Spin relaxation in a zinc-blende (110) symmetric quantum well with δ doping

Hiroshi Akera, Hidekatsu Suzuura, and Yoshiyuki Egami
Division of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo, Hokkaido 060-8628, Japan
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The spin relaxation of a two-dimensional electron system (2DES) formed in a symmetric quantum well is studied theoretically when the quantum well is parallel to the (110) plane of the zinc-blende structure, the spin polarization is perpendicular to the well, and electrons occupy only the ground subband. The spin-relaxation rate is calculated as a function of the distribution of donor impurities which are placed in the well layer. Considered processes of the spin relaxation are (1) the intrasubband process by impurity-potential-induced spin-orbit interaction (SOI), which is the Elliott-Yafet mechanism in the 2DES, and (2) virtual intersubband processes consisting of a spin flip by (2a) well-potential-induced SOI or (2b) the Dresselhaus SOI, and a scattering from an impurity. It is shown that all of the above processes disappear when all impurities are located on the center plane of the well. Even if impurities are distributed over three (110) atomic layers, the spin-relaxation rate is two orders of magnitude lower than that for the uniform distribution over the well width of 7.5 nm. In GaAs/AlGaAs type-I quantum wells, the processes (1) and (2a) interfere constructively, being dominant over (2b) for the well width of $\sim$10 nm, while in some type-II quantum wells, they can interfere destructively.

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I. INTRODUCTION

Employing the spin degree of freedom in semiconductors is a promising approach to the development of hybrid devices which perform all of the information processing, communications, and storage [1]. The prerequisite spin polarization can be created in nonmagnetic semiconductors by the spin-orbit interaction (SOI) through the spin Hall effects of the extrinsic origin [2–6] and of the intrinsic one [7,8], which have been confirmed in experiments [9–11]. However, the same SOI becomes a driving force of the spin relaxation in various mechanisms [12,13]. In this paper, we show theoretically that the spin relaxation due to a spin flip by the SOI with a scattering at an impurity vanishes for a two-dimensional electron system (2DES) formed in a symmetric quantum well with a δ doping [14,15] on the center plane of the well.

Two major mechanisms of the spin relaxation in n-doped semiconductors are the Dyakonov-Perel mechanism [16–18] and the Elliott-Yafet mechanism [19–21]. The Dyakonov-Perel mechanism is due to the spin precession around a SOI-induced effective magnetic field whose direction and magnitude depend on the momentum of each electron. In addition to the Dresselhaus SOI [22] due to the inversion asymmetry in the crystal structure, the Rashba SOI [23–26] produces the effective magnetic field in a 2DES formed in a quantum well with the inversion asymmetry, and the Dyakonov-Perel mechanism due to such SOIs is a major mechanism of the spin relaxation. Fortunately, the Dyakonov-Perel mechanism can be turned off for the spin direction perpendicular to the 2DES by preparing a symmetric quantum well on a substrate oriented parallel to the (110) plane of the zinc-blende structure. This is because the Dresselhaus SOI in symmetric quantum wells parallel to the (110) plane gives an effective magnetic field perpendicular to the 2DES regardless of electron momentum [18,27] and the Rashba SOI is absent in symmetric quantum wells.

The suppression of the spin relaxation in (110) symmetric quantum wells was observed for the first time by Ohno *et al.* [28] using the pump-probe method: the spin-relaxation time in GaAs (110) symmetric quantum wells is more than an order of magnitude longer than that in (100) quantum wells [29]. The spin relaxation remaining in their undoped sample was ascribed to the Bir-Aronov-Pikus mechanism [30] due to the electron-hole exchange interaction. Holes are introduced in the pump-probe experiment when the sample is excited optically for the purpose of generation and detection of the spin polarization. The Bir-Aronov-Pikus mechanism, however, can be neglected in the later measurement by Müller *et al.* [31] with use of the spin noise spectroscopy, which can avoid the introduction of holes. Since the quantum well used in this spin noise measurement was modulation doped, the observed low spin-relaxation rate of $(24\ \mathrm{ns})^{-1}$ was attributed to the Dyakonov-Perel mechanism due to the random Rashba field produced by density fluctuations of donors located in barrier layers [32]. In this paper, we instead consider a doping in a well layer, and therefore the random Rashba field is outside the scope of this paper.

A doping in a well layer has the advantage of efficient generation of the spin polarization by the extrinsic spin Hall effect. In fact, the spin accumulation produced by the spin Hall effect has been observed in AlGaAs (110) quantum wells, in which Si donors are doped uniformly in the well layer [11]. The observed spin Hall effect has been explained by the theory of the extrinsic spin Hall effect [33,34], in which donor impurities in the well layer play a major role in creating the spin polarization.

Such previous studies suggest that one promising way to achieve large spin polarizations is to employ an n-doped (110) symmetric quantum well, which can produce the spin polarization by the extrinsic spin Hall effect and, at the same time, can avoid the spin relaxation due to the Dyakonov-Perel mechanism. An important task in this direction will be to find a method to suppress the spin relaxation caused by donor impurities introduced in the well layer.

It is known that impurities give rise to the spin relaxation called the Elliott-Yafet mechanism, in which spin-flip scatterings are caused by the combined action of the impurity potential and the SOI. This mechanism is likely to be dominant for the relaxation of the spin polarization perpendicular to

the 2DES in a (110) symmetric quantum well, in which the Dyakonov-Perel mechanism does not work. In quantum wells, the Elliott-Yafet mechanism is modified by the subband structure: in addition to intrasubband spin-flip processes [35,36], intersubband spin-flip processes due to SOI matrix elements between states in different subbands contribute to the spin relaxation.

The importance of such intersubband processes in various spin dynamics has been suggested in recent studies. Döhrmann *et al.* [37] have proposed a spin-relaxation mechanism due to intersubband spin-flip transitions, which are induced by the Dresselhaus SOI and the impurity potential, between the ground subband and the first-excited subband to explain their observed result of the spin-relaxation time in a (110) symmetric quantum well at higher temperatures such that the first-excited subband is occupied by electrons. Bernardes *et al.* [38] have theoretically studied roles of the intersubband matrix element of the SOI induced by the well potential in a symmetric quantum well and have derived the formula of the spin Hall conductivity in this system. Zhou and Wu [39] have calculated the spin-relaxation time of the 2DES occupying only the ground subband in a (110) symmetric quantum well by considering a virtual intersubband process through the first-excited subband in terms of the Dresselhaus SOI with the impurity potential.

In this paper, we theoretically study the spin relaxation in an n-doped (110) symmetric quantum well for the spin orientation perpendicular to the well. We consider the 2DES occupying only the ground subband and study spin-flip scatterings through both intrasubband and intersubband processes. The intrasubband spin-flip scattering is caused by the SOI due to the impurity potential. The intersubband spin-flip scattering is a virtual process through one of the excited subbands, which consists of an intersubband spin-flip process due to the SOI and an intersubband scattering process due to the impurity potential [39]. We take into account both the well-potential-induced SOI and the Dresselhaus SOI for the intersubband spin-flip process. In particular, we investigate the dependence of the spin-flip scattering rate on the position of δ doping [14,15], which can introduce impurities within an atomic layer in the well.

