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Author(s)	Yakubo, K.
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Floating of extended states in a random magnetic field with a finite mean

K. Yakubo*
Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan
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Effects of a uniform magnetic field on two-dimensional (2D) electrons subject to a random magnetic field (RMF) are studied by a multifractal scaling analysis. For sufficiently strong uniform field ($\overline{B}\gg\delta b$), the RMF system is equivalent to a quantum Hall system (QHS), namely, the spectral density of states splits into subbands, and states only at the subband centers are extended with the localization-length exponent $\nu=2.31\pm0.01$, where $\overline{B}$ is the averaged magnetic field and δb is the characteristic amplitude of the spatially fluctuating field. In the case of $\overline{B}\lesssim\delta b$, subbands overlap each other and energies of extended states shift upwards with keeping its universality class. This behavior conflicts with a recent theoretical prediction and demonstrates that 2D systems in RMF's even with small finite means are rather close to QHS's.

Quantum transport and eigenstate properties of two-dimensional (2D) fermions subject to a random magnetic field (RMF) have generated great theoretical and experimental interest in recent years. This problem is deeply related to the composite-fermion picture of a half-filled Landau level.[1] Strongly correlated 2D electrons can be mapped onto a fermion gas in an effective gauge field. While this gauge field globally cancels out the external magnetic field at half-filling, the inhomogeneity of the local filling factor due to the random potential by impurities induces spatial fluctuations of the total field. Thus, one can treat correlated electrons at half-filling as noninteracting charged fermions moving in a RMF. In addition to this theoretical motivation, experimental works on direct realizations of 2D RMF systems also encourage theoretical and numerical investigations.[2–4]

Most of the previous works on 2D RMF systems have studied the case of the vanishing averaged field, i.e., $\overline{B}\equiv\int B(\mathbf{r})d\mathbf{r}/L^2=0$, because the RMF with zero mean corresponds to the composite-fermion picture of the half-filled Landau level. The key question of this problem is whether extended states exist in the system. In the early stage, except for Ref. 5, a number of numerical works found a mobility edge above which electronic states are extended.[6–10] Recent large-scale and precise simulations claim that localization lengths of electrons in the midrange of the spectrum are extremely long but finite and extended states found in previous works are actually localized in the thermodynamic limit.[11–14] This result is also supported by the analytical argument based on the nonlinear σ model.[15] There is, however, no conclusive evidence of the absence of the mobility edge, and the localization problem of this system is still controversial.

It is also intriguing to study 2D electrons in a RMF with a finite mean ($\overline{B}\neq0$) in connection with magnetotransport around half-filling. If $\overline{B}$ is much larger than δb, which is the characteristic amplitude of fluctuations of $B(\mathbf{r})$ and the correlation length ξ_B of the fluctuating magnetic field is much longer than the cyclotron radius (l_c) for $\overline{B}$, a semiclassical approximation gives a significant insight into electronic states.[16] Kalmeyer *et al.*[7] and Huckestein[17] have suggested that the RMF with $\overline{B}\gg\delta b$ is equivalent to the random scalar potential of $V(\mathbf{r})=(n+\frac{1}{2})\hbar\omega_c(\mathbf{r})$, where $\omega_c(\mathbf{r})=eb(\mathbf{r})/mc$, $B(\mathbf{r})=\overline{B}+b(\mathbf{r})$, and n is the Landau level index. This means that the RMF system with $\delta b/\overline{B}\ll1$ and $l_c/\xi_B\ll1$ behaves as a quantum Hall system (QHS), namely, the energy spectrum of the 2D RMF system takes a subband structure, states only at the subband centers are extended, and the critical behavior is described by the same localization-length exponent ν with that characterizing the quantum Hall transition.

