Production spectra of the $\Sigma NN$ quasibound states in $^3\text{He}(K^-, \pi^\pm)$ reactions

Toru Harada · Yoshiharu Hirabayashi

Received: date / Accepted: date

Abstract We theoretically demonstrate the inclusive and semiexclusive spectra in the $^3\text{He}(K^-, \pi^\pm)$ reactions at 600 MeV/c ($4^\circ$) within a distorted-wave impulse approximation, using a coupled $(2N-\Lambda)+(2N-\Sigma)$ model with a spreading potential. It is shown that a signal of a $^3\Lambda$He quasibound state is clearly observed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of a $^3n$ quasibound state is relatively reduced in the $\pi^+$ spectrum. The mechanism of $\Sigma$ production for these spectra is discussed.

Keywords Sigma hypernuclei · Quasibound states · Production

1 Introduction

One of the most important subjects on strangeness nuclear physics is to understand properties of a $\Sigma$ hyperon in nuclei as well as the nature of $\Sigma N$ interaction, e.g., the $\Sigma^-$ hyperon is expected to play an essential role in the description of neutron stars [?]. Many efforts for $\Sigma$ hypernuclear studies on $s$- and $p$-shell nuclei have been carried out in $(K^-, \pi^\pm)$ reactions at CERN, BNL and KEK. However, it has been known that there is no observation of a $\Sigma$ nuclear state [?], except $^4\text{He}$, which is established to be a quasibound (or unstable bound) state experimentally [?,?], as predicted by Ref. [?]. Moreover, Saha et al. [?] reported that the $\Sigma$-nucleus potential has a strong repulsion in the real part with a sizable imaginary part, analyzing nuclear $(\pi^-, K^+)$ spectra on C, Si, Ni, In and Bi targets. This repulsion originates from the $\Sigma N^3S_1, I=3/2$ channel that corresponds to a quark Pauli-forbidden state in the baryon-baryon system [?].

Presented at the 20th International IUPAP Conference on Few-Body Problems in Physics, 20 - 25 August, 2012, Fukuoka, Japan

T. Harada
Osaka Electro-Communication University, Neyagawa, Osaka, 572-8530, Japan.
J-PARC Branch, KEK Theory Center, IPNS, KEK, Tokai, Ibaraki, 319-1106, Japan.
Tel.: +81-72-825-4584
E-mail: harada@isc.osakac.ac.jp

Y. Hirabayashi
Information Initiative Center, Hokkaido University, Sapporo, 060-0811, Japan.
On the other hand, several theoretical predictions [3, 4, 5] have suggested a possible candidate of a \( \Sigma NN \) quasibound state: Koike and Harada [3] found that there are \( \Sigma NN \) quasibound states with \( S=1/2, T=1 \) \((\frac{3}{2})^+\text{He}, \text{\frac{3}{2}}^{-}\text{H} \) and \( \frac{3}{2}^{-}\text{n} \) due to the coupling through the \( \Sigma N \) potential which strongly admixes \( J=1 \), \( I=0 \) states in the \( NN \) pair. Recently, Garciaizco et al. [4] showed that a narrow \( \Sigma NN \) quasibound state exists near \( \Sigma \) threshold in the \( S=1/2, T=1 \) channel by \( \Lambda NN-\Sigma NN \) Faddeev calculations. However, it has been recognized that there is no evidence of a narrow structure for the \( \Sigma NN \) quasibound state \((\frac{3}{2}^{-}\text{n}) \) below the \( \Sigma \) threshold by the \( ^3\text{He}(K^-, \pi^+) \) reaction at BNL-E774 experiments [5]. These contradictory arguments are still not settled: Is there a quasibound state in \( \Sigma NN \) systems?

In this article, we theoretically demonstrate the inclusive and semiexclusive spectra in \( ^3\text{He}(K^-, \pi^+) \) reactions at 600 MeV/c \((4^+ \) within a distorted-wave impulse approximation (DWIA), using a coupled \((2N-A)+(2N-\Sigma)\) model with a spreading potential. Here we focus on behavior of a signal of the \( \Sigma NN \) quasibound state in the \( \pi^- \) and \( \pi^+ \) spectra in order to study the mechanism of \( \Sigma \) production for these spectra.