The organization of the paper is as follows. In Sec. II, we describe the Hamiltonian, which includes the SOIs originating from the impurity potential and the well potential in addition to the Dresselhaus SOI. In Sec. III, we show that spin-flip scatterings are absent when impurities are placed only in the atomic layer at the center of the well (center δ doping). In Sec. IV, we investigate spin-flip scatterings for off-center δ dopings by calculating the spin-flip scattering rate as a function of the position of δ doping. We also calculate the spin-flip scattering rate for impurity distributions having nonzero widths. In Sec. V, conclusions are given, in which it is also shown that the spin current caused by the extrinsic spin Hall effect does not disappear in the center δ doping.

II. HAMILTONIAN

We consider electron states in a quantum-well structure which is formed by two different zinc-blende semiconductors.

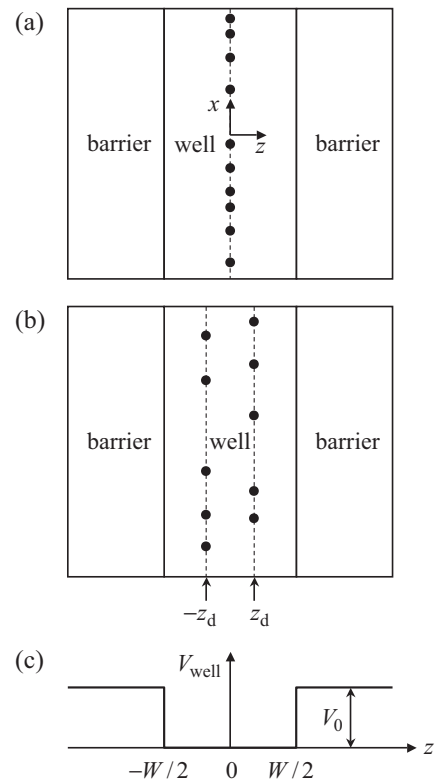

FIG. 1. (a) δ doping at the well center ($z = 0$). Filled circles (•) represent impurities. (b) δ dopings at $z = \pm z_d$. (c) The well potential $V_{well}(z)$ with height V_0 and width W.

The Hamiltonian is

$$H = H_0 + H_1. \tag{1}$$

The unperturbed Hamiltonian H_0 is

$$H_0 = \frac{\hat{\boldsymbol{p}}^2}{2m} + V_{well}(z), \tag{2}$$

where $\hat{\boldsymbol{p}} = (\hat{p}_x, \hat{p}_y, \hat{p}_z) = -i\hbar\nabla = -i\hbar(\nabla_x, \nabla_y, \nabla_z)$ and m is the effective mass of the conduction band. The well potential $V_{well}(z)$, which is the potential due to the offset of the conduction band at the interface between two constituent semiconductors [40], is given for the width W and the height $V_0(>0)$ by

$$V_{well}(z) = \begin{cases} 0 & (|z| < W/2) \\ V_0 & (|z| > W/2), \end{cases} \tag{3}$$

and is illustrated in Fig. 1. Each eigenstate of H_0 is labeled by the subband index, $n = 0,1,2,\ldots$, as well as the wave vector in the xy plane, $\boldsymbol{k} = (k_x, k_y)$, and the z component of the spin, $\sigma = \uparrow, \downarrow$. The corresponding eigenenergy depends only on n and $k = |\boldsymbol{k}|$ and is denoted by ε_{nk} or $\varepsilon_{n\boldsymbol{k}}$. We assume that only the ground subband with $n = 0$ is occupied by electrons.

The perturbation H_1 is

$$H_1 = V_{imp}(\boldsymbol{r}) + H^{so}_{imp} + H^{so}_{well} + H^{so}_{D}. \tag{4}$$

Here $V_{imp}(\boldsymbol{r})$ with $\boldsymbol{r} = (x, y, z)$ is the potential due to randomly distributed impurities. H^{so}_{imp} is the SOI due to $V_{imp}(\boldsymbol{r})$, given by

$$H^{so}_{imp} = -\frac{\eta}{\hbar}\boldsymbol{\sigma} \cdot (\nabla V_{imp} \times \hat{\boldsymbol{p}}), \tag{5}$$

while $H^{\rm so}_{\rm well}$ is that caused by the well potential for an electron in each of the valence bands, defined by

$$H^{\rm so}_{\rm well} = -\frac{\eta b_{\rm off}}{\hbar}\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla} V_{\rm well}\times\hat{\boldsymbol{p}}), \tag{6}$$

where $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ is the Pauli spin matrix and η is the effective coupling constant of the SOI for an electron in the conduction band of the semiconductor in the well layer. The factor $b_{\rm off}$ is a dimensionless constant reflecting the difference in the band offset between the conduction band and each of the valence bands. The formula of $b_{\rm off}$ is given in the Appendix. The last term of H_1 is the Dresselhaus SOI in the zinc-blende structure,

$$H^{\rm so}_{\rm D} = -\frac{\gamma}{2\hbar^3}\boldsymbol{\sigma}\cdot\boldsymbol{h}(\hat{\boldsymbol{p}}), \tag{7}$$

where γ is the coupling constant of the Dresselhaus SOI and $\boldsymbol{h} = (h_x,h_y,h_z)$ can be understood as an effective magnetic field. In (110) quantum wells, $\boldsymbol{h}$ is given by

$$\begin{aligned} h_x &= \left(-\hat{p}_x^2 - 2\hat{p}_y^2 + \hat{p}_z^2\right)\hat{p}_z,\\ h_y &= 4\hat{p}_x\hat{p}_y\hat{p}_z,\\ h_z &= \hat{p}_x\left(\hat{p}_x^2 - 2\hat{p}_y^2 - \hat{p}_z^2\right), \end{aligned} \tag{8}$$

where the Cartesian unit vectors are taken as

$$\begin{aligned} \boldsymbol{e}_x &= (-\boldsymbol{e}_{[100]} + \boldsymbol{e}_{[010]})/\sqrt{2},\\ \boldsymbol{e}_y &= \boldsymbol{e}_{[001]},\\ \boldsymbol{e}_z &= (\boldsymbol{e}_{[100]} + \boldsymbol{e}_{[010]})/\sqrt{2}, \end{aligned} \tag{9}$$

with $\boldsymbol{e}_{[100]}$, $\boldsymbol{e}_{[010]}$, and $\boldsymbol{e}_{[001]}$ the unit vectors along the crystal axes.

III. ABSENCE OF SPIN-FLIP SCATTERINGS IN THE CENTER δ DOPING

Typical impurity-doping profiles are the uniform doping and the modulation doping. The uniform doping in a well layer has been used in measurements [11,28] and a calculation [35] of the spin-relaxation time in (110) quantum wells, while the modulation doping in barriers has also been employed in measurements [31,37,41] and calculations [32,39].

