CLOSED SUBMANIFOLDS WITH CONSTANT 
$v$-TH MEAN CURVATURE RELATED WITH A VECTOR 
FIELD IN A RIEMANNIAN MANIFOLD 

By 

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Introduction. H. Liebmann (1900) [1], proved the following theorem: The only ovaloids with constant mean curvature $H$ in an Euclidean space $E^3$ are the spheres.

Extension of this theorem to a convex hypersurface in an $n$-dimensional Euclidean space $E^n$ has been given by W. Süss (1929) [2], (cf. also [3], p. 118, and [4]). Then H. Hope (1951) [5], and A. D. Alexandrov (1958) [6], have shown the results that the convexity is not necessary for the validity of the Liebmann–Süss theorem.

Recently the analogous problem for closed hypersurfaces in an $n$-dimensional Riemannian manifold $R^n$ has been discussed by the present author [8], [9], [10], A. D. Alexandrov [7], K. Yano [13], T. Ōtsuki [15], M. Tani [16], and K. Nomizu [17], [18]. And also for a submanifold of codimension 2 in an odd dimensional sphere, M. Okumura has treated the analogue [19].

In the previous papers [11], [12], which are common works by T. Nagai, H. Kôjyo and the present author, we have given a certain extension of this problem to an $m$-dimensional closed submanifold $V^m$ ($1 \leq m \leq n - 1$) in the $n$-dimensional Riemannian manifold $R^n$ admitting a vector field $\xi$. However we have given there a restriction such that at each point on $V^m$, the vector $\xi$ lies in the vector space spanned by the tangent space of $V^m$ and the Euler–Schouten vector $n$.

The purpose of this paper is to give more general results except this restriction.

§ 1. Some integral formulas for a submanifold. We suppose an $n$-dimensional Riemannian manifold $R^n$ ($n \geq 3$) of class $C^r$ ($r \geq 3$) which admits an one-parameter continuous group $G$ of transformations generated by an infinitesimal transformation

\[ \bar{x}^i = x^i + \xi^i(x) \partial \tau \]

1) Numbers in brackets refer to the references at the end of the paper.
(where $x^i$ are local coordinates in $R^n$ and $\xi^i$ are the components of a contravariant vector $\xi$). If $\xi$ is a Killing vector, a homothetic Killing vector, a conformal Killing vector, etc. ([14], p. 32), then the group $G$ is called isometric, homothetic, conformal, etc.

In $R^n$, we consider a domain $M$. If the domain $M$ is simply covered by the orbits of the transformations generated by $\xi$, and $\xi$ is everywhere of class $C^r$ and $\neq 0$ in $M$; then we call $M$ a regular domain with respect to the vector field $\xi$.

Let us denote by $V^m$ an $m$-dimensional closed orientable submanifold of class $C^3$ imbedded in a regular domain $M$ with respect to the vector field $\xi$.

Let us consider a differential form of $m-1$ degree at a point of $V^m$, defined by

$$ (n, n, \cdots, n, \xi, dx, \cdots, dx)_{\text{def.}} = \sqrt{g} (n, n, n, \xi, dx, \cdots, dx) $$

$$ = \sqrt{g} \begin{vmatrix} n, n, \cdots, n, \xi, \frac{\partial x}{\partial u^\alpha}, \cdots, \frac{\partial x}{\partial u^{m-1}} \end{vmatrix} du^\alpha \wedge \cdots \wedge du^{m-1}, $$

where the symbol ($\begin{vmatrix} \end{vmatrix}$) means a determinant of order $n$ whose columns are the components of respective vectors, $dx$ is a displacement along $V^m$, $g$ is the determinant of the metric tensor $g_{ij}$ of $R^n$. Then the exterior differential of the differential form (1.3) divided by $m!$ becomes as follows

$$ \frac{1}{m!} d (n, n, \cdots, n, \xi, dx, \cdots, dx) = \frac{1}{m!} \{ (\delta n, n, \cdots, n, \xi, dx, \cdots, dx) + (n, \delta n, \cdots, n, \xi, dx, \cdots, dx) + \cdots + (n, \cdots, n, \delta n, \xi, dx, \cdots, dx) \} $$

where $u^\alpha$ are local coordinates of $V^m$. Throughout the present paper Latin indices run from 1 to $n$ and Greek indices from 1 to $m$. We assume that at any point on $V^m$ the vector $\xi$ is not on its tangent space.