Increasing the ratio $\delta b/\overline{B}$, effects of the subband mixing become crucial. In particular, it is quite interesting how energies of extended states move with increasing δb or decreasing $\overline{B}$, because the problem of the RMF with zero mean (i.e., $\overline{B}=0$ and $\delta b\neq0$) is regarded as the limiting case of $\delta b/\overline{B}\rightarrow\infty$. The similar problem has been vigorously studied for the quantum Hall transition. In a two-dimensional electron gas with a random scalar potential and strong uniform magnetic field, the number of extended states below the Fermi energy ($N_{\rm ext}$) increases, with decreasing the magnetic field as $N_{\rm ext}=mcE_F/\hbar eB$, if subbands are well separated from each other. The scaling theory, however, predicts that all states in 2D orthogonal systems that correspond to the limit of $B\rightarrow0$ are localized.[18] Khmelnitskii[19] and Laughlin[20] solved this paradox by clarifying that the subband mixing pushes upwards energies of extended states. This ''floating-up'' scenario has been supported by both numerical[21,22] and experimental[23–25] studies. Taking into account the equivalence between the RMF system and the QHS in the semiclassical limit, information on electronic states in the RMF with a large δb is significant to probe the nature of electrons in the RMF with $\overline{B}=0$. Chang, Yang, and Hong[26] have predicted, using a semiclassical treatment and a perturbative approach,[27] that extended states in a RMF system with a long-range correlation do not float up, even if subband mixing becomes noticeable. Their conclusion is, however, valid only if δb is small compared to $\overline{B}$ and $\xi_B\gg l_c$. The behavior of extended states is unclear when δb becomes still larger, or in the case of a short-range correlation.

The purpose of this paper is to examine numerically the behavior of extended states of 2D electrons in a short-range RMF with a finite mean. The multifractal scaling analysis[28] is employed to obtain the localization-length exponent and

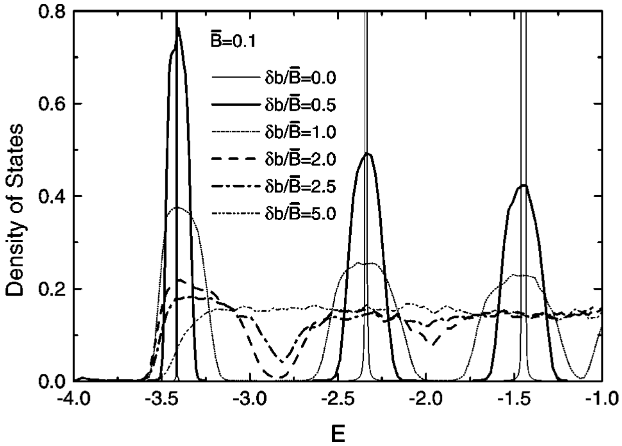

FIG. 1. Densities of states of 2D electrons in RMF's with several values of $\delta b/\bar{B}$. The system sizes are 500×500 for $\delta b=0$ and 300×300 for other values of $\delta b/\bar{B}$. $\bar{B}$ is chosen to be 0.1 for all cases.

critical energies for several values of $\delta b/\bar{B}$. This technique is based on the idea that amplitudes of the critical wave function have a multifractal distribution, and not even at the critical point but in the critical region, the amplitude distribution is also multifractal in a scale less than the correlation length ξ. We clarified that the 2DEG in a RMF with $\delta b/\bar{B} \ll 1$ is equivalent to a QHS as predicted by Kalmeyer *et al.*[7] and Huckestein.[17] For $\delta b/\bar{B} \gtrsim 1$, we found the floating up of extended states in contrast to the result by Chang, Yang, and Hong.[26]

A model of noninteracting electrons on a square lattice subject to a RMF $B(\mathbf{r})=\bar{B}+b(\mathbf{r})$ is considered, where $\bar{B}$ and $b(\mathbf{r})$ are the average field and the fluctuation around $\bar{B}$. The RMF constitutes the only type of disorder present. The system is described by the tight-binding Hamiltonian

$$H=-\sum_{\langle ij\rangle}|i\rangle t_{ij}\langle j|+\text{H.c.}, \tag{1}$$

where

$$t_{ij}=\exp\left[2\pi i\frac{\phi_{ij}}{\phi_0}\right]. \tag{2}$$

The symbol ϕ_0 is the unit of flux quanta and ϕ_{ij} is the line integral of a random vector potential along the link (ij). The magnetic flux through a plaquette is given by $\phi=\Sigma_\square \phi_{ij}$, where the summation runs over four links around the plaquette. Considering the correspondence of ϕ_{ij} to the RMF $B(\mathbf{r})=\bar{B}+b(\mathbf{r})$, in the Landau gauge, $\phi_{ij}=0$ for the link (ij) in the direction of the x axis and

$$\phi_{ij}=-\bar{B}ax_i+\delta\phi_{ij}, \tag{3}$$

in the direction parallel to the y axis. Here x_i is the x coordinate of the ith site and a is the lattice constant. The quantity $\delta\phi_{ij}$ is uniformly distributed in the range of $[-a^2\delta b, a^2\delta b]$. Hereafter we measure $\bar{B}$ and δb in units of ϕ_0/a^2. Periodic boundary conditions have been considered for both x and y directions.