In this paper, we adopt the δ doping [14,15] in the well layer ($|z| < W/2$). By the method of δ doping, it is possible, in principle, to dope donor impurities in a particular atomic layer. We choose a doping symmetric with respect to the well center ($z = 0$): a δ doping on two atomic layers at $z = \pm z_{\rm d}$ ($z_{\rm d} < W/2$). Such a symmetric doping keeps the impurity potential averaged over the plane symmetric. Note, however, that the impurity potential $V_{\rm imp}$ is not symmetric because of the random distribution of impurities in the plane.

First we consider a δ doping with $z_{\rm d} = 0$ in which all impurities are on the center plane of the well [Fig. 1(a)]. In such a δ doping, the impurity potential $V_{\rm imp}$ is even in z. Terms with σ_z in $H^{\rm so}_{\rm imp}$ and $H^{\rm so}_{\rm D}$ are also even in z, while terms with σ_x or σ_y in $H^{\rm so}_{\rm imp}$, $H^{\rm so}_{\rm well}$, and $H^{\rm so}_{\rm D}$ are odd in z, since $\hat{p}_z \to -\hat{p}_z$ when $z \to -z$.

Such a symmetry with respect to $z = 0$ leads to the absence of spin-flip scatterings with initial and final states in the same subband, which is valid in any order of the impurity potential and the SOI [42]. Note that since we have assumed that only the ground subband is occupied by electrons and consider only elastic processes, both the initial and final states should be in the ground subband.

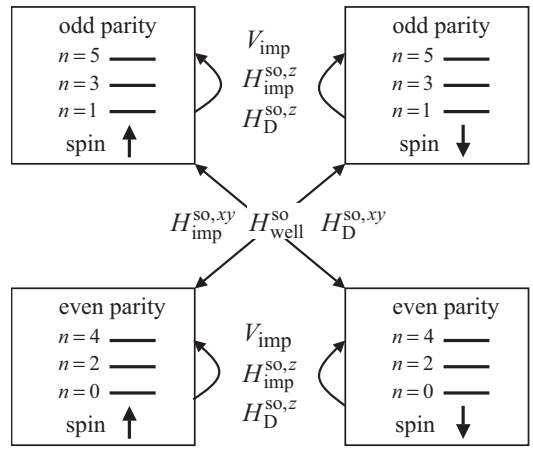

FIG. 2. Each electron state has a parity in addition to a spin (z component) since the well potential $V_{\rm well}$ is even in z. When $z_{\rm d} = 0$ [Fig. 1(a)] and the impurity potential $V_{\rm imp}$ is also even in z, some perturbation terms change the parity and the spin at the same time, while all others change neither the parity nor the spin. $H^{\rm so,xy}_{\rm imp}$ and $H^{\rm so,z}_{\rm imp}$ ($H^{\rm so,xy}_{\rm D}$ and $H^{\rm so,z}_{\rm D}$) denote terms with σ_x or σ_y and terms with σ_z, respectively, of $H^{\rm so}_{\rm imp}$ ($H^{\rm so}_{\rm D}$).

The absence of such spin-flip scatterings is illustrated in Fig. 2. First note that each wave function associated with z in a symmetric quantum well has a parity: even parity for $n =$ even and odd parity for $n =$ odd. Therefore, each electron state is characterized by the parity and the spin σ. Terms in the perturbation H_1 with $z_{\rm d} = 0$, which are odd in z and include σ_x or σ_y, change the parity and the spin at the same time, while all of the others are even in z with σ_z and change neither the parity nor the spin. Since the initial and final states of the considered spin-flip scattering processes have the same parity and the opposite spin, such processes do not occur by the perturbation H_1 with $z_{\rm d} = 0$.

IV. SPIN-FLIP SCATTERINGS IN OFF-CENTER δ DOPINGS

A. Spin-relaxation time in terms of the spin-flip scattering rate

Next we investigate the spin relaxation in the case of off-center δ dopings with $z_{\rm d} \neq 0$ [Fig. 1(b)]. In this section, we derive the formula of the spin-relaxation time, which is given in terms of the spin-flip scattering rate of electrons.

The spin polarization, or the z component of the total spin angular momentum of the 2DES, is given by

$$S_z = \sum_{n\boldsymbol{k}} \frac{\hbar}{2}(f_{n\boldsymbol{k}\uparrow} - f_{n\boldsymbol{k}\downarrow}), \tag{10}$$

in terms of the occupation probability, $f_{n\boldsymbol{k}\sigma}$, of a state with two-dimensional momentum $\boldsymbol{k}$ and spin σ in the nth subband. Our assumption that electrons occupy only the ground subband

with $n = 0$ is expressed by

$$f_{n\boldsymbol{k}\sigma} = 0 \ \ (n \geqslant 1). \tag{11}$$

We assume that $f_{0\boldsymbol{k}\sigma}$ is given by the Fermi distribution function with the spin-dependent chemical potential, μ_σ:

$$f_{0\boldsymbol{k}\sigma} = f_\sigma(\varepsilon_{0k}) = \left[\exp\left(\frac{\varepsilon_{0k} - \mu_\sigma}{k_B T}\right) + 1\right]^{-1}. \tag{12}$$

Then the spin polarization becomes

$$S_z = \frac{\hbar}{2}\int_{\varepsilon_0}^{\infty} d\varepsilon D[f_\uparrow(\varepsilon) - f_\downarrow(\varepsilon)], \tag{13}$$

where D is the constant density of states per spin of the ground subband and ε_0 is the energy at the bottom of the ground subband at $\boldsymbol{k} = 0$.

The spin polarization S_z changes at each of the spin-flip scatterings. With use of the transition rate $W_{0\boldsymbol{k}'\bar{\sigma}\leftarrow 0\boldsymbol{k}\sigma}$ of a spin-flip scattering from $0\boldsymbol{k}\sigma$ to $0\boldsymbol{k}'\bar{\sigma}$ ($\bar{\sigma}$ is the spin opposite to σ), the time derivative of S_z is

$$\frac{dS_z}{dt} = \sum_{\boldsymbol{k},\boldsymbol{k}'} \hbar(-W_{0\boldsymbol{k}'\downarrow\leftarrow 0\boldsymbol{k}\uparrow} f_{0\boldsymbol{k}\uparrow} + W_{0\boldsymbol{k}'\uparrow\leftarrow 0\boldsymbol{k}\downarrow} f_{0\boldsymbol{k}\downarrow}). \tag{14}$$

Here we define the total spin-flip scattering rate of an electron in a state $0\boldsymbol{k}\sigma$ by

$$P^{\mathrm{sf}}_{0\boldsymbol{k}\sigma} = \sum_{\boldsymbol{k}'} W_{0\boldsymbol{k}'\bar{\sigma}\leftarrow 0\boldsymbol{k}\sigma}, \tag{15}$$

and write the equation for the time derivative of S_z as

$$\frac{dS_z}{dt} = \sum_{\boldsymbol{k}} \hbar\left(-P^{\mathrm{sf}}_{0\boldsymbol{k}\uparrow} f_{0\boldsymbol{k}\uparrow} + P^{\mathrm{sf}}_{0\boldsymbol{k}\downarrow} f_{0\boldsymbol{k}\downarrow}\right). \tag{16}$$