We shall indicate by $n^i (p=m+1, \cdots, n)$ the contravariant unit vectors normal $V^m$ and suppose that they are mutually orthogonal. Let $n$ be in the vector space spanned by $m+1$ independent vectors $\frac{\partial x^i}{\partial u^\alpha} (\alpha=1, \cdots, m)$ and $\xi$ and be the unit vector normal $V^m$. Then, we may consider $n$ as one of the unit normal vectors of $V^m$, that is, $n^i = n^i$.
where \( \delta v \) means \( v_{,a} du^a \) and the symbol ";" the operation of D-symbol due to van der Waerden–Bortolotti ([20] p. 254).

Let \( C_j^i \) be \( \sum_{p=m+1}^{n} n_i^j n^j (n = n) \) and \( i (\lambda = 1, \cdots, m) \) mutually orthogonal unit tangent vectors of \( V^m \). Then we have

\[
 n_{;a}^i = C_{j;k}^i n^j \frac{\partial x^k}{\partial u^a} = -\sum_{i-1}^{m} (i_{j;k} n^j \frac{\partial x^k}{\partial u^a}) n^{i}.
\]

Therefore we may put

\[
 n_{;a}^i = \gamma_{a}^i \frac{\partial x^i}{\partial u^a}.
\]

Since we have

\[
 g_{ij} \left( \frac{\partial x^i}{\partial u^p} \right) n^j = -g_{ij} \frac{\partial x^i}{\partial u^p} n^{j},
\]

we obtain

\[
 (1.5) \quad n_{;a}^i = -b_{a}^i \frac{\partial x^i}{\partial u^a} \quad (p = \xi, m + 2, \cdots, n)
\]

where \( b_{a}^i \) means \( g^{i\beta} b_{a\beta} \) and \( b_{a\beta} = \left( \frac{\partial x^i}{\partial u^p} \right) n_{;p} \), and \( g^{i\beta} \) is the contravariant metric tensor of \( V^m \).

From (1.5) the first term of the right-hand member of (1.4) becomes

\[
 (1.6) \quad \frac{1}{m!} \left( (\partial n, n, \cdots, n, \xi, dx, \cdots, dx) \right) = (-1)^{(n-m)(n-1)} H_1 n_i \xi^i dA,
\]

where \( dA \) is the area element of \( V^m \) and \( H_1 \) means the first mean curvature of \( V^m \) with respect to the normal direction \( n_i \). Similarly, for every integer \( p \) satisfying \( m + 2 \leq p \leq n \) we have

\[
 (1.7) \quad \frac{1}{m!} \left( (n, \cdots, \partial n, \cdots, n, \xi, dx, \cdots, dx) \right) = (-1)^{(n-m)(n-1)} H_1 n_i \xi^i \frac{\partial x^i}{\partial u^a} dA = 0
\]

because \( \xi \) lies in the vector space spanned by \( m + 1 \) independent vectors \( \frac{\partial x^i}{\partial u^a} \) \((\alpha = 1, \cdots, m) \) and \( n \).

On the other hand the last term of the right-hand member of 1.4 becomes

\[
 (1.8) \quad \frac{1}{m!} \left( (n, \cdots, n, \delta \xi, dx, \cdots, dx) \right) = (-1)^{(n-m)(n-1)} \frac{1}{2m} (L g_{ij}) \frac{\partial x^i}{\partial u^a} \frac{\partial x^j}{\partial u^a} g^{i\beta} dA,
\]
where $L g_{ij}$ is the Lie derivative of $g_{ij}$ with respect to $\xi$ ([14], p. 5).