Densities of states $D(E)$ are shown in Fig. 1 for several values of $\delta b/\bar{B}$, which is calculated by the forced oscillator method.[29,30] The value of $\bar{B}$ is fixed to be 0.1 for all results. The system sizes are 500×500 for $\delta b=0$ and 300×300 for other values of δb. For $\delta b/\bar{B}\ll 1$, the density of states well separates into subbands like a QHS. This has been predicted by the semiclassical theory that is valid for $\delta b/\bar{B}\ll 1$ and $\xi_B \gg l_c$.[17] Increasing $\delta b/\bar{B}$, subbands become broad, and the lowest subband starts to merge with the second subband. It should be noted that the density of states is completely symmetric around the band center $E=0$ due to the particle-hole symmetry of the RMF system without any diagonal disorder on a square lattice.

In order to examine the critically of electronic states, we analyze wave functions by the multifractal finite-size scaling method.[28] In a conventional finite-size scaling for quasi-one-dimensional systems, critical properties are obtained via a response function, and rich information involved in the amplitude distribution of a wave function is discarded. On the contrary, the present numerical technique utilizes such information to reveal the critical behavior of the metal-insulator transition (MIT). This scaling analysis is based on the fact that the spatial distribution of the squared wave function at criticality is multifractal. The multifractality is maintained not even at the critical point if the length scale is less than the correlation (or localization) length ξ. Analyzing how the multifractality changes with the length scale, one can obtain critical properties of the MIT. The amplitude distribution of wave function is characterized by the quantity F_q:

$$F_q=\ln\left[\sum_{\text{box}(l)}\left(\sum_{i\in\text{box}(l)}|\psi_i|^2\right)^q\right], \tag{4}$$

where ψ_i is the wave function at the site i, and is normalized as $\Sigma_i|\psi_i|^2=1$ (the summation is taken over all sites in the system), and q is an arbitrary constant. The symbols $\Sigma_{\text{box}(l)}$ and $\Sigma_{i\in\text{box}(l)}$ represent summations over small boxes with a linear size l into which one divides the whole system with a size L and over sites in the box, respectively. If we apply the scaling hypothesis to the amplitude distribution of the wave function near the critical point, the quantity F_q can be written as

$$F_q=f_q(L^{1/\nu}|E-E_c|, l^{1/\nu}|E-E_c|), \tag{5}$$

where f_q is a two-argument scaling function that depends on q, E_c is the critical energy, and ν is the localization-length exponent. Since the length L is always larger than l, the scaling function $f_q(x,y)$ is defined in the regime of $x\geqslant y$. Considering the multifractality of the critical wave function, the asymptotic forms of the scaling function $f_q(x,y)$ for localized states are[28]

$$f_q(x,y)\sim\begin{cases}0 & \text{if } x\gg 1 \text{ and } y\gg 1\\ \nu\tau(q)\ln y & \text{if } x\gg 1 \text{ and } y\ll 1\\ \nu\tau(q)(\ln y-\ln x) & \text{if } x\ll 1 \text{ and } y\ll 1,\end{cases} \tag{6}$$

where $\tau(q)$ (mass exponent) characterizes the multifractality as $F_q=\tau(q)\ln(l/L)$ for the critical wave function. For extended states, the asymptotic forms are given by