Since $f_{0\boldsymbol{k}\sigma} = f_\sigma(\varepsilon_{0k})$, it is convenient to separate the summation with respect to $\boldsymbol{k}$ into the integration with respect to energy ε and the summation over the constant energy surface:

$$\sum_{\boldsymbol{k}} \cdots = \int_{\varepsilon_0}^{\infty} d\varepsilon \sum_{\boldsymbol{k}} \delta(\varepsilon - \varepsilon_{0\boldsymbol{k}}) \cdots . \tag{17}$$

In addition, we introduce the average of $P^{\mathrm{sf}}_{0\boldsymbol{k}\sigma}$ over the constant energy surface as

$$\bar{P}^{\mathrm{sf}}(\varepsilon) = \frac{1}{D}\sum_{\boldsymbol{k}} \delta(\varepsilon - \varepsilon_{0\boldsymbol{k}}) P^{\mathrm{sf}}_{0\boldsymbol{k}\sigma}, \tag{18}$$

which is shown to be independent of spin [43]. Equation (16), with Eqs. (17) and (18), reduces to

$$\frac{dS_z}{dt} = (-\hbar)\int_{\varepsilon_0}^{\infty} d\varepsilon D \bar{P}^{\mathrm{sf}}(\varepsilon)[f_\uparrow(\varepsilon) - f_\downarrow(\varepsilon)]. \tag{19}$$

Here we assume a degenerate 2DES satisfying $k_B T \ll \varepsilon_{\mathrm{F}} - \varepsilon_0$ (ε_{F} denotes the Fermi energy) and a small spin polarization satisfying $|\mu_\uparrow - \mu_\downarrow| \ll \varepsilon_{\mathrm{F}} - \varepsilon_0$. Then, $f_\uparrow(\varepsilon) - f_\downarrow(\varepsilon)$ is negligibly small except in the close vicinity of ε_{F}, and Eq. (19) becomes

$$\frac{dS_z}{dt} = (-\hbar)\bar{P}^{\mathrm{sf}}(\varepsilon_{\mathrm{F}})\int_{\varepsilon_0}^{\infty} d\varepsilon D[f_\uparrow(\varepsilon) - f_\downarrow(\varepsilon)] = -\frac{1}{\tau_{\mathrm{s}}} S_z, \tag{20}$$

with

$$\frac{1}{\tau_{\mathrm{s}}} = 2\bar{P}^{\mathrm{sf}}(\varepsilon_{\mathrm{F}}). \tag{21}$$

Here, τ_{s} is the spin-relaxation time. We have shown here that $1/\tau_{\mathrm{s}}$ is equal to twice the spin-flip scattering rate averaged over the Fermi surface of the 2DES [44].

B. Intrasubband and intersubband processes giving spin-flip scatterings

The transition rate appearing in the formula of the spin-flip scattering rate, given by Eq. (15), is

$$W_{0\boldsymbol{k}'\bar{\sigma}\leftarrow 0\boldsymbol{k}\sigma} = \frac{2\pi}{\hbar}|\langle 0\boldsymbol{k}'\bar{\sigma}|T|0\boldsymbol{k}\sigma\rangle|^2 \delta(\varepsilon_{0k'} - \varepsilon_{0k}). \tag{22}$$

Here, $\langle 0\boldsymbol{k}'\bar{\sigma}|T|0\boldsymbol{k}\sigma\rangle$ is the transition matrix element.

In deriving the transition matrix element, we take into account both intrasubband and intersubband processes with a spin flip by one of the SOIs, $H^{\mathrm{so}}_{\mathrm{imp}}$, $H^{\mathrm{so}}_{\mathrm{well}}$, and $H^{\mathrm{so}}_{\mathrm{D}}$. We retain terms of the transition matrix element in the lowest order both in the spin-orbit coupling strength represented by η and γ and in the impurity potential V_{imp}. A spin flip occurs due to the SOI and therefore requires at least the first order in η or in γ. From the argument in Sec. III, in order to have a spin-flip scattering process with both the initial and final states in the ground subband, we need to break the symmetry with respect to $z = 0$ by introducing V_{imp} with $z_{\mathrm{d}} \neq 0$. Therefore, the first order of V_{imp} is at least required. All of the processes, which are of the first order in the SOI and of the first order in V_{imp}, are represented in Fig. 3. The intrasubband process in Fig. 3(a) is due to $H^{\mathrm{so}}_{\mathrm{imp}}$. The intersubband processes in Figs. 3(b) and 3(c) are virtual processes through one of the excited subbands caused by $H^{\mathrm{so}}_{\mathrm{well}}$ and $H^{\mathrm{so}}_{\mathrm{D}}$, respectively, combined with V_{imp}.

The transition matrix element for a spin-flip scattering from $0\boldsymbol{k}\sigma$ to $0\boldsymbol{k}'\bar{\sigma}$ consists of three terms, each corresponding to each process in Fig. 3:

$$\langle 0\boldsymbol{k}'\bar{\sigma}|T|0\boldsymbol{k}\sigma\rangle = T^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}_{\mathrm{intra}} + T^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}_{\mathrm{inter,well}} + T^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}_{\mathrm{inter,D}}, \tag{23}$$

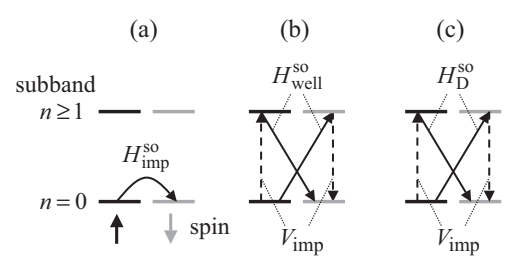

FIG. 3. (a) Intrasubband process by impurity-potential-induced SOI, $H^{\mathrm{so}}_{\mathrm{imp}}$. (b) Intersubband processes by well-potential-induced SOI, $H^{\mathrm{so}}_{\mathrm{well}}$, combined with impurity potential, V_{imp}. (c) Intersubband processes by the Dresselhaus SOI, $H^{\mathrm{so}}_{\mathrm{D}}$, combined with V_{imp}. In (b) and (c), the summation is taken over excited subbands with odd parity (n = odd).

where

$$T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = \langle 0\boldsymbol{k}'\bar{\sigma}|H_{\text{imp}}^{\text{so}}|0\boldsymbol{k}\sigma\rangle, \tag{24}$$

$$T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = \sum_{n=1,3,\ldots} \frac{\langle 0\boldsymbol{k}'\bar{\sigma}|H_{\text{well}}^{\text{so}}|n\boldsymbol{k}'\sigma\rangle\langle n\boldsymbol{k}'\sigma|V_{\text{imp}}|0\boldsymbol{k}\sigma\rangle}{\varepsilon_0-\varepsilon_n} + \sum_{n=1,3,\ldots} \frac{\langle 0\boldsymbol{k}'\bar{\sigma}|V_{\text{imp}}|n\boldsymbol{k}\bar{\sigma}\rangle\langle n\boldsymbol{k}\bar{\sigma}|H_{\text{well}}^{\text{so}}|0\boldsymbol{k}\sigma\rangle}{\varepsilon_0-\varepsilon_n}, \tag{25}$$

and $T_{\text{inter,D}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}$ is obtained by replacing $H_{\text{well}}^{\text{so}}$ in $T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}$ with H_{D}^{so}. Here the summation is taken over positive odd numbers and ε_n is the energy at the bottom of the nth subband.

