1 \right]^{\frac{\beta}{\gamma+1-\beta}} - \frac{\theta\,(1/v_1)}{1+\tau_1^c} - \frac{\theta\,(1/v_1)\,(pq_1/q_2)}{1+\tau_2^c},$$

$$\frac{\dot{k}_2}{k_2} = A^{\frac{\gamma+1}{\gamma+1-\beta}} \left[\left(1-\tau_2^w\right)(1-a)\, v_2 \right]^{\frac{\beta}{\gamma+1-\beta}} - \frac{(1-\theta)(1/v_2)}{1+\tau_2^c} - \frac{(1-\theta)\,(1/v_2)\,(q_2/pq_1)}{1+\tau_1^c}.$$

From (22), (40) and (42) the implicit price of capital in each country follows:

$$\dot{q}_i/q_i = \rho - \left(1-\tau_i^r\right) aA \left[\left(1-\tau_i^w\right)(1-a)\, Av_i \right]^{\frac{\beta}{\gamma+1-\beta}}, \qquad i = 1, 2.$$

Using (22), the dynamic equations of p, k_i and q_i derived above can be summarized as the following dynamic system with respect to v_1, v_2 and h $(\equiv pq_1/q_2)$:

$$\dot{v}_1 = v_1 \left[\rho + \left(\frac{1}{a} - \left(1-\tau_1^r\right) \right) r_1\left(v_1\right) \right] - \frac{\theta}{1+\tau_1^c} - \frac{\theta h}{1+\tau_2^c}, \tag{49}$$

$$\dot{v}_2 = v_2 \left[\rho + \left(\frac{1}{a} - \left(1-\tau_2^r\right) \right) r_2\left(v_2\right) \right] - \frac{1-\theta}{1+\tau_2^c} - \frac{(1-\theta)(1/h)}{1+\tau_1^c}, \tag{50}$$

$$\frac{\dot{h}}{h} = \frac{\dot{p}}{p} + \frac{\dot{q}_1}{q_1} - \frac{\dot{q}_2}{q_2} = \tau_1^r r_1\left(v_1\right) - \tau_2^r r_2\left(v_2\right). \tag{51}$$

In our model, the initial capital stock holdings of both countries, $k_1(0)$ and $k_2(0)$, are historically given. However, in the absence of financial frictions in the world bond market, the initial asset positions, $b_1(0)$ and $b_2(0)$, may be adjusted instantaneously, so that the initial level of relative price that satisfies the equilibrium condition of the bond market, $b_1(0) + p(0)\, b_2(0) = 0$, is not predetermined. Therefore, all the state variables, $v_1(t)$, $v_2(t)$ and $h(t)$, are forward-looking variables, implying that the local determinacy of the equilibrium path near the balanced-growth equilibrium requires that the linearly approximated system of (49), (50) and (51) have three unstable roots.

3.3 Balanced-Growth Equilibrium

Along the BGP v_i and h stay constant over time. When $\dot{v}_1 = \dot{v}_2 = \dot{h} = 0$ in (49), (50) and (51), the following conditions hold:

$$\rho + \left[\frac{1}{a} - (1 - \tau_1^r)\right] r_1(v_1) - \frac{\theta/v_1}{1+\tau_1^c} - \frac{\theta h/v_1}{1+\tau_2^c} = 0, \tag{52}$$

$$\rho + \left[\frac{1}{a} - (1 - \tau_2^r)\right] r_2(v_2) - \frac{(1-\theta)/v_2}{1+\tau_2^c} - \frac{(1-\theta)(1/v_2 h)}{1+\tau_1^c} = 0, \tag{53}$$

$$\tau_1^r r_1(v_1) = \tau_2^r r_2(v_2). \tag{54}$$

Using the expression $r_i(v_i) \equiv aA\left[(1-\tau_i^w)(1-a)Av_i\right]^{\frac{\beta}{\gamma+1-\beta}}$, $i = 1, 2$, we can rewrite (54) as

$$\tau_1^r\left[(1-\tau_1^w)(1-a)Av_1\right]^{\frac{\beta}{\gamma+1-\beta}} = \tau_2^r\left[(1-\tau_2^w)(1-a)Av_2\right]^{\frac{\beta}{\gamma+1-\beta}}.$$

This equation amounts to the following relation between v_1 and v_2:

$$\Phi v_1 = v_2, \tag{55}$$

where

$$\Phi \equiv \left(\frac{\tau_1^r}{\tau_2^r}\right)^{\frac{1+\gamma-\beta}{\beta}} \frac{1-\tau_1^w}{1-\tau_2^w}.$$

Equation (55) shows that in the steady state the value of capital (in terms of utility) of country 2 relative to that of country 1 depends on the relative magnitudes of factor income tax rates in both countries. Such a direct link between the values of capitals held in both countries stems from free mobility of financial assets between the two countries. Moreover, if $\tau_1^w = \tau_2^w$, then it follows from (55) that

$$\begin{aligned} \text{sign } (v_1 - v_2) &= \text{sign } (\tau_2^r - \tau_1^r) \quad \text{if } 1+\gamma > \beta, \\ \text{sign } (v_1 - v_2) &= \text{sign } (\tau_1^r - \tau_2^r) \quad \text{if } 1+\gamma < \beta. \end{aligned}$$

That is, if the tax rates on wage income in both countries are the same and if labor externalities are small enough to satisfy $1 + \gamma > \beta$, then a higher capital income tax rate in

country 1 leads to a lower relative value of capital in country 1. This intuitively plausible result, however, fails to hold, if labor externalities are sufficiently large to satisfy $\beta > 1 + \gamma$. Similarly, if $\tau_1^r = \tau_2^r$, then

$$\text{sign } (v_1 - v_2) = \text{sign } (\tau_1^w - \tau_2^w).$$

Notably, it follows from (44) and (54) that the relative price on the BGP varies in the following manner:

$$\frac{\dot{p}}{p} = (1 - \tau_1^r) r_1 (v_1) - (1 - \tau_2^r) r_2 (v_2) = g_1^k - g_2^k, \tag{56}$$

where the last equality follows from $g_i^k = (1 - \tau_i^r) r_i (v_i) - \rho$. Since this condition coincides with (33), the growth rate and price change differentials between the home and foreign countries on the BGP are characterized in the same manner as in the previous model without financial integration. However, the presence of international borrowing and lending opportunities gives rise to the key difference in policy effects between these two models*: growth performance of each country depends not only on her own tax policy but also on the tax policy of the foreign country,* which will be shown later.