From (1.6), (1.7) and (1.8), (1.4) is rewritten as follows

$$
\frac{1}{m!} d ((n, \cdots, n, \xi, dx, \cdots, dx)) = (-1)^{(n-m)(n-1)} \left\{ H_n n_{i} \xi^{i} dA \right.
+ \frac{1}{2m} \left( L_{g_{ij}} \frac{\partial x^{i}}{\partial u^{\alpha}} \frac{\partial x^{j}}{\partial u^{\beta}} g^{a\beta} dA \right) \right\}.
$$

Integrating both members of (1.9) over the whole submanifold and applying Stokes' theorem, we obtain

$$
\frac{1}{m!} \int_{\partial V} ((n, n, \cdots, n, \xi, dx, \cdots, dx)) = (-1)^{(n-m)(n-1)} \left\{ \int_{V} H_n n_{i} \xi^{i} dA + \frac{1}{2m} \int_{V^{m\xi}} g^{*ij} L q_{ij} dA \right\},
$$

where $\partial V^{m}$ means the boundary of $V^{m}$ and $g^{*ij}$ is $\frac{\partial x^{i}}{\partial u^{a}} \frac{\partial x^{j}}{\partial u^{\beta}} q^{a\beta}$. Making use of the fact that $V^{m}$ is closed, we have

$$(I'') \int_{V^{m}} H_n n_{i} \xi^{i} dA + \frac{1}{2m} \int_{V^{m\xi}} g^{*ij} L q_{ij} dA = 0 .$$

If the manifold $R^{n}$ assumes of constant Riemann curvature which includes an Euclidean space, then we consider the following differential form of $m-1$ degree

$$
((n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx))
$$

$$(1.10) \quad \text{def.} \quad \sqrt{g} \quad (n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)
$$

for a fixed integer $\nu$ satisfying $m-1 \geq \nu \geq 1$.

As well-known, a submanifold $V^{m}$ in $R^{n}$ has the following property:

$$
b_{a\beta ;\gamma} - b_{a\gamma ;\beta} = - R_{ijkl} n^{i} \frac{\partial x^{j}}{\partial u^{a}} \frac{\partial x^{k}}{\partial u^{\beta}} \frac{\partial x^{l}}{\partial u^{\gamma}} \quad (20), \text{p. 226),}
$$

where $R_{ijkl}$ is the curvature tensor of $R^{n}$. Since $R^{n}$ is of constant Riemann curvature, we have

$$
n_{i,a \beta} - n_{i,a} = 0 .
$$

Consequently differentiating exteriorly the differential form (1.10), we have
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\[ d((n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = ((\delta n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) + \cdots + ((n, \delta n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) + ((n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)), \]

because \(((n, n, \cdots, n, \xi, \delta \delta n, \delta n, \cdots, \delta n, dx, \cdots, dx)) = 0\) from (1.11).

On substituting \(n^{i}_{;a} = -\frac{\partial x^{i}}{\partial u^{\beta}}\) into the first term of the right-hand member of (1.12), we get

\[ ((\delta n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = m!(-1)^{(n-m)(n-1)-\nu}H_{\nu+1}n^{i}\xi^{i}dA, \]

where \(H_{\nu+1}\) denotes the \(\nu+1\)-th mean curvature of \(V^{m}\) with respect to the normal direction \(n^{i}\) and if we indicate by \(k_{1}, k_{2}, \cdots, k_{m}\) the principal curvatures of \(V^{m}\) for the normal vector \(n\), \(H_{\nu+1}\) is defined to be the \(\nu+1\)-th elementary symmetric function of \(k_{\alpha}\) (\(\alpha = 1, \cdots, m\)) divided by the number of terms, that is,

\[ \left(\begin{array}{c} m \\ \nu + 1 \end{array}\right) H_{\nu+1} = \sum_{a_{1} < a_{2} < \cdots < a_{\nu+1}} k_{a_{1}}k_{a_{2}}\cdots k_{a_{\nu+1}}. \]

Also, by virtue of (1.5) we can see that the vectors

\[ n \times \delta n \times n \times \cdots \times n \times \delta n \times \cdots \times \delta n \times dx \times \cdots \times dx, \]

\[ n \times \delta n \times n \times \cdots \times n \times \delta n \times \cdots \times \delta n \times dx \times \cdots \times dx, \]

\[ \cdots \]

and

\[ n \times n \times \cdots \times n \times \delta n \times \delta n \times \cdots \times \delta n \times dx \times \cdots \times dx \]

have the same direction to the covariant vectors \(n, n, \cdots, n\) respectively.