We assume that the impurity potential is the sum of contributions from each impurity,

$$V_{\text{imp}}(\boldsymbol{r}) = \sum_j v(\boldsymbol{r}-\boldsymbol{r}_j), \tag{26}$$

where j labels each impurity, $\boldsymbol{r}_j = (x_j,y_j,z_j)$ is the position of the jth impurity, and $v(\boldsymbol{r})$ is the potential created by an impurity when it is located at $\boldsymbol{r}=0$. Then the intrasubband contribution becomes

$$T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = \frac{\eta}{2S}K_{1\sigma}\sum_j e^{-i\boldsymbol{q}\cdot\boldsymbol{\rho}_j}\langle 0|[\nabla_z\tilde{v}(q,z-z_j)]|0\rangle, \tag{27}$$

where $|n\rangle$ with $n=0,1,2,\ldots$ represents the wave function along the z direction of the nth subband, $\boldsymbol{\rho}_j=(x_j,y_j)$, $\boldsymbol{q}=(q_x,q_y)=\boldsymbol{k}'-\boldsymbol{k}$, S is the area of the 2DES, and

$$K_{1\sigma} = (k_y+k_y') - is_\sigma(k_x+k_x'), \tag{28}$$

with $s_\sigma = 1$ ($\sigma=\uparrow$) and $s_\sigma=-1$ ($\sigma=\downarrow$). $\tilde{v}(q,z)$ with $q=(q_x^2+q_y^2)^{1/2}$ is the two-dimensional Fourier transform of $v(\boldsymbol{r})$,

$$\tilde{v}(q,z) = \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\, e^{-i\boldsymbol{q}\cdot\boldsymbol{\rho}}\, v(\boldsymbol{r}), \tag{29}$$

with $\boldsymbol{\rho}=(x,y)$. Since $v(\boldsymbol{r})$ depends on $\boldsymbol{\rho}$ only through $|\boldsymbol{\rho}|$, its two-dimensional Fourier transform has no dependence on the direction of $\boldsymbol{q}$, and is real. The intersubband contributions are

$$T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = \frac{\eta}{S}K_{1\sigma}b_{\text{off}}\sum_j e^{-i\boldsymbol{q}\cdot\boldsymbol{\rho}_j} \times \sum_{n=1,3,\ldots}\frac{\langle 0|(\nabla_z V_{\text{well}})|n\rangle\langle n|\tilde{v}(q,z-z_j)|0\rangle}{\varepsilon_0-\varepsilon_n}, \tag{30}$$

and

$$T_{\text{inter,D}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = \frac{\gamma}{S}iK_{2\sigma}\sum_j e^{-i\boldsymbol{q}\cdot\boldsymbol{\rho}_j} \times \sum_{n=1,3,\ldots}\frac{\langle 0|\nabla_z|n\rangle\langle n|\tilde{v}(q,z-z_j)|0\rangle}{\varepsilon_0-\varepsilon_n}, \tag{31}$$

where

$$K_{2\sigma} = \frac{k_x^2-(k_x')^2}{2} + k_y^2-(k_y')^2 - 2is_\sigma(k_xk_y-k_x'k_y'). \tag{32}$$

We first consider the case of $b_{\text{off}}=1$, where $T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}$ and $T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}$ can be joined into

$$T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} + T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = \frac{\eta}{2S}K_{1\sigma}\sum_j e^{-i\boldsymbol{q}\cdot\boldsymbol{\rho}_j} \times \langle\psi_0(z,z_j)|\{\nabla_z[V_{\text{well}}+\tilde{v}(q,z-z_j)]\}|\psi_0(z,z_j)\rangle. \tag{33}$$

Here, $\psi_0(z,z_j)$ is the ground-state wave function of a fictitious Hamiltonian, which includes a fictitious potential from a single impurity at z_j, $\tilde{v}(q,z-z_j)$,

$$\left[\frac{\hat{p}_z^2}{2m} + V_{\text{well}}(z) + \tilde{v}(q,z-z_j)\right]\psi_0(z,z_j) = \tilde{\varepsilon}_0(z_j)\psi_0(z,z_j), \tag{34}$$

where $\tilde{\varepsilon}_0(z_j)$ is the corresponding energy eigenvalue. Note that the right-hand side of Eq. (33) is to be evaluated in the first order of $\tilde{v}(q,z-z_j)$. We can show that each term of the right-hand side of Eq. (33) is zero, since the average of the force induced by any potential $V(z)$ is zero when the average is taken with respect to the wave function $\psi(z)$ for each bound eigenstate of the Hamiltonian, $\hat{p}_z^2/2m+V(z)$, that is,

$$\langle\psi\,|(\nabla_z V)|\,\psi\rangle = 0. \tag{35}$$

The vanishing of $T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} + T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}$ at $b_{\text{off}}=1$ leads to its simplified formula at nonzero b_{off}:

$$T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} + T_{\text{inter,well}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma} = (1-b_{\text{off}})T_{\text{intra}}^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}. \tag{36}$$

This equation shows that the intrasubband and intersubband terms interfere destructively when $b_{\text{off}}>0$ and the interference becomes completely destructive at $b_{\text{off}}=1$. The formula of b_{off} given in the Appendix shows that b_{off} can take a value close to unity in some type-II quantum wells.

The same equality as Eq. (35) has been employed by Ando [45,46] to show that the spin splitting, linear in the in-plane momentum of the 2DES, due to the SOI is absent when the SOI is proportional to $\nabla_z V$ where $V(z)$ is the confining potential of the 2DES, even if $V(z)$ has no inversion symmetry. Later, the $\boldsymbol{k}\cdot\boldsymbol{p}$ theory developed for heterostructures [47–49] showed that the spin splitting is present when differences in the band gap and the spin-orbit splitting between the well and barrier layers are considered. This is because $b_{\text{off}}\neq 1$, in general, and therefore the combined SOI due to the band offset and the electrostatic potential is not proportional to $\nabla_z V$ (see the Appendix).