To examine the existence of the balanced-growth equilibrium, it is useful to rewrite (52) as

$$h = \frac{1 + \tau_2^c}{\theta} \left[\rho v_1 + \left(\frac{1}{a} - (1 - \tau_1^r) \right) r_1 (v_1) v_1 - \frac{\theta}{1 + \tau_1^c} \right].$$

Similarly, (53) is rewritten as

$$h = \frac{1 - \theta}{1 + \tau_1^c} \left[\rho v_2 + \left(\frac{1}{a} - (1 - \tau_2^r) \right) r_2 (v_2) v_2 - \frac{1 - \theta}{1 + \tau_2^c} \right]^{-1}.$$

Using (22) and (55), the above equations can be respectively expressed in the following manner:

$$\begin{aligned} h &= \frac{1 + \tau_2^c}{\theta} \left[\left(\frac{1}{a} - 1 + \tau_1^r \right) a \left[(1 - \tau_1^w)(1 - a) \right]^{\frac{\beta}{1+\gamma-\beta}} (A v_1)^{\frac{1+\gamma}{1+\gamma-\beta}} + \rho v_1 - \frac{\theta}{1 + \tau_1^c} \right] \\ &\equiv F(v_1), \end{aligned} \tag{57}$$

$$
\begin{aligned}
h &= \frac{1-\theta}{1+\tau_1^c}\left[\left(\frac{1}{a}-1+\tau_2^r\right)a\left[\left(1-\tau_2^w\right)(1-a)\right]^{\frac{\beta}{1+\gamma-\beta}}\left(A\Phi v_1\right)^{\frac{1+\gamma}{1+\gamma-\beta}}+\rho\Phi v_1-\frac{1-\theta}{1+\tau_2^c}\right]^{-1} \\
&\equiv G\left(v_1\right), \qquad (58)
\end{aligned}
$$

Equations (57) and (58) jointly determine the steady-state values of v_1 ($\equiv q_1 k_1$) and h ($\equiv pq_1/q_2$). The steady-state level of v_2 ($\equiv q_2 k_2$) is then given by (55) as a residual.

It is easy to confirm that if $1+\gamma>\beta$, then $F(v_1)$ monotonically increases with v_1, while $G(v_1)$ monotonically decreases with v_1. Thus there is a unique set of steady-state levels of v_1, v_2 and h; see Figures 9 and 10. If $1+\gamma<\beta$, then we find:

$$
\lim_{v_1\to 0} F(v_1)=+\infty, \quad \lim_{v_1\to+\infty} F_1(v_1)=+\infty, \quad \lim_{v_1\to 0} G(v_1)=0, \quad \lim_{v_1\to+\infty} G(v_1)=0.
$$

Taken together, it is also seen that $F(v_1)$ is a U-shaped function, while $G(v_1)$ is an inverse U-shaped function; consequently, $F(v_1)$ has a unique minimum and $G(v_1)$ has a unique maximum. Therefore, if it exists, there are dual BGPs; see Figure 11.

As for the local determinacy of the balanced-growth equilibrium, we need to check the local behavior of (49), (50) and (51) around the steady state. The Jacobian of the linearized dynamic system is given by

$$
J=\begin{bmatrix}
\rho+\left[\frac{1}{a}-\left(1-\tau_1^r\right)\right]\left[r_1'(v_1)v_1+r_1(v_1)\right] & 0 & -\frac{\theta}{1+\tau_2^c} \\
0 & \rho+\left[\frac{1}{a}-\left(1-\tau_2^r\right)\right]\left[r_2'(v_2)v_2+r(v_2)\right] & \frac{1-\theta}{(1+\tau_1^c)h^2} \\
h\tau_1^r r_1'(v_1) & -h\tau_2^r r_2'(v_2) & 0
\end{bmatrix}. \qquad (59)
$$

In the above v_1, v_2 and h denote their steady-state values. Appendix A demonstrates that if $1+\gamma>\beta$, all of the characteristic roots of J have positive real parts, so that the unique BGP of the world economy holds local determinacy. In the case of $1+\gamma<\beta$, the Jacobian J has at least one stable root and, hence, local indeterminacy always holds on both of the BGPs. The following proposition summarizes the characterization of the BGP of the world economy:

Proposition 3 *Suppose that international borrowing and lending are allowed. Then if* $1+\gamma>\beta$, *the world economy has a unique BGP that exhibits local determinacy. If* $1+\gamma<\beta$,

then the world economy may have dual BGPs, both of which are locally indeterminate.[13]

3.4 Global Impacts of Taxation

We now investigate impacts of tax policy in the world economy with financial integration. To examine the growth effects of taxation, it is helpful to use (57) and (58) . The new steady-state level of v_1 is determined by condition $F(v_1) = G(v_1)$ in response to each tax rate change. Once the steady-state value of v_1 is given, the corresponding level of v_2 is determined by (55) . Thus we may inspect impacts of taxation by observing how changes in tax rates shift the graphs of $F(v_1)$ and $G(v_1)$.

(i) *The case of* $1+\gamma > \beta$

Figures 9 and 10 depict the graphs of $h = F(v_1)$ and $h = G(v_1)$ under $1+\gamma > \beta$. As the figures show, in this case there is a unique balanced-growth equilibrium for the feasible region of $h > 0$.