Thus we obtain

\[ ((n, \delta n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = 0, \]
$$((n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = 0,$$

(1.14)

$$((n, n, \cdots, n, \delta n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = 0,$$

because $\xi$ lies in the vector space spanned by $m + 1$ independent vectors $\frac{\partial x^\xi}{\partial u^\alpha}$ ($\alpha = 1, \cdots, m$) and $n$.

From that the vector $n \times n \times \cdots \times n \times \delta n \times \cdots \times \delta n \times dx \times \cdots \times dx$ is orthogonal to the normal vectors $n, n, \cdots$ and $n$, and $\delta n^\xi = -b_{a}^\theta \frac{\partial x^\xi}{\partial u^\beta} du^a$, the last term of the right-hand member of (1.12) becomes as follows

$$((n, n, \cdots, n, \delta \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = m!(-1)^{(n-m)(n-1)} \frac{1}{2m} H^a_{\xi} L g_{a\beta} dA,$$

(1.15)

where

$$L g_{a\beta} = (L g_{ij}) \frac{\partial x^i}{\partial u^\alpha} \frac{\partial x^j}{\partial u^\beta}$$

and

$$K^a_{\xi} = \frac{1}{(m-1)!} a_{s_1 \cdots s_m-1} b_{s_1} \cdots b_{s_m} g_{a \beta} \epsilon^{a_1 \cdots a_{m-1}} \epsilon^{s_1 \cdots s_m-1},$$

and $\epsilon^{a_1 \cdots a_{m-1}}$ denotes the $\epsilon$-symbol of the submanifold $V^m$. Accordingly we have

$$\frac{1}{m!} d ((n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = (-1)^{(n-m)(n-1)} \left\{ H^a_{\xi} n^a \xi dA + \frac{1}{2m} H^a_{\xi} L g_{a\beta} dA \right\}.$$

(1.16)

Integrating both members of (1.16) over the whole submanifold $V^m$ and applying Stokes' theorem, we have

$$\frac{1}{m!} \int_{\partial V^m} ((n, n, \cdots, n, \xi, \delta n, \cdots, \delta n, dx, \cdots, dx)) = (-1)^{(n-m)(n-1)} \left\{ \int_{V^m} H^a_{\xi} n^a \xi dA + \frac{1}{2m} \int_{V^m} H^a_{\xi} L g_{a\beta} dA \right\}.$$

Thus, for a closed orientable submanifold $V^m$ we obtain

$$(\Pi'') \quad \int_{V^m} H^a_{\xi} n^a \xi dA + \frac{1}{2m} \int_{V^m} H^a_{\xi} L g_{a\beta} dA = 0.$$

If $m = n - 1$, that is, $V^m$ is the hypersurface in $R^n$, the formulas (I') and
(II") are coincide with the formulas (I) and (II) given in the previous paper [8]. Especially if the vector $n$ is coincide with the Euler-Schouten unit vector $n$ at each point on $V^m$, then the formulas (I") and (II") become the formulas (I') and (II') given in the previous paper [12].

§ 2. The integral formulas concerning some special transformations. In this section, we shall discuss the formulas (I") and (II") for a special infinitesimal transformation. Let the group $G$ of transformations be conformal, that is, $\xi^i$ satisfies an equation: $Lg_{ij}\equiv\xi_{i;j}+\xi_{j;i}=2\phi g_{ij}$ ([14], p. 32). Then we obtain

$$g^{*ij}Lg_{ij}=2m\phi, \quad H^s_i Lg_{si}=2m\phi H_u.$$  

Therefore (I") and (II") are rewritten in the following forms:

$$(I'\prime\prime)_c \quad \int_{V^m} H_1 n_i \xi^i dA + \int_{V^m} \phi dA = 0,$$

$$(II'\prime\prime)_c \quad \int_{V^m} H_\nu n_i \xi^i dA + \int_{V^m} \phi H_i dA = 0 \quad (1 \leq \nu \leq m-1)$$