C. Spin-flip scattering rate averaged over impurity in-plane positions

In calculating the spin-flip scattering rate averaged with respect to the direction of $\boldsymbol{k}$, $\bar{P}^{\text{sf}}(\varepsilon)$, defined by Eq. (18) with Eq. (15), we perform another averaging of $\bar{P}^{\text{sf}}(\varepsilon)$ over various impurity configurations with the same doping position z_{d}. This is performed by taking the average of $|\langle 0\boldsymbol{k}'\bar{\sigma}|T|0\boldsymbol{k}\sigma\rangle|^2$ over uncorrelated in-plane positions of impurities,

$$\left(\prod_j\frac{1}{S}\int_S dx_jdy_j\right)|\langle 0\boldsymbol{k}'\bar{\sigma}|T|0\boldsymbol{k}\sigma\rangle|^2. \tag{37}$$

Then the spin-flip scattering rate $\bar{P}^{\text{sf}}(\varepsilon)$ is obtained to be

$$\bar{P}^{\text{sf}}(\varepsilon) = P_0\int_0^{2\pi}d\theta\big[(kW)^2(1+\cos\theta)t_{\text{pot}}(q,z_{\text{d}})^2 + a_{\text{D}}(kW)^4(1-\cos 2\theta)t_{\text{inter}}^{\text{D}}(q,z_{\text{d}})^2\big], \tag{38}$$

where $\varepsilon = \hbar^2 k^2/2m + \varepsilon_0$, $q = k\sqrt{2(1-\cos\theta)}$, θ is the angle of $\boldsymbol{k}'$ with respect to $\boldsymbol{k}$, and

$$P_0 = \frac{\pi}{2\hbar}\frac{(e^2/\epsilon)^2 n_{\rm imp}}{\varepsilon_0}\left(\frac{\eta}{W^2}\right)^2, \tag{39}$$

with ϵ the static dielectric constant of the semiconductor and $n_{\rm imp}$ the area density of impurities. The dimensionless parameter $a_{\rm D}$ is defined by

$$a_{\rm D} = \frac{17}{32}\left(\frac{\gamma}{\eta W \varepsilon_0}\right)^2, \tag{40}$$

with the ratio between γ, the coupling constant of the Dresselhaus SOI, and η, that of the potential-induced SOIs. The dimensionless quantity $t_{\rm pot}(q,z_{\rm d})$ comes from terms of the transition matrix element caused by the potential-induced SOIs and is given, using $T^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}_{\rm intra} + T^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}_{\rm inter,well}$ in Eq. (36), by

$$t_{\rm pot}(q,z_{\rm d}) = \frac{\epsilon}{2e^2}\langle 0|[\nabla_z \tilde{v}(q,z-z_{\rm d})]|0\rangle(1-b_{\rm off}). \tag{41}$$

On the other hand, $t^{\rm D}_{\rm inter}(q,z_{\rm d})$ is the contribution of the intersubband process due to $H^{\rm so}_{\rm D}$ and is given, from $T^{\boldsymbol{k}'\bar{\sigma}\boldsymbol{k}\sigma}_{\rm inter,D}$ in Eq. (31), by

$$t^{\rm D}_{\rm inter}(q,z_{\rm d}) = \varepsilon_0\frac{\epsilon}{e^2}\sum_{n=1,3,\ldots}\frac{\langle 0|\nabla_z|n\rangle\langle n|\tilde{v}(q,z-z_{\rm d})|0\rangle}{\varepsilon_0-\varepsilon_n}. \tag{42}$$

We simplify the calculation of $t_{\rm pot}(q,z_{\rm d})$ and $t^{\rm D}_{\rm inter}(q,z_{\rm d})$ by taking the limit [50] of $V_0 \to \infty$, which gives

$$\varepsilon_n = \frac{\hbar^2}{2m}\left[\frac{(n+1)\pi}{W}\right]^2, \tag{43}$$

and

$$\langle 0|\nabla_z|n\rangle = -\frac{4}{W}\frac{n+1}{(n+1)^2-1} \quad (n = \text{odd}). \tag{44}$$

The potential of each donor impurity, $v(\boldsymbol{r})$, is modeled by a screened Coulomb potential,

$$v(\boldsymbol{r}) = -\frac{e^2}{\epsilon r}\exp(-k_{\rm s} r), \tag{45}$$

where $r = |\boldsymbol{r}|$ and $k_{\rm s}$ is the inverse of the screening length. Its two-dimensional Fourier transform is

$$\tilde{v}(q,z) = -\frac{2\pi e^2}{\epsilon Q}\exp(-Q|z|), \tag{46}$$

with $Q = (q^2 + k_{\rm s}^2)^{1/2}$.

D. Calculated spin-flip scattering rate as a function of the impurity distribution

We present the spin-flip scattering rate calculated for a quantum well made of GaAs and $Al_{0.4}Ga_{0.6}As$ with the width $W = 75$ Å, as in the sample employed in the measurement by Ohno *et al.* [28]. We use the following values of parameters for GaAs: $\gamma = 27.5$ eV Å^3 (Table III of Ref. [51]) and $m = 0.067m_0$ with m_0 the electron rest mass. We obtain the value of $b_{\rm off}$ in Eq. (A9) and that of η in Eq. (A5) to be $b_{\rm off} = -0.82$ for GaAs/$Al_{0.4}Ga_{0.6}As$ quantum well and $\eta = 5.28$ Å^2 for GaAs by using the band parameters [52] of GaAs, AlAs, AlGaAs, and GaAs/AlAs. By substituting the values of γ, m, and η with $W = 75$ Å into Eq. (40), we obtain $a_{\rm D} = 0.26$.

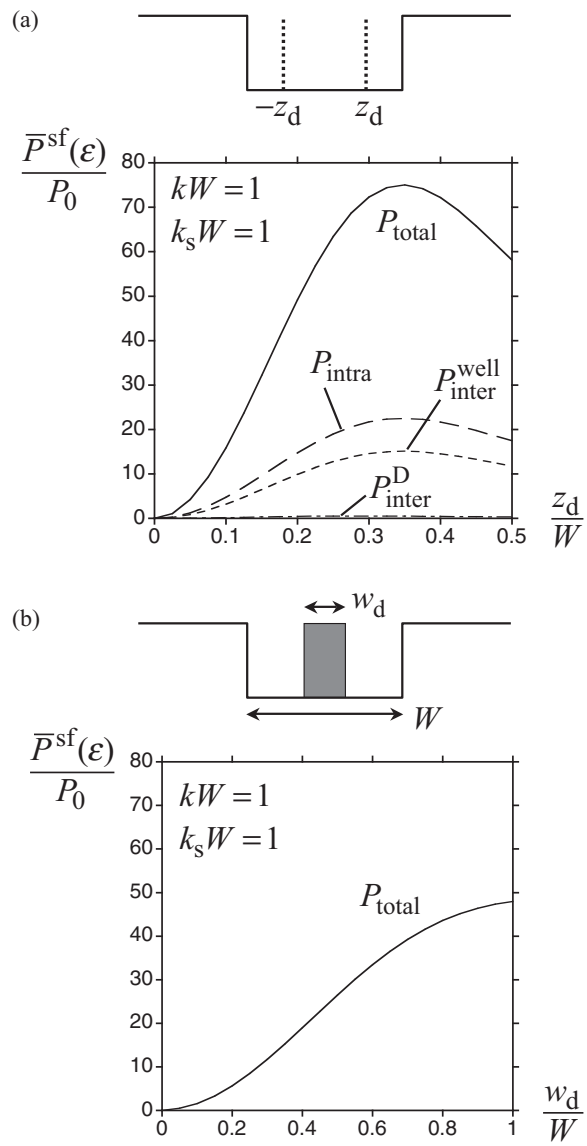

FIG. 4. Spin-flip scattering rate $\bar{P}^{\rm sf}(\varepsilon)$ divided by P_0 [Eq. (39)]. $k = |\boldsymbol{k}|$ with $\boldsymbol{k} = (k_x, k_y)$ and $k_{\rm s}$ is the inverse of the screening length. (a) Dependence on the position of δ doping, $z_{\rm d}$. Contributions from each spin-flip scattering process in Fig. 3 are also shown. (b) Dependence on the width of the doped layer, $w_{\rm d}$, with all three processes in Fig. 3 considered.