2 Calculations

Now we consider hypernuclear final states in \((K^-, \pi^+)\) reactions on a \(^3\text{He} \) target, as shown in Table ???. The model wavefunctions of \(2N-Y\) systems are assumed to be written as

\[
\Psi(^3\text{He}) = \phi(pp)\psi_A + \phi(pn)\psi^{(t)}_{\Sigma^+} + \phi(np)\psi^{(s)}_{\Sigma^+} + \phi(pp)\psi_{\Sigma^0}, \tag{1}
\]

for the \( \pi^- \) spectrum, and those as

\[
\Psi(^3\text{n}) = \phi(nn)\psi_A + \phi(pn)\psi^{(t)}_{\Sigma^-} + \phi(np)\psi^{(s)}_{\Sigma^-} + \phi(np)\psi_{\Sigma^0}, \tag{2}
\]

for the \( \pi^+ \) spectrum. Here \( \phi(N_1N_2) \) and \( \phi([N_1N_2]) \) denote the \( 2N \) wavefunctions with \( J=1 \), \( I=0 \) state, respectively, and \( \theta_A \), \( \psi^{(t)}_{\Sigma^\pm} \) and \( \psi_{\Sigma^0} \) denote relative wavefunctions between \( 2N \) and \( Y \) (\( =A, \Sigma^\pm \) or \( \Sigma^0 \)), respectively.

According to the KAT theory [7], we calculate the effective \( 2N-Y \) potential which is derived from a two-body \( YN \) potential microscopically. The effective \( 2N-Y \) potential is written by

\[
\hat{U}_{ce'} = \langle \phi(c)\mid \hat{V}^{\text{ex}}\hat{F}^{\text{ex}}\mid \phi(c') \rangle, \tag{3}
\]

where \( \hat{V}^{\text{ex}}\hat{F}^{\text{ex}} \) is an external operator which is constructed from the multiple-scattering operators with \( YN \) \( g \)-matrices and on/off-shell correlation functions in nuclei [8]. In order to estimate them, we solve the Bethe-Goldstone equation for the \( YN \) system in nuclear medium, taking appropriate values of \( E_s \) and \( k_f \) parameters, so that we can reproduce the binding energies of \( B^{\text{exp}}_A(\Lambda \text{H}) = 0.13 \) MeV in experimental data.
and $B_{\Sigma}^{(3)\text{He}}$ obtained in three-body calculations [?]. For a spreading (imaginary) potential that describes $2N$-breakup processes due to the $\Sigma N \rightarrow \Lambda N$ conversion, we determine the strength of its potential to reproduce the width of $\Gamma_{\Sigma}^{\text{cal}}(\frac{3}{2}\text{He})$ [?].

Figure 1 displays real parts of the effective $2N$-$Y$ potential $\hat{U}_{cc}(R)$ for $^3\text{He} (J^{\pi} = 1/2^+)$ at $E_A=70$ MeV which corresponds to the $\Sigma$ threshold region, as a function of a relative distance $R$ between $2N$ and $Y$. Here we used the Nijmegen model F simulated (NF$_S$) for $YN$ [?], which was often used in full few-body calculations of $A= 2$-$6$ hypernuclei [?]. We find that the coupling components of \{pn\}$\Sigma^+ - \{pp\}\Sigma^0$, \{pn\}$\Sigma^+ - \{pn\}\Sigma^+$ and \{pn\}$\Sigma^+ - \{pp\}\Sigma^0$ are quite large. This nature originates from the fact that the $\Sigma N$ potential has a strong spin-isospin dependence, as suggested by recent $YN$ potential models [?].