and we can see

$$(I'\prime\prime)_h \quad \int_{V^m} H_1 n_i \xi^i dA + c \int_{V^m} dA = 0,$$

$$(II'\prime\prime)_h \quad \int_{V^m} H_\nu n_i \xi^i dA + \int_{V^m} H_i dA = 0 \quad (1 \leq \nu \leq m-1)^2)$$

in case of $\phi=$constant ($\equiv c$) ($G$ being homothetic), and

$$(I'\prime\prime)_1 \quad \int_{V^m} H_1 n_i \xi^i dA = 0,$$

$$(II'\prime\prime)_1 \quad \int_{V^m} H_\nu n_i \xi^i dA = 0 \quad (1 \leq \nu \leq m-1)$$

in case of $\phi=0$ ($G$ being isometric).

Especially if our manifold $R^n$ is an Euclidean space $E^n$ and if $\xi$ is the homothetic Killing vector field on $E^n$ with components $\xi^i=x^i$, $x^i$ being rectangular coordinates with a point in the interior of $V^m$ as origin in the space $E^n$, then the orbits of the transformations generated by $\xi$ are the lines through

2) In this case, $R^n$ becomes an Euclidean space, because if $R^n$ with constant Riemann curvature admits an one-parameter group $G$ of homothetic transformations, then either $R^n$ is $E^n$ or the group $G$ is isometric.
the origin and we have

\[ Lg_{ij} = 2g_{ij}. \]

Consequently, from \((I'')_{h}\) and \((II'')_{h}\) we obtain

\[(I^*) \quad \int_{V^{m} \xi} H_{1} p dA + \int_{V^{m}} dA = 0, \]

\[(II^*) \quad \int_{V^{m} \xi} H_{\nu+1} p dA + \int_{V^{m} \xi} H_{\nu} dA = 0, \]

where \(p = n_{i} x^{l} \xi^{t}. \) This means that the formulas \((I^*)\) and \((II^*)\) are generalization of those formulas given by C. C. Hsiung [4] for a closed orientable hypersurface in an \(n\)-dimensional Euclidean space \(E^{n} \).

\[\S 3.\] Some properties of a closed orientable submanifold related with a vector field. In this section we suppose again that the group \(G\) is conformal. Then we shall prove the following four theorems for a closed orientable submanifold \(V^{m}\) in a Riemannian manifold \(R^{n}\) with constant Riemann curvature.

**Theorem 3.1.** If in \(R^{n}\), there exists such a group \(G\) of conformal transformations as \(\rho\) is positive (or negative) at each point of \(V^{m}\) and if \(H_{1}\) is constant, then every point of \(V^{m}\) is umbilic with respect to the normal vector \(n\), where \(\rho\) denotes \(n_{i} x^{l} \xi^{t}. \)

**Proof.** Multiplying the formula \((I'')_{c}\) by \(H_{1} = \text{const.}, \) we have

\[ \int_{V^{m} \xi} H_{1}^{2} \rho dA + \int_{V^{m}} \phi H_{1} dA = 0. \]

On the other hand, from \((II'')_{c}\) we have

\[ \int_{V^{m} \xi} H_{2} \rho dA + \int_{V^{m} \xi} \phi H_{1} dA = 0. \]

Consequently it follows that

\[ \int_{V^{m} \xi} (H_{1}^{2} - H_{2}) \rho dA = 0. \]

From our assumption about \(\rho\), this holds if and only if \(H_{1}^{2} - H_{2} = 0\), since

\[ H_{1}^{2} - H_{2} = \frac{1}{m^{2}(m-1)} \sum_{a < b} (k_{a} - k_{b})^{2} \geq 0. \]

Therefore at each point of \(V^{m}\) we obtain
\[ k_1 = k_2 = \cdots = k_m. \]

Accordingly every point of \( V^m \) is umbilic with respect to \( n \).