Figure 4(a) presents the calculated $z_{\rm d}$ dependence of $\bar{P}^{\rm sf}(\varepsilon)$ for $kW = 1$ and $k_{\rm s}W = 1$, where $k_{\rm F}W = 1$ with $W = 75$ Å corresponds to the electron density of 2.8×10^{11} cm^{-2}. Each of the curves labeled $P_{\rm intra}$, $P^{\rm well}_{\rm inter}$, and $P^{\rm D}_{\rm inter}$ shows the value of $\bar{P}^{\rm sf}(\varepsilon)$ when one of the processes, (a)–(c), respectively, in Fig. 3, is considered. $P_{\rm intra}$, the intrasubband contribution, and $P^{\rm well}_{\rm inter} = b^2_{\rm off}P_{\rm intra}$, the intersubband contribution due to $H^{\rm so}_{\rm well}$, are comparable in magnitude since $b_{\rm off} = -0.82$. $P^{\rm D}_{\rm inter}$, the intersubband contribution due to $H^{\rm so}_{\rm D}$, is about 0.03 of $P^{\rm well}_{\rm inter}$ in magnitude at the maximum. The value of $\bar{P}^{\rm sf}(\varepsilon)$ when all three processes are considered is also plotted in Fig. 4(a) as a curve labeled $P_{\rm total}$. Since $b_{\rm off}$ is negative, the intrasubband

and intersubband terms of $t_{\rm pot}(q,z_{\rm d})$ interfere constructively and therefore $P_{\rm total}$ in Fig. 4(a) is nearly four times larger than each of $P_{\rm intra}$ and $P_{\rm inter}^{\rm well}$. $\bar{P}^{\rm sf}(\varepsilon)$ as a function of $z_{\rm d}$ increases in the vicinity of the well center, while it decreases near the well boundary because the expectation value and the matrix element of the screened Coulomb potential, given by Eq. (45), are reduced in magnitude.

Next we consider impurity distributions with nonzero widths: impurities are distributed uniformly within a layer in $-w_{\rm d}/2 < z < w_{\rm d}/2$. We change the width of the doped layer, $w_{\rm d}$, with the total number of impurities kept constant. Figure 4(b) shows a calculated result of $\bar{P}^{\rm sf}(\varepsilon)$ as a function of $w_{\rm d}$. $\bar{P}^{\rm sf}(\varepsilon)$ remains small for small values of $w_{\rm d}$ and increases monotonically with $w_{\rm d}$. Suppose that a possible diffusion of impurities from the δ-doped layer gives an impurity distribution over three (110) atomic layers. Then $w_{\rm d}$ is twice the atomic layer distance: $w_{\rm d} = a/\sqrt{2} = 4.0$ Å for GaAs with $a = 5.65$ Å. $\bar{P}^{\rm sf}(\varepsilon)$ for this value of $w_{\rm d}$ is found to be two orders of magnitude smaller than that for $w_{\rm d} = W$ (uniform distribution in the full width of the well) when $W = 75$ Å.

Figure 5(a) demonstrates the dependence of $\bar{P}^{\rm sf}(\varepsilon)$ on the electron momentum, k: $\bar{P}^{\rm sf}(\varepsilon)$ increases as kW becomes larger. The origin of this increase is the factor $(kW)^2$ in front of $t_{\rm pot}(q,z_{\rm d})^2$ in Eq. (38), which is partly suppressed by the k dependence of $\tilde{v}(q,z)$ in Eq. (46) through $q^2 = 2k^2(1-\cos\theta)$.

Figure 5(b) shows the dependence of $\bar{P}^{\rm sf}(\varepsilon)$ on the inverse of the screening length, $k_{\rm s}$: $\bar{P}^{\rm sf}(\varepsilon)$ decreases with $k_{\rm s}$. This comes from the $k_{\rm s}$ dependence of $\tilde{v}(q,z)$ in Eq. (46) through $Q = (q^2 + k_{\rm s}^2)^{1/2}$. $\bar{P}^{\rm sf}(\varepsilon)$ approaches a constant value as $k_{\rm s} \to 0$, since $Q \to q$ then.

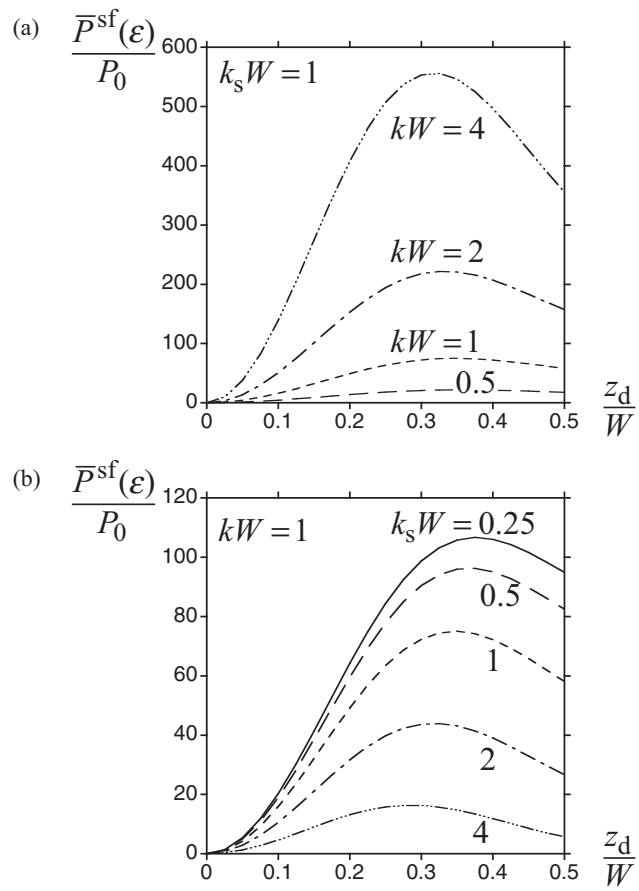

FIG. 5. Spin-flip scattering rate $\bar{P}^{\rm sf}(\varepsilon)$ divided by P_0 [Eq. (39)], as a function of the position of δ doping, $z_{\rm d}$. (a) Dependence on $k = |\boldsymbol{k}|$. (b) Dependence on $k_{\rm s}$, the inverse of the screening length.

V. CONCLUSIONS

We have theoretically investigated the dependence of the spin-relaxation rate on the impurity distribution in a zinc-blende (110) symmetric quantum well for the spin orientation perpendicular to the well, by calculating the spin-flip scattering rate. First we have considered a δ doping on the center plane of the well at $z = 0$ and shown that the symmetry with respect to $z = 0$ of the impurity potential and the well potential leads to the vanishing of all spin-flip scattering processes when only the ground subband is occupied by electrons.