First, suppose that country 1 (the home country) raises the consumption tax rate, τ_1^c. Unlike the previous case without capital mobility, this change directly diffuses to the steady-state condition for country 2 (the foreign country), because (58) involves τ_1^c. It is easy to see that an increase in τ_1^c shifts the graphs of $h = F(v_1)$ and $h = G(v_1)$ upward and downward in Figure 9, respectively. As illustrated in Figure 9, the steady-state level of v_1 unambiguously falls, although the impact on h is ambiguous. Nevertheless, we should note that using (55) the balanced-growth rates of capital in both countries (34) can be respectively rewritten as

$$g_1^k = (1-\tau_1^r)\, aA\left[(1-\tau_1^w)(1-a)Av_1\right]^{\frac{\beta}{1+\gamma-\beta}} - \rho, \tag{60}$$

$$g_2^k = (1-\tau_2^r)\, aA\left[(1-\tau_2^w)(1-a)A\Phi v_1\right]^{\frac{\beta}{1+\gamma-\beta}} - \rho. \tag{61}$$

Since a higher τ_1^c lowers the steady-state value of v_1 under $1+\gamma > \beta$, the relations between g_i^k $(i = 1, 2)$ and v_1 given above reveal that a higher τ_1^c depresses the balanced-growth rates of both countries. In the similar manner, a rise in τ_2^c also lowers the growth rates of both countries on the BGP of the world economy through the reduction in v_2.

Intuitively, the impact of a rise in τ_1^c on the home country is basically the same as in

[13] Remember that in the world economy without financial integration, there may exist four BGPs. The BGP of the world economy with international borrowing and lending involves two BGPs at most.

the previous model without capital mobility; that is, a higher τ_1^c reduces an equilibrium level of employment and thus the long run growth rate of the home country. Moreover, this tax increase also lowers the long-run growth rate of the foreign country. This cross-country effect stems from the existence of condition (55) on the BGP. The values of capital stocks held in both countries, v_1 and v_2, are directly and positively linked by this condition, unlike the previous model without financial integration.

Next, consider the effects of a change in the wage income tax. As before, a higher τ_1^w reduces the level of employment and thus the output of country 1, which leads to a fall in the growth rate of capital of country 1 under $1+\gamma > \beta$. This makes the graph of $h = F(v_1)$ given by (57) shift downward, as illustrated in Figure 10. At the same time, a higher τ_1^w also distorts (55), which makes the graph of $h = G(v_1)$ given by (57) shift upward. As Figure 10 demonstrates, therefore, a rise in τ_1^w ends up increasing v_1 and v_2. However, it turns out from (60) and (61) that the growth rates of capital of both countries fall, because the direct negative effect of the increase in τ_1^w on the growth rate overweighs the indirect effect of increasing v_1 and v_2.

Finally, assume that the home country raises the capital income tax rate, τ_1^r. Since the increase in τ_1^r makes the after-tax rate of return to capital in the home country, $(1-\tau_1^r)r_1$, smaller, which discourages the incentive of saving (40). This impact is depicted by an upward shift of the graph of $h = F(v_1)$ in Figure 9. At the same time, a higher τ_1^r also distorts condition (57), which makes the graph of $h = G(v_1)$ given by (57) shift downward. As illustrated in Figure 9, the capital values of both countries, v_1 and v_2, fall, which in turn reduces the levels of employment and the long run growth rates of both countries.

(ii) The case of $1+\gamma < \beta$

Figure 11 depicts the case of $1+\gamma < \beta$. Again, there exist dual BGPs in general. We can demonstrate that comparative statics results on the high-growth BGP with a lower v_1 are the same as those in Proposition 4; see Proposition 5 (i). It is also seen that on the low-growth BGP with a higher v_1, most of the results are reversed; see Proposition 5 (ii). The further detailed relationships between the growth effects and the relative slope of the graphs of $h = F(v_1)$ and $h = G(v_1)$ in Figure 11 are found in Appendix C.

As previously shown, an increase in τ_1^c reduces the demand for consumption goods, $x_1 + py_1$, which makes the the graph of curve $h = F(v_1)$ shifts upward. In addition, it is seen

from (53) that this reduction also depresses the export of consumption goods of country 2. As a consequet, the graph of $h = G(v_1)$ shifts downward. As shown in Figure 11, the value of capital, v_1, rises at the high-growth BGP (with lower v_1), while v_1 falls at the low-growth BGP (with higher v_1). When the increase in v_1 raises labor supply, the rightward shift of the Frisch labor supply curve reduces employment in the case of $1+\gamma < \beta$, which in turn lowers the growth rate of capital at the high-growth BGP. In contrast, the decreased v_1 raises employment, the growth rate of capital falls at the low-growth BGP.

The following proposition summarizes our findings:

Proposition 4 *Suppose that $1+\gamma > \beta$. Then the BGP of the world economy is uniquely given and the following cross-country and own-country growth effects of tax reforms hold:*

$$\begin{aligned} dg_i^k/d\tau_i^c &< 0, \quad dg_i^k/d\tau_i^w < 0, \quad dg_i^k/d\tau_i^r < 0, \\ dg_j^k/d\tau_i^c &< 0, \quad dg_j^k/d\tau_i^w < 0, \quad dg_j^k/d\tau_i^r > 0, \quad i,j = 1,2,\ i \neq j. \end{aligned}$$

Proposition 5 *Suppose that $1+\gamma < \beta$ and that the world economy has dual BGPs.*
(i) If the world economy stays on the BGP with higher growth rates of both countries, then the growth effects of taxation are given by

$$\begin{aligned} dg_i^k/d\tau_i^c &< 0, \quad dg_i^k/d\tau_i^w < 0, \quad dg_i^k/d\tau_i^r \gtrless 0, \\ dg_j^k/d\tau_i^c &< 0, \quad dg_j^k/d\tau_i^w < 0, \quad dg_j^k/d\tau_i^r \lesseqgtr 0, \quad i,j = 1,2,\ i \neq j. \end{aligned}$$