**Theorem 3.2.** If in \( R^n \), there exists such a group \( G \) of conformal transformations as \( \rho \) is positive (or negative) at each point of \( V^m \), and if the principal curvatures \( k_1, k_2, \ldots, k_m \) at each point of \( V^m \) are positive and \( H_\nu \) is constant for any \( \nu \) \((1 < \nu \leq m-1)\), then every point of \( V^m \) is umbilic with respect to the normal vector \( n \).

**Proof.** Multiplying the formula \((I'')_c\) by \( H_\nu = \text{const.} \), we obtain

\[
(3.1) \quad \int_{\nu^m} H_1 H_\nu \rho dA + \int_{\nu^m} \phi H_\nu dA = 0.
\]

By value of \((II'')_c\) and \((3.1)\), we have

\[
\int_{\nu^m} (H_1 H_\nu - H_{\nu+1}) \rho dA = 0.
\]

From our assumption, this holds if and only if \( H_1 H_\nu - H_{\nu+1} = 0 \), since

\[
H_1 H_\nu - H_{\nu+1} = \nu! (m - \nu - 1)! \sum_{a_1 \cdots a_{\nu-1}} k_{a_1} \cdots k_{a_{\nu-1}} (k_{\alpha_\nu} - k_{\alpha_{\nu+1}})^2 \geq 0.
\]

Then at each point of \( V^m \), we obtain

\[
k_1 = k_2 = \cdots = k_m.
\]

Accordingly every point of \( V^m \) is umbilic with respect to \( n \).

**Theorem 3.3.** If in \( R^n \), there exists such a group \( G \) of conformal transformations as \( \rho \) is positive (or negative) at each point of \( V^m \), for which \( H_1 \rho + \phi \geq 0 \) (or \( \leq 0 \)) at all points of \( V^m \), then every point of \( V^m \) is umbilic with respect to \( n \).

**Proof.** If we express the formula \((I'')_c \) as follows

\[
\int_{\nu^m} (H_1 \rho + \phi) dA = 0,
\]

then from our assumption we have the relation:

\[
(3.2) \quad \phi = -H_1 \rho.
\]

Substituting \((3.2)\) into \((II'')_c \) for \( \nu = 1 \), we have
\[
\int_{V^{m}_{\xi}}(H_{1}^{2}-H_{2})\rho dA = 0 .
\]
Thus, we can see the conclusion.

**Theorem 3.4.** If \( H_{1} \) is positive (or negative) at all points of \( V^{m} \) and if \( R^{n} \) admits such a group \( G \) of conformal transformations as \( \phi \) is positive (or negative), for which either \( \rho \geq \frac{-\phi}{H_{1}} \) or \( \rho \leq \frac{-\phi}{H_{1}} \) at all points of \( V^{m} \), then every point of \( V^{m} \) is umbilic with respect to \( n \).

**Proof.** The formula \((1'')_{c}\) is rewritten as follows
\[
\int_{V^{m}}H_{1}\left(\rho + \frac{\phi}{H_{1}}\right)dA = 0 .
\]
By virtue of our assumptions \( H_{1} > 0 \) (or \( < 0 \)) and \( \rho + \frac{\phi}{H_{1}} \geq 0 \) (or \( \leq 0 \)) at all points of \( V^{m} \), we have the following relation
\[
(3.3) \quad \rho = -\frac{\phi}{H_{1}} .
\]
Substituting (3.3) into \((II'')_{c}\) for \( \nu = 1 \), we obtain
\[
\int_{V^{m}}\frac{\phi}{H_{1}}(H_{1}^{2}-H_{2})dA = 0 ,
\]
which holds if and only if \( H_{1}^{2} - H_{2} = 0 \). Thus we obtain the conclusion.

**Remark I.** If \( V^{m} \) is the hypersurface in \( R^{n} \), these four theorems are coincide with the theorems given in the previous paper \[8\]. Especially if the vector \( n \) is coincide with the Euler-Schouten unit vector \( \xi \) at each point of \( V^{m} \), then these four theorems become those theorems given in the previous paper \[12\].

**Remark II.** In all these sections we have treated the normal unit vector \( n \) with respect to the vector field \( \xi \) and the mean curvature \( H_{\nu} \). These are the notions due to R. E. Stong \[21\].

**References**


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