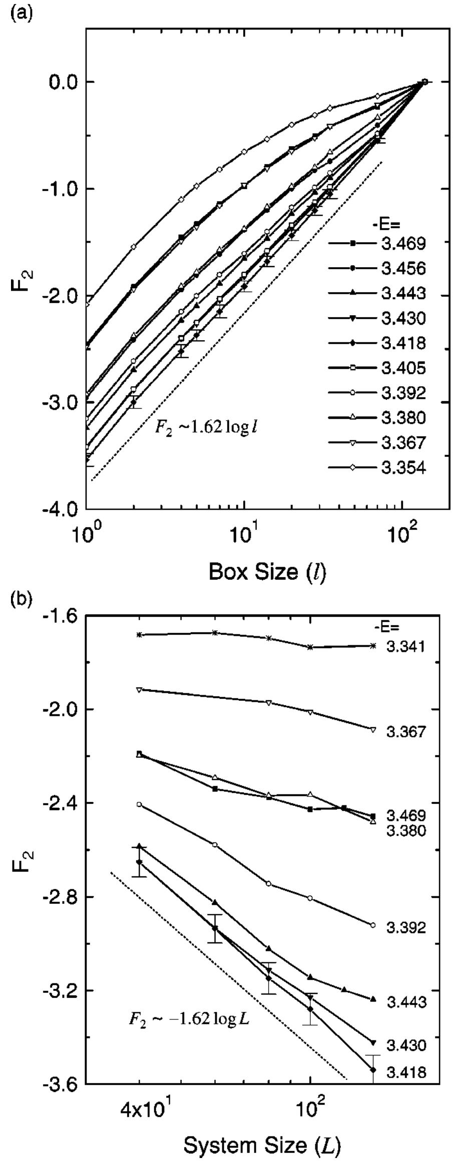

FIG. 2. (a) Box-size dependences of F_2 for several values of energies and $\delta b/\overline{B}=0.5$ with $\overline{B}=0.1$. The system size L is 140. The dotted straight line shows $F_2=1.62\ln l+c_1$, where c_1 is a constant. (b) System-size dependences of F_2 for several values of energies and $\delta b/\overline{B}=0.5$ with $\overline{B}=0.1$. The box size l is 1. The dotted straight line shows $F_2=-1.62\ln L+c_2$. Typical error bars are indicated only for $E=-3.418$ in both figures.

$$f_q(x,y) \sim \begin{cases} 2\nu(q-1)(\ln y-\ln x) & \text{if } x\gg 1 \text{ and } y\gg 1\\ \nu[2(1-q)\ln x+\tau(q)\ln y] & \text{if } x\gg 1 \text{ and } y\ll 1\\ \nu\tau(q)(\ln y-\ln x) & \text{if } x\ll 1 \text{ and } y\ll 1. \end{cases} \tag{7}$$

Using this scaling analysis, one obtains the critical energy E_c, the localization-length exponent ν, and the exponent $\tau(q)$ at the same time, while exponents ν and $\tau(q)$ are calculated separately in previous analyses. Values of ν and E_c can be calculated for an arbitrary value of q. Since these quantities do not depend on q, we can obtain ν and E_c multiply for different q's. The scaling analysis with a fixed value

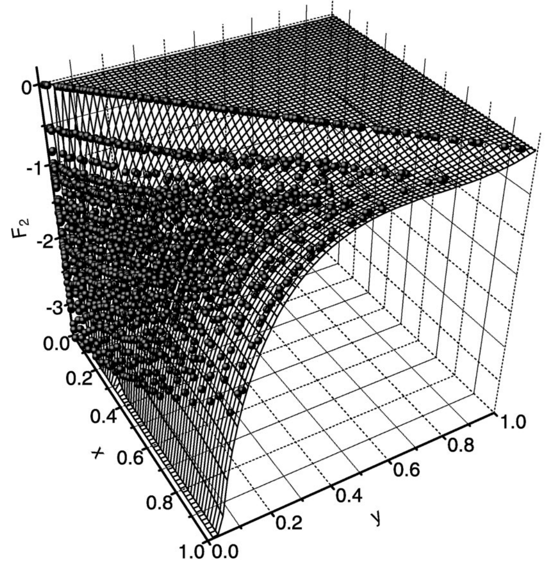

FIG. 3. The calculated scaling function $f_2(x,y)$ for $\delta b/\overline{B}=0.5$. Filled circles are rescaled data of F_2.

of q implies that a part of the amplitudes with a specific intensity is analyzed. In our scaling analysis, box sizes play a role of the scaling measure in addition to system sizes. This feature releases us from calculating many eigenstates for different system sizes. In fact we have treated only two system sizes in the case of $\delta b/\overline{B}=2.5$.