(ii) If the world economy stays on the BGP with lower growth rates of both countries, then the growth effects of taxation are given by

$$\begin{aligned} dg_i^k/d\tau_i^c &> 0, \quad dg_i^k/d\tau_i^w > 0, \quad dg_i^k d\tau_i^r \gtrless 0, \\ dg_j^k/d\tau_i^c &> 0, \quad dg_j^k/d\tau_i^w > 0, \quad dg_{ji}^k/d\tau_i^r \lesseqgtr 0, \quad i,j = 1,2,\ i \neq j. \end{aligned}$$

4 Conclusion

The central message of this paper is that the growth effects of tax reforms in the global economy heavily depends on the trade structure; i.e., the availability of the opportunities of

international borrowing and lending. By use of a two-country model of endogenous growth with variable labor supply, we have examined the growth effects of taxation under alternative specifications of trade structures. We have shown that the presence of international borrowing and lending plays a significant role as to how a change in tax policy in one country affects the other country's growth performance. Our study demonstrates that when inspecting growth effects of fiscal actions in an open-economy setting, we should carefully consider what kind of specification of trade structure can capture the reality well.

Our discussion has relied on specific functional forms of preferences and technologies. To obtain results that are useful policy recommendations, we should use more general modelling than that employed in this paper. As we have seen, even in our simple setting analytical examination of policy impacts are rather complex. This fact suggests that we should explore numerical consideration to generalize our analysis. Such a generalization is an urgent task in our future study.

Appendix A

This appendix confirms the determinacy conditions given by Proposition 3. First, note from (22) that we obtain

$$r'(v_i) = \frac{\beta}{1+\gamma-\beta}\frac{r(v_i)}{v_i}, \quad i = 1, 2.$$

Taking this relation into account, we see that the coefficient matrix J in (59) is expressed as

$$J = \begin{bmatrix} \Gamma_1(v_1) & 0 & -\frac{\theta}{1+\tau_2^c} \\ 0 & \Gamma_2(v_2) & \frac{1-\theta}{(1+\tau_1^c)h^2} \\ h\tau_1^r r_1'(v_1) & -h\tau_2^r r_2'(v_2) & 0 \end{bmatrix},$$

where v_1, v_2 and h take their steady-state values and

$$\begin{aligned} \Gamma_i(v_i) &\equiv \rho + \left[\frac{1}{a} - (1-\tau_i^r)\right]\frac{1+\gamma}{1+\gamma-\beta} r_i(v_i), \quad i = 1, 2, \\ r_i'(v_i) &= \frac{\beta}{\gamma+1-\beta} a A^{\frac{1+\gamma}{1+\gamma-\beta}} \left[(1-\tau_i^w)(1-a)\right]^{\frac{\beta}{1+\gamma-\beta}} v_i^{\frac{2\beta-\gamma-1}{1+\gamma-\beta}}, \quad i = 1, 2. \end{aligned}$$

We can confirm that if $1+\gamma > \beta$, then $r_i'(v_i) > 0$ and $\Gamma_i(v_i) > 0$, $i = 1, 2$, while if $1+\gamma < \beta$, then $r_i'(v_i) < 0$ and

$$\begin{aligned} \text{sign } [\Gamma_i(v_i)] &= \text{sign } \left[\rho + \left[\frac{1}{a} - (1-\tau_i^r)\right] \frac{1+\gamma}{\gamma+1-\beta} a A^{\frac{1+\gamma}{1+\gamma-\beta}} \left[(1-\tau_i^w)(1-a) v_i\right]^{\frac{\beta}{\gamma+1-\beta}}\right], \\ i &= 1, 2. \end{aligned}$$

The characteristic equation of J is written as

$$\phi(\lambda) = \lambda^3 - (\text{Tr } J)\lambda^2 + a_2\lambda - |J| = 0, \tag{62}$$

where λ denotes the characteristic root of J and

$$\begin{aligned} \text{Tr } J &\equiv \Gamma_1 + \Gamma_2, \\ |J| &\equiv \frac{\theta h}{1+\tau_2^c}\Gamma_2\tau_1^r r_1' + \frac{1-\theta}{(1+\tau_1^c)h}\Gamma_1\tau_2^r r_2', \\ a_2 &\equiv \Gamma_1\Gamma_2 + \frac{\theta h}{1+\tau_1^r}\tau_1^r r_1' + \frac{1-\theta}{(1+\tau_1^c)h}\tau_2^r r_2', \end{aligned}$$

where Tr J and $|J|$ represent the trace and determinant of matrix J, respectively. According to the Routh–Hurwitz criterion, the number of the roots of $\phi(\lambda) = 0$ with positive real parts equals the number of changes in signs of the following sequence:

$$\left\{1,\ -\text{Tr } J,\ a_2 - \frac{|J|}{\text{Tr } J},\ \ -|J|\right\}. \tag{63}$$

Note that

$$a_2 - \frac{|J|}{\text{Tr } J} = \frac{1}{\Gamma_1+\Gamma_2}\left[\Gamma_1\Gamma_2(\Gamma_1+\Gamma_2) + \frac{\theta h}{1+\tau_2^c}\tau_1^r r_1'\Gamma_1 + \frac{1-\theta}{(1+\tau_1^c)h}\tau_2^r r_2'\Gamma_2\right].$$

First, suppose that $1+\gamma > \beta$. In this case it holds that $r_i' > 0$, Tr $J > 0$, $|J| > 0$ and $a_2 - \frac{|J|}{\text{Tr } J} > 0$. Hence, the sequence (63) changes signs three times, which means that all of the characteristic roots of J have positive real parts. Since v_1, v_2 and h are not predetermined variables, this means that the BGP is locally determinate in the case of $1+\gamma > \beta$. On the other hand, if $1+\gamma < \beta$ and if $\Gamma_i < 0$ $(i = 1, 2)$, then we see that Tr $J < 0$ and $|J| > 0$. This shows that the characteristic equation (62) has two roots with negative real parts, so

that the BGP is locally indeterminate. In addition, if $\Gamma_1 > 0$ and $\Gamma_2 > 0$, then Tr $J > 0$ and $|J| < 0$. Therefore, the characteristic equation (62) has one negative, real root. Again, there is a continuum of converging paths around the balanced growth equilibrium. As a result, regardless of whether the BGP displays determinacy or indeterminacy, equilibrium indeterminacy holds under $1 + \gamma < \beta$.