Let us consider the production spectra of the $\Sigma NN$ quasibound states in $^3\text{He}(K^-, \pi^\pm)$ reactions. The inclusive spectrum of the double-differential cross section within the DWIA [?] is rewritten as

$$\frac{d^2\sigma}{d\Omega dE_\pi} = \beta(-\frac{1}{\pi})\text{Im} \sum_{c'c} \langle F_{c'}|\hat{G}(\omega)|F_c\rangle,$$

where $\hat{G}(\omega)$ is the complete Green’s function for the $2N$-$Y$ system, and $\beta$ is a kinematical factor for the translation from $K^- N$ to $K^- ^3\text{He}$ systems. The production function is written by $F_c = T_{\pi Y}(\chi_{\pi}^{(-)}|\phi_{c'}^-(\tau)|\Psi_A)$, where $T_{\pi Y}$ is a Fermi-averaged amplitude for $K^- N \rightarrow \pi Y$ in nuclear medium, which is obtained from the elementary amplitude by Gopal et al. [?]. $\chi_{\pi}^{(-)}$ and $\chi_{K^\pm}^{(+)}$ are meson distorted waves obtained with
the help of the eikonal approximation, and $\langle \phi(c)|\Psi_A \rangle$ is a wave function for a struck nucleon in the $^3\text{He}$ target. The recoil effects are taken into account.

The complete Green’s function $\hat{G}(\omega)$ describes all information concerning $(2N-L)+(2N-\Sigma)$ coupled-channel dynamics. We obtain it as a numerical solution of the multichannels radial coupled equations with the $2N-Y$ potential $\hat{U}$, which is written as

$$\hat{G}(\omega) = \hat{G}^{(0)}(\omega) + \hat{G}^{(0)}(\omega)\hat{U}\hat{G}(\omega),$$

(5)

where $\hat{G}^{(0)}(\omega)$ is a free Green’s function. Therefore, we evaluate the inclusive $\pi^-$ spectrum from Eq. (5), and also the semiexclusive spectra of (a)-(d) in Table ?? with the identity

$$\text{Im}\hat{G}(\omega) = \hat{\Omega}^{(-)}\{\text{Im}\hat{G}^{(0)}_A(\omega)\}\hat{\Omega}^{(-)} + \hat{\Omega}^{(-)}\{\text{Im}\hat{G}^{(0)}_{2\Sigma^\pm}(\omega)\}\hat{\Omega}^{(-)}$$

$$+ \hat{\Omega}^{(-)}\{\text{Im}\hat{G}^{(0)}_{2\Sigma^\pm}(\omega)\}\hat{\Omega}^{(-)} + \hat{G}(\omega)\{\text{Im}\hat{U}\}\hat{G}(\omega),$$

(6)

where $\hat{\Omega}^{(-)} = 1 + \hat{U}\hat{G}(\omega)$ is the Möller wave operator, and $\hat{G}^{(0)}_{2N-Y}(\omega)$ denotes the free Green’s function for the $2N-Y$ channel [?].

3 Results and discussion

Figures ?? shows the calculated spectrum of the $^3\text{He}(K^-,\pi^-)$ reaction at 600 MeV/c (4°) near the $d+\Sigma^+$ threshold, together with the components of $pp\Lambda$, $d\Sigma^+$, $pn\Sigma^+$, $pp\Sigma^0$ and $\Lambda-\Sigma$ conversion, which will be carried out at forthcoming J-PARC facilities. It is recognized that a clear enhancement just below the $d+\Sigma^+$ threshold in the $\pi^-$ spectrum is connected with dominance of the secondary process $[^3\Sigma_2^+\text{He}] \to pp\Lambda$, where
the produced Σ hyperon in the real or virtual $^{3}_n$He state subsequently interacts with a second nucleon, and it is converted to a Λ via the $ΣN→ΛN$ processes inducing 2N-nuclear breakup due to the mass difference $m_Σ − m_Λ ≃ 70$ MeV. We confirm that a pole of the quasibound state $^{3}_n$He with $S=1/2$, $T=1$ resides on the second Riemann sheet in the $Σ$ channel, and gives rise to a resonance in the $Λ$ channel. The pole position corresponds to a complex eigenvalue of the 2N-$Y$ system on the complex energy plane. This complex eigenvalue represents $E(\text{pole})^{^{3}_n\text{He}} = +1.2 − i 3.1$ MeV (7) for NF$_S$, where the real part of $E(\text{pole})^{^{3}_n\text{He}}$ is measured from the $d+Σ^−$ threshold, and its width becomes $Γ = 6.2$ MeV.