Eigenstates of the Hamiltonian Eq. (1) are calculated by the forced oscillator method.[29,30] This numerical technique enables us to compute eigenvalues and eigenvectors of very large matrices. Values of $\delta b/\overline{B}$ treated here are 0.5, 2.0, and 2.5 with fixing $\overline{B}=0.1$. For each $\delta b/\overline{B}$, we calculated eigenstates at 23 energy points in the lowest subband. The system sizes are $L=40$, 60, 80, 100, 120, and 140 for $\delta b/\overline{B}=0.5$ and 2.0, and $L=60$ and 120 for $\delta b/\overline{B}=2.5$. The quantity F_q given by Eq. (4) is averaged over 64 realizations of wave functions for $L<100$ and 32 for $L\geqslant 100$. Figure 2 shows the l and L dependences of F_2 for several energies ($\delta b/\overline{B}$ is fixed at 0.5). The l dependences shown in Fig. 2(a) are results for $L=140$. Increasing energy from the lower side of the lowest subband center, profiles of $F_2(\ln l)$ change from convex curves to a straight line and again turn to convex curves. This implies that correlation lengths of states near the subband center are long compared to those of other states. The fact that F_2 at $E=-3.418$ is almost proportional to $\ln l$ with the slope of 1.62 shows that the critical energy E_c is closed to this energy and the exponent $\tau(2)$ is 1.62. As shown in Fig. 2(b), the L dependence of F_2 also supports this estimation. In this case, $F_2\propto -1.62\ln L$ at $E=-3.418$.

Using these data of F_q, we obtain the exponent ν and the critical energy E_c by fitting F_q to the scaling function $f_q(L^{1/\nu}|E-E_c|, l^{1/\nu}|E-E_c|)$. The function $f_q(x,y)$ is expanded in powers of x and y up to the third order. Since F_q is always zero for $l=L$, we impose restriction on the expansion coefficients so that $f_q=0$ for $x=y$. The calculated scaling function $f_q(x,y)$ for $q=2$ and $\delta b/\overline{B}=0.5$ is presented in Fig. 3. Rescaled data of F_2 (filled circles in Fig. 3) collapse on a single scaling surface. The results of the scaling analysis

TABLE I. Numerical results of E_c, ν, and τ for several values of $\delta b/\overline{B}$. Quantities of χ^2, numbers of data N, and values of goodness-of-fit Q are also listed. The value of q is set to be $q=2$ for these results.

$\delta b/\overline{B}$	E_c	ν	$\tau(2)$	χ^2	N	Q
0.5	-3.41274 ± 0.00035	2.308 ± 0.013	1.614 ± 0.026	1433	1474	0.75
2.0	-3.2774 ± 0.0018	2.353 ± 0.016	1.608 ± 0.019	1499	1531	0.69
2.5	-3.2131 ± 0.0017	2.244 ± 0.068	1.609 ± 0.017	626	616	0.34

for $q=2$ are summarized in Table I. Errors, calculated by the bootstrap procedure, indicate their 95.4% confidence intervals. In the bootstrap procedure, we analyzed 10^4 synthetic data sets produced by duplicate sampling from the original data. In the case of $\delta b/\overline{B}=0.5$ for which subbands are well separated as shown in Fig. 1, the critical energy is very close to the center of the lowest subband [$E_{\rm center}\approx-3.41$, where $D(E)$ becomes maximum]. The localization-length exponent ν and the exponent $\tau(q)$ agree well with those for the integer quantum Hall transition, where $\nu=2.35\pm0.03$ (Ref. 31) and $\tau(2)=1.62\pm0.02$.[32] These results indicate that the RMF system is equivalent to the QHS when $\delta b/\overline{B}\ll1$. While this equivalency has been predicted by the semiclassical theory for long-range RMF systems,[7,17] our results show the same consequence even for short-range RMF systems.

As shown in Fig. 1, the mixing between the lowest and the second lowest subbands just starts at $\delta b/\overline{B}=2.0$. In this case the value of the critical energy E_c is clearly shifted upwards, while the exponents ν and $\tau(2)$ are nearly the same with those for $\delta b/\overline{B}=0.5$. Thus, the delocalization transition in the RMF system with $\delta b/\overline{B}\sim1$ belongs to the same universality class with that of the quantum Hall transition. Increasing further $\delta b/\overline{B}$ ($\delta b/\overline{B}=2.5$), E_c moves to still higher energy with keeping ν and $\tau(2)$. We conclude from these results that the subband mixing gives rise to the ''floating up'' of the critical energy. This conflicts with the perturbation theory justified for $\delta b/\overline{B}\lesssim1$.[26] The value of ν is somewhat small compared to those for $\delta b/\overline{B}=0.5$ and 2.0. However, the quantity Q describing the goodness-of-fit of the scaling function for $\delta b/\overline{B}=2.5$ is less than Q's for smaller $\delta b/\overline{B}$, which is presumably caused by narrowing the critical region by the subband mixing. Thus, the value of ν for $\delta b/\overline{B}=2.5$ has less reliability than those for $\delta b/\overline{B}=0.5$ and 2.0, and we suppose that the universality class does not change even if subbands merge to this extent.