Appendix B

In Propositions 4 and 5, the magnitudes of own-country and cross-country growth effects of the labor income and consumption taxes are given by the following:[14]

$$\frac{dg_1^k}{d\tau_1^c} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1+\tau_1^c)^2 v_1 h}\frac{\beta}{1+\gamma-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_1^k}{d\tau_2^c} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1+\tau_2^c)^2 v_1}\frac{\beta}{\gamma+1-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_2^k}{d\tau_1^c} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1+\tau_1^c)^2 v_1 h}\frac{\beta}{1+\gamma-\beta}(1-\tau_2^r)r_2,$$

$$\frac{dg_2^k}{d\tau_2^c} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1+\tau_2^c)^2 v_1}\frac{\beta}{\gamma+1-\beta}(1-\tau_2^r)r_2,$$

$$\frac{dg_1^k}{d\tau_1^w} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1-\tau_1^w)(1+\tau_1^c) v_1 h}\frac{\beta}{1+\gamma-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_1^k}{d\tau_2^w} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1-\tau_2^w)(1+\tau_2^c) v_1}\frac{\beta}{1+\gamma-\beta}(1-\tau_1^r)r_1,$$

$$\frac{dg_2^k}{d\tau_1^w} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1-\tau_1^w)(1+\tau_1^c) v_1 h}\frac{\beta}{1+\gamma-\beta}(1-\tau_2^r)r_2,$$

$$\frac{dg_2^k}{d\tau_2^w} = \frac{\Omega}{|D|}\frac{\theta(1-\theta)}{(1-\tau_2^w)(1+\tau_2^c) v_1}\frac{\beta}{1+\gamma-\beta}(1-\tau_2^r)r_2,$$

where

$$\Omega \equiv \frac{1}{(1+\tau_1^c)h} + \frac{1}{1+\tau_2^c} > 0 \text{ and } |D| \equiv -\frac{\gamma+1-\beta}{\beta}\frac{v_2}{\tau_1^r r_1 h}|J|.$$

[14] Although we have also computed own-country and cross-country growth effects of capital income taxation, we omit their results because their expressions are very complicated and in order to save a space. The results are available upon request to the corresponding author.

A close inspection of the above results imply that

$$\frac{dg_i^k}{d\tau_j^c}, \frac{dg_i^k}{d\tau_j^w} \gtreqless 0 \text{ if and only if } |J| \lesseqgtr 0, \;\; i,j = 1,2.$$

Moreover, when $(1-\tau_1^r)r_1 > (<)(1-\tau_2^r)r_2$, i.e., $r_1(v_1) > (<) r_2(\Phi v_1)$ on the BGP due to (54), then $dg_1^k/d\tau_1^w > (<) dg_2^k/d\tau_1^w$, $dg_1^k/d\tau_2^w > (<) dg_2^k/d\tau_2^w$, $dg_1^k/d\tau_1^c > (<) dg_2^k/d\tau_1^c$, $dg_1^k/d\tau_2^c > (<) dg_2^k/d\tau_2^c$. It is seen that from (20) and (22), if $r_1 > r_2$, then $l_1 > l_2$. Given a higher r_1 (i.e. a higher R in (42)), the magnitude of each tax effect on l_1 is larger than that on l_2. Then, the growth effect of each tax change in the home country is larger than in the foreign country.

Appendix C

In this appendix, we show how the signs of $|D|$ evaluated at the respective BGPs are related to the graphs of $F(v_1)$ and $G(v_1)$ in Figure 11. To this end, we first differentiate (57) and (58) with respect to v_1 to get

$$F'(v_1) = \frac{1+\tau_2^c}{\theta}\left(\frac{1}{a} - 1 + \tau_1^r\right)\frac{1+\gamma}{1+\gamma-\beta} r_1 + \rho \equiv \frac{1+\tau_2^c}{\theta}\Gamma_1, \tag{64}$$

$$G'(v_1) = \frac{1-\theta}{1+\tau_1^c}\frac{-\Phi\Gamma_2}{\left[\left(\frac{1}{a} - 1 + \tau_2^r\right) a\left[(1-\tau_2^w)(1-a)\right]^{\frac{\beta}{1+\gamma-\beta}}(A\Phi v_1)^{\frac{1+\gamma}{1+\gamma-\beta}} + \rho\Phi v_1 - \frac{1-\theta}{1+\tau_2^c}\right]^2}, \tag{65}$$

where $\Gamma_2 \equiv \left(\frac{1}{a} - 1 + \tau_2^r\right)\frac{1+\gamma}{1+\gamma-\beta} r_2 + \rho$. An inspection of Figure 11 reveals that $F'(v_1) < 0$ and $G'(v_1) > 0$ evaluated at the lower v_1 (i.e., the high-growth BGP). This observation, together with (64) and (65), imply that $\Gamma_1 < 0$ and $\Gamma_2 < 0$. Moreover, since it follows from Appendix A and $r_i'(v_i) < 0$ that

$$|J| = \frac{\theta h}{1+\tau_2^c}\Gamma_2\tau_1^r r_1'(v_1) + \frac{1-\theta}{(1+\tau_1^c)h}\Gamma_1\tau_2^r r_2'(v_2) > 0,$$

and since $sign[|J|] = sign[|D|]$ when $1+\gamma < \beta$ (see Appendix B), we can find that $|D| > 0$ evaluated at the high-growth BGP.