Next we have considered the presence of impurities in positions deviated from the well center. We have found that the spin-flip scattering rate remains small for narrow impurity distributions centered at $z = 0$: the spin-flip scattering rate for the distribution width of 4 Å [twice the distance between adjacent GaAs (110) atomic layers] is estimated to be two orders of magnitude smaller than that for the uniform distribution over the well width of 75 Å. Since such a reduction due to the symmetry of the impurity potential does not occur in the spin current caused by the extrinsic spin Hall effect [53], we conclude theoretically that the δ doping on the center plane of the well has an advantage of achieving a larger spin polarization than the uniform doping in the well layer.

In the calculation, we have taken into account all processes, which are in the first order of the SOI and, at the same time, in the first order of the impurity potential. We have found that the intersubband spin-flip scattering process due to the well-potential-induced SOI gives a contribution comparable to the intrasubband process. In type-II quantum wells, the interference between these two processes can be destructive, which may result in a strong suppression of the spin relaxation. In type-I quantum wells made of GaAs and AlGaAs, however, these two processes interfere constructively, giving an enhanced spin-flip scattering rate, while the third contribution from the intersubband process caused by the Dresselhaus SOI makes only a negligible contribution.

APPENDIX

Here we derive the formula of $b_{\rm off}$ appearing in Eq. (6), by following the $\boldsymbol{k}\cdot\boldsymbol{p}$ theory developed for heterostructures [38,47,54]. The potential acting on an electron is due to either the band offset or the electrostatic potential. The potential due to the band offset at the interface of heterostructures depends on the band which the electron occupies. Without specifying whether it is due to the band offset or the electrostatic potential until Eq. (A3) below, we denote the potential acting on an electron in the conduction band by $V_{\rm c}(z)$, that in the heavy-hole plus light-hole bands by $V_{\rm v}(z)$, and that in the split-off band by $V_{\rm so}(z)$. The SOI for an electron in the conduction band is induced by position dependences of $V_{\rm v}(z)$ and $V_{\rm so}(z)$ through

the mixing between the conduction and valence bands by the $\boldsymbol{k}\cdot\boldsymbol{p}$ term, and is given for an electron with momentum (k_x,k_y) by [38,47,54]

$$H_{\rm so} = \frac{P^2}{3}\left[\frac{\nabla_z V_{\rm v}}{E_{\rm g}^2} - \frac{\nabla_z V_{\rm so}}{(E_{\rm g}+\Delta_{\rm so})^2}\right]\left(\sigma_x k_y - \sigma_y k_x\right), \quad \text{(A1)}$$

where $E_{\rm g}$ is the band gap and $\Delta_{\rm so}$ is the spin-orbit splitting. P is the Kane matrix element [55] given by

$$P = -i\frac{\hbar}{m_0}\langle S|\hat{p}_x|X\rangle . \quad \text{(A2)}$$

Here m_0 is the electron rest mass, while $|S\rangle$ and $|X\rangle$ are the s-type wave function at the conduction-band bottom and the p-type wave function at the valence-band top, respectively.

First consider the case of the electrostatic potential. In this case, $V_{\rm c}(z)$, $V_{\rm v}(z)$, and $V_{\rm so}(z)$ are all equal to the electrostatic potential energy $V_{\rm es}(z)$,

$$V_{\rm c}(z) = V_{\rm v}(z) = V_{\rm so}(z) = V_{\rm es}(z). \quad \text{(A3)}$$

Then Eq. (A1) becomes

$$H_{\rm so} = \eta\,(\nabla_z V_{\rm es})(\sigma_x k_y - \sigma_y k_x), \quad \text{(A4)}$$

with

$$\eta = \frac{P^2}{3}\left[\frac{1}{E_{\rm g}^2} - \frac{1}{(E_{\rm g}+\Delta_{\rm so})^2}\right], \quad \text{(A5)}$$

which gives the formula of the effective coupling constant of the SOI appearing in Eq. (5).

Next we consider the case where the potentials are due to the band offset. In a quantum well with width W,

$$\begin{aligned} V_{\rm c}(z) &= \Delta E_{\rm c} h(z) = V_{\rm well}(z),\\ V_{\rm v}(z) &= \Delta E_{\rm v} h(z),\\ V_{\rm so}(z) &= \Delta E_{\rm so} h(z), \end{aligned} \quad \text{(A6)}$$

with

$$h(z) = \begin{cases} 0 & (|z| < W/2)\\ 1 & (|z| > W/2). \end{cases} \quad \text{(A7)}$$

Here, $\Delta E_{\rm c}(=V_0)$, $\Delta E_{\rm v}$, and $\Delta E_{\rm so}$ are band offsets of the corresponding bands, defined by the offset of the energy in the barrier layers relative to that in the well layer. Introducing $E_{\rm c}$ ($E_{\rm c}^{\rm b}$) the energy of the conduction-band bottom, $E_{\rm v}$ ($E_{\rm v}^{\rm b}$) that of the valance-band top, and $E_{\rm so}$ ($E_{\rm so}^{\rm b}$) that of the split-off-band top in the well layer (the barrier layers), we have $E_{\rm g} = E_{\rm c} - E_{\rm v}$, $\Delta_{\rm so} = E_{\rm v} - E_{\rm so}$, $\Delta E_{\rm c} = E_{\rm c}^{\rm b} - E_{\rm c}$, $\Delta E_{\rm v} = E_{\rm v}^{\rm b} - E_{\rm v}$, and $\Delta E_{\rm so} = E_{\rm so}^{\rm b} - E_{\rm so}$. In this case, Eq. (A1) becomes

$$H_{\rm so} = \eta b_{\rm off}(\nabla_z V_{\rm c})(\sigma_x k_y - \sigma_y k_x), \quad \text{(A8)}$$

with the formula of $b_{\rm off}$:

$$b_{\rm off} = \frac{\Delta E_{\rm v} E_{\rm g}^{-2} - \Delta E_{\rm so}(E_{\rm g}+\Delta_{\rm so})^{-2}}{\Delta E_{\rm c}\left[E_{\rm g}^{-2} - (E_{\rm g}+\Delta_{\rm so})^{-2}\right]}. \quad \text{(A9)}$$

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[43] To show that $\bar{P}^{\rm sf}(\varepsilon)$ in Eq. (18) is independent of σ, we write

$$\sum_{\boldsymbol{k}}\delta(\varepsilon-\varepsilon_{0\boldsymbol{k}})P^{\rm sf}_{0\boldsymbol{k}\sigma}=\sum_{\boldsymbol{k},\boldsymbol{k}'}\delta(\varepsilon-\varepsilon_{0\boldsymbol{k}})W_{0\boldsymbol{k}'\bar{\sigma}\leftarrow 0\boldsymbol{k}\sigma},$$

and use the detailed balance $W_{0\boldsymbol{k}'\bar{\sigma}\leftarrow 0\boldsymbol{k}\sigma}=W_{0\boldsymbol{k}\sigma\leftarrow 0\boldsymbol{k}'\bar{\sigma}}$ and the energy conservation $\varepsilon_{0\boldsymbol{k}}=\varepsilon_{0\boldsymbol{k}'}$.

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