We have performed scaling analyses also for several values of q. Results for $\delta b/\overline{B}=2.0$ are listed in Table II. The critical energy E_c and the exponent ν should be independent of q. In fact, E_c takes an almost constant value. On the contrary, the exponent ν decreases slightly for large values of q. However, the quantity Q decreases with q. (Note that errors in Tables I and II are estimated by the bootstrap method.) Similar tendencies appear in the case of $\delta b/\overline{B}=0.5$ and 2.5. Therefore, we believe that E_c and ν obtained for $q=2$ are the most reliable values. The exponent τ clearly depends on q. The q dependence of τ agrees well with that of the quantum Hall transition,[33,34] which supports the coincidence of the universality classes between the delocalization transition in the RMF system with a strong uniform field and the quantum Hall transition.

In conclusion, we have studied numerically 2D electronic states in a RMF with a finite mean. A scaling analysis based on the multifractality of the critical wave function reveals that (i) the RMF system with $\delta b/\overline{B}\ll1$ is equivalent to a QHS, i.e., the density of states takes a subband structure and states only at the subband centers are extended with $\nu=2.31\pm0.01$ and $\tau(2)=1.61\pm0.03$, (ii) for $\delta b/\overline{B}\gtrsim1$, energies of extended states move upwards, and (iii) the values of ν and τ do not depend on $\delta b/\overline{B}$, which means that the ''floating up'' occurs with keeping its universality class. The floating up of extended states is also found in QHS's. These results suggest that the behavior of 2D electrons in a RMF is quite similar to that in a QHS. When increasing $\delta b/\overline{B}$ much larger than 2.5, it becomes difficult to determine the values of E_c, ν and τ precisely. Thus, it remains unclear whether the similarity between RMF systems and QHS's holds even for $\delta b/\overline{B}\gg1$. In the limiting case of $\delta b/\overline{B}\to\infty$, namely, the RMF with zero mean, the value of $\tau(2)$ [equivalent to the fractal dimension D_2 of the wave function because of $\tau(q)=(q-1)D_q$] is about 1.8.[11,35] This value apparently differs from the present value, which suggests that the universality class must change at a certain value of $\delta b/\overline{B}$ ($\gg1$). Our previous work has predicted that all states are localized in the case of $\overline{B}=0$.[11] The present result that energies of the extended states shift upwards as increasing $\delta b/\overline{B}$ does not conflict with our previous conclusion, if a set of several extended states changes to that of localized ones at a large $\delta b/\overline{B}$ due to merging the Chern numbers of these floating extended states.[22,36] In order to clarify this mechanism, further investigation of the Chern number is necessary. We believe that our numerical results give significant information

TABLE II. Numerical results of E_c, ν, and τ for several values of q. Values of χ^2 and Q are also listed. The number of data for each case is 1531. The value of $\delta b/\overline{B}$ is set to be $\delta b/\overline{B}=2$ for these results.

q	E_c	ν	$\tau(q)$	χ^2	Q
1.5	-3.2766 ± 0.0019	2.335 ± 0.016	0.8426 ± 0.0071	1503	0.66
2.0	-3.2774 ± 0.0018	2.353 ± 0.016	1.608 ± 0.019	1499	0.69
2.5	-3.2778 ± 0.0017	2.344 ± 0.016	2.317 ± 0.037	1508	0.63
3.0	-3.2781 ± 0.0017	2.331 ± 0.016	2.988 ± 0.057	1520	0.55
3.5	-3.2783 ± 0.0017	2.310 ± 0.016	3.634 ± 0.080	1537	0.42
4.0	-3.2784 ± 0.0017	2.294 ± 0.016	4.26 ± 0.10	1554	0.31

about electronic states in RMF's and are useful to understand transport properties of 2D electrons in RMF's.

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*Email address: yakubo@eng.hokudai.ac.jp

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