On the other hand, since $F'(v_1) > 0$ and $G'(v_1) < 0$ evaluated at the higher v_1 (i.e., the

low-growth BGP) in Figure 11, it turns out that $\Gamma_1 > 0$ and $\Gamma_2 > 0$. Similarly, since we can show that $|J| < 0$, we obtain $|D| < 0$ evaluated at the low-growth BGP.

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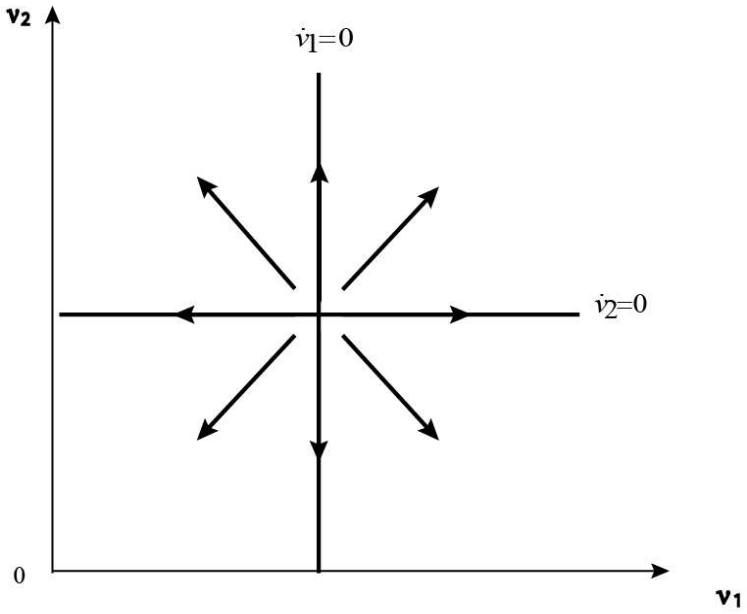

Figure 1: The case of $1+\gamma > \beta$

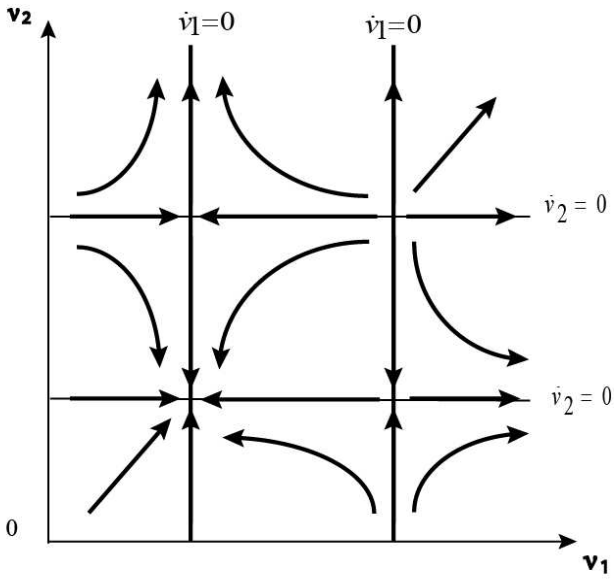

Figure 2: The case of $1+\gamma < \beta$

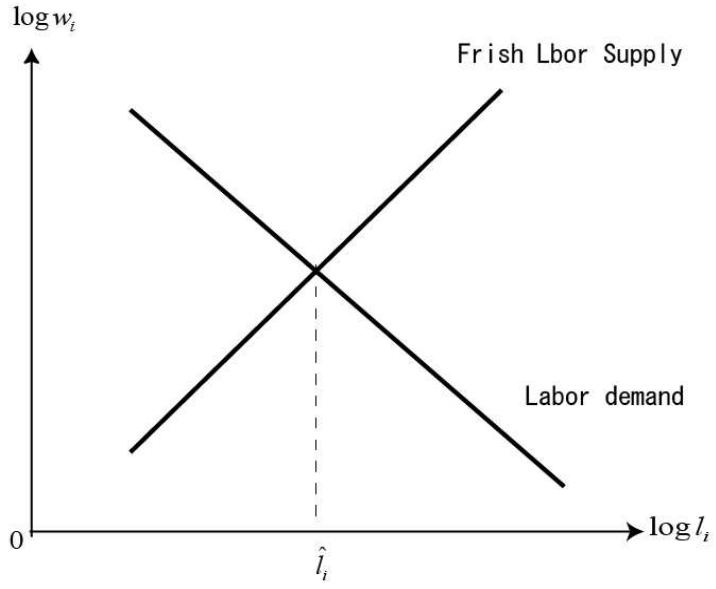

Figure 3: ： The case of $1 + \gamma > \beta$

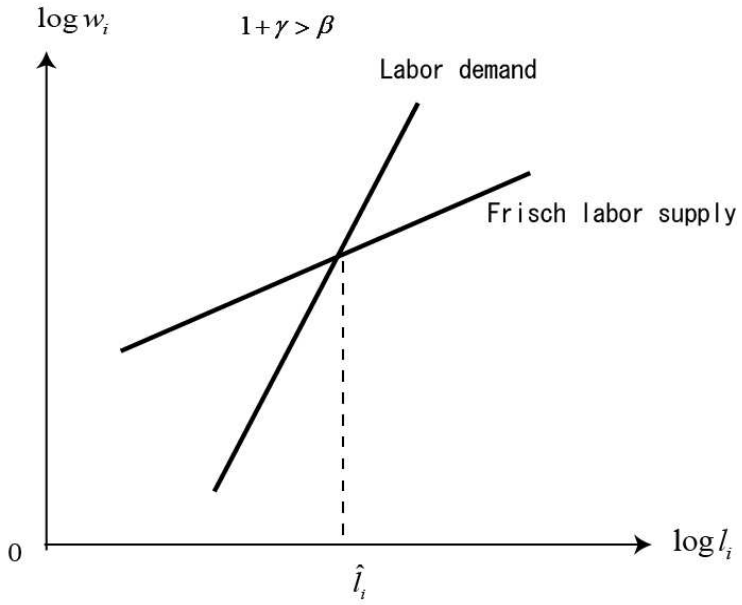

Figure 4: The case of $1 + \gamma < \beta$

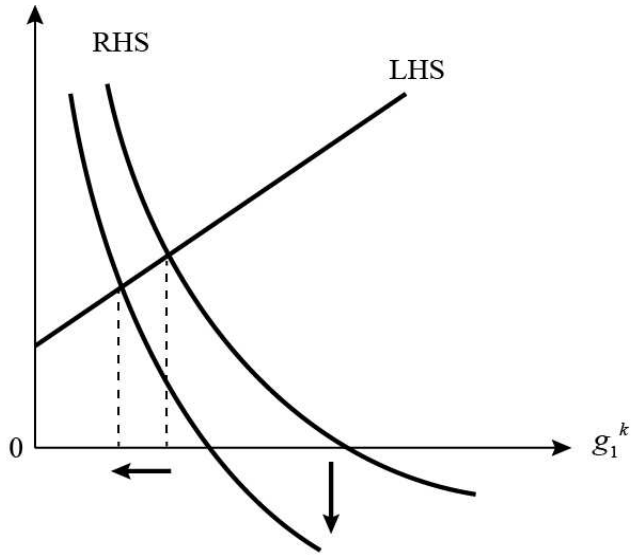

Figure 5: Effects of changes in τ_1^c and τ_1^w under $1+\gamma > \beta$

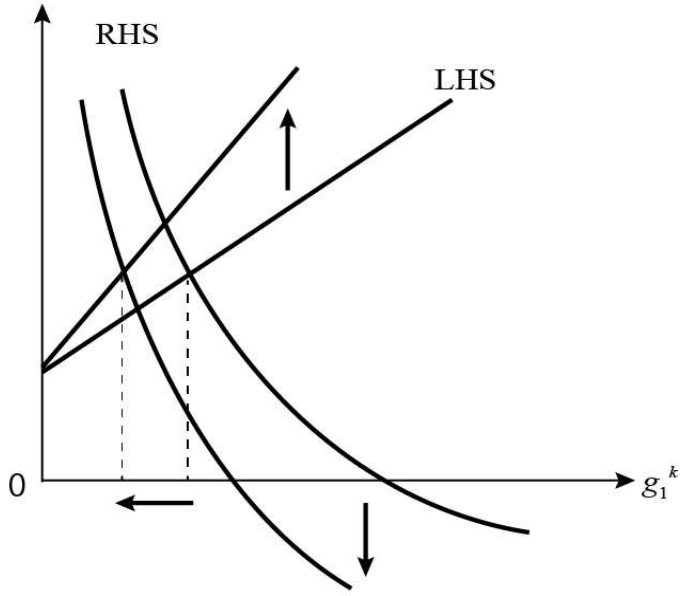

Figure 6: Effect of a change in τ_1^r under $1+\gamma > \beta$

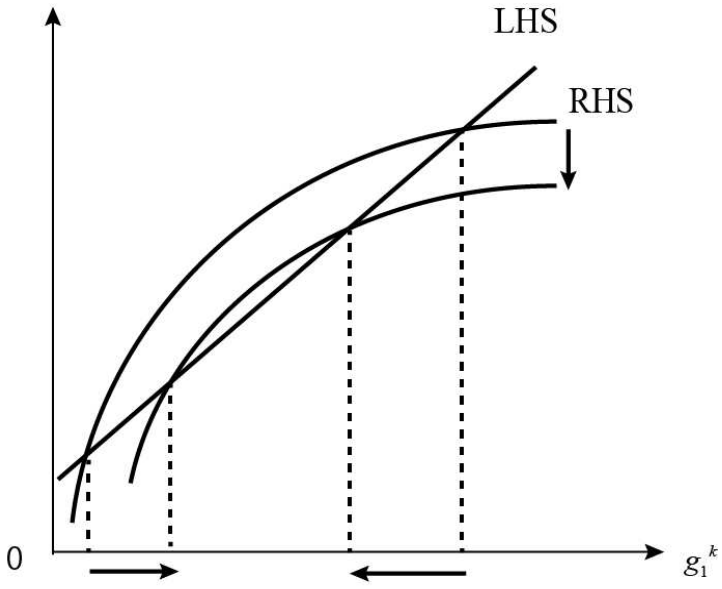

Figure 7: Effects of changes in τ_1^c and τ_1^w under $1+\gamma<\beta$

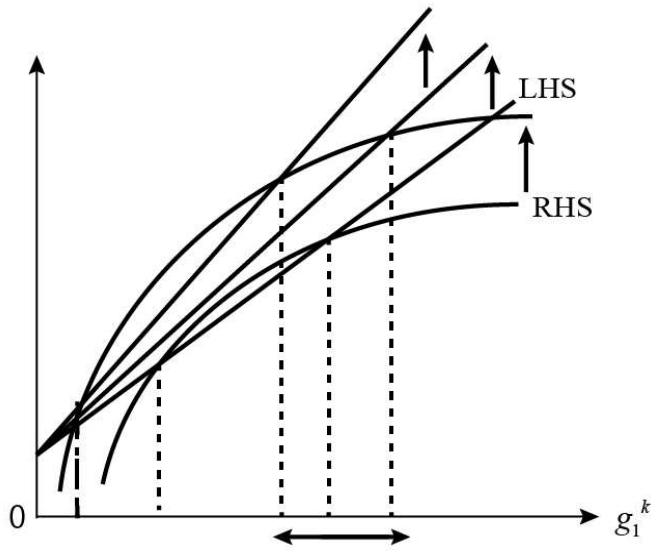

Figure 8: Effect of a change in τ_1^r under $1+\gamma<\beta$

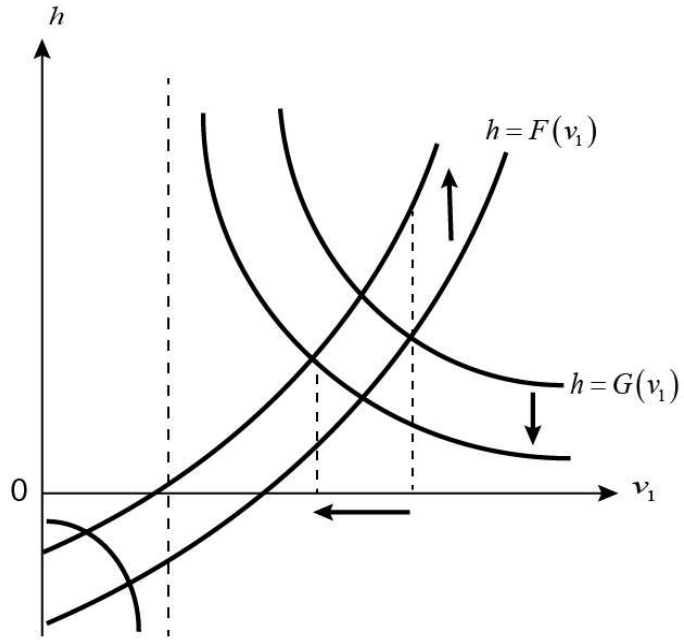

Figure 9: Effects of changes in τ_1^c and τ_1^r change in τ_1^r under $1+\gamma > \beta$

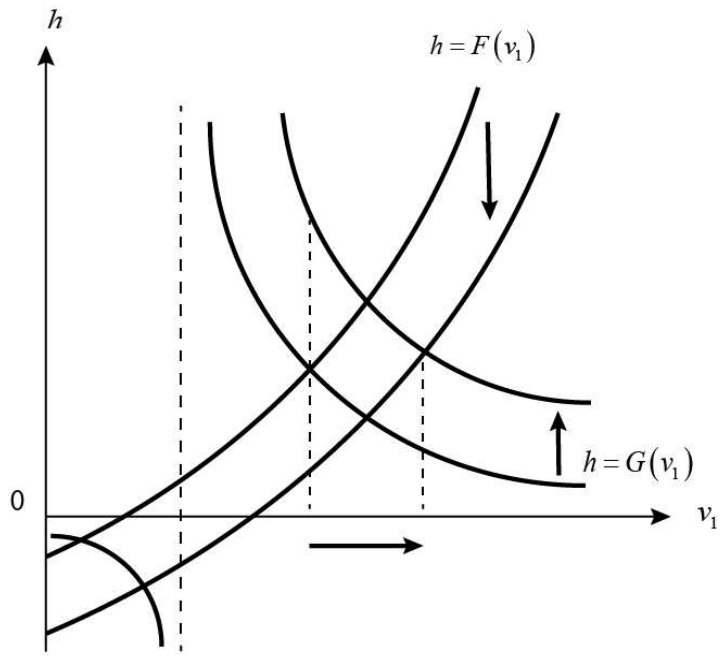

Figure 10: Effect of a change in τ_1^w under $1+\gamma > \beta$

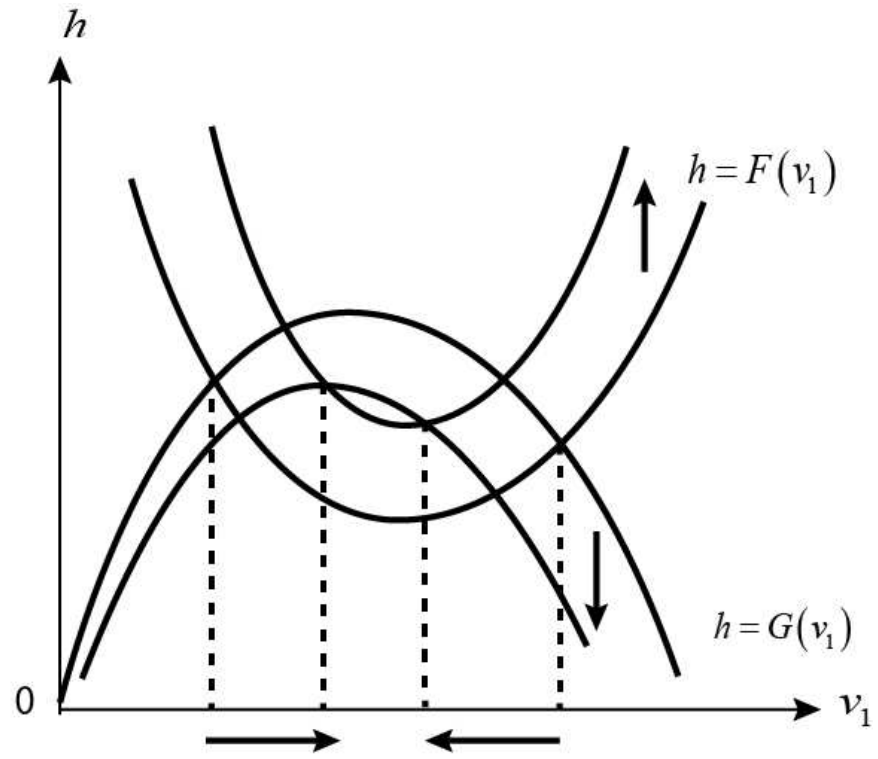

Figure 11: Effect of a change in τ_1^c under $1+\gamma<\beta$