On the other hand, the $(K^−, π^+)$ reaction on a nuclear target seems to be appropriate to search a bound state in the $Σ$ bound region. The reason is because (1) this reaction can only populate a $Σ^−$ configuration in final states by the double-charge exchange reaction, so that the contribution of a $Λ$ can be removed out from the $π^+$ spectrum, and (2) it has a substitutional mechanism under the near-recoilless condition so as to produce $^{3}_n$He which belongs to a $S=1/2$ isotriplet state from the $^{3}_n$He target, as well as $^{3}_n$He. Therefore, we often expect that a signal of the corresponding peak can be clearly observed in the $π^+$ spectrum, rather than the $π^−$ one.

Figure ?? shows the calculated spectrum of the $^{3}_n\text{He}(K^−, π^+)$ reaction at 600 MeV/c $(4^°)$, together with the experimental data form BNL-E774 [?]. However, we find that no enhancement below the $d+Σ^−$ threshold is observed in the $π^+$ spectrum although there exists a quasibound state in $^{3}_n$He. The shape of the calculated spectrum seems to agree with that of the E774 data [?].

In order to understand the behavior of the $π^+$ spectrum, we discuss interference effects among configurations of the NN core states in the $Σ$ production amplitude, because the 2N-$Y$ potential should admix $^1S_0$ and $^3S_1$ states in the $NN$ pair [?].

**Fig. 3** Calculated spectrum of the $^{3}_n\text{He}(K^−, π^+)$ reaction at 600 MeV/c $(4^°)$ near the $Σ$ threshold, together with the experimental data form BNL-E774 [?]. The dashed line denotes the contribution of the $Λ-Σ^−$ conversion via the $^{3}_n$He quasibound state.
depending on the nature of the $\Sigma N$ potential. We get the production amplitude as

$$\langle (\Sigma NN)^0 | \pi^+ | 3^\text{He}K^- \rangle \approx \frac{7}{8} |^{1}_2 \langle T = 2 |^{3}_1 \text{He} \rangle + \frac{2\sqrt{3} - \sqrt{2}}{4} |^{3}_1 n |^{3}_1 \text{He} \rangle + \frac{2\sqrt{3} + \sqrt{2}}{4} |^{3}_1 n^* |^{3}_1 \text{He} \rangle \rangle,$$

where $|^{3}_1 n \rangle = \alpha | T = 1 \rangle + \beta | T = 0 \rangle$ as a ground state of $(\Sigma NN)^0$. Here we assumed $\alpha = -\beta = 1/\sqrt{2}$ for simplicity. We find that a cross section for $\frac{3}{2}^- n$ as a ground state is relatively reduced by a factor $(2\sqrt{3} - \sqrt{2})/4 = 0.51$, whereas that for $\frac{3}{2} n^*$ as an excited state is enhanced by a factor $(2\sqrt{3} + \sqrt{2})/4 = 1.22$. This mechanism is inevitable whenever we consider the $3^\text{He}(K^-, \pi^+)$ reaction, and it gives a similar spectrum to the E774 data, as seen in Fig. ??.

4 Summary

We theoretically have demonstrated the inclusive and semiexclusive spectra in the $3^\text{He}(K^-, \pi^+)$ reactions at 600 MeV/c $(4\pi)$ within the DWIA, using the coupled $(2N-A)+(2N-\Sigma)$ model with the spreading potential. The effective $2N$-$Y$ potential derived from the KAT theory has a strong spin-isospin dependence, and gives us quasibound states with $S=1/2$, $T=1$ ($\frac{3}{2}^1 \text{He}, \frac{3}{2}^1 \Sigma, \frac{1}{2}^1 n$). Our result shows that a signal of the $\frac{3}{2}^1 \text{He}$ quasibound state is clearly observed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of the $\frac{1}{2}^- n$ quasibound state is relatively reduced in the $\pi^+$ spectrum because of the admixture of the $\frac{3}{2}^1 S_0$ and $\frac{3}{2}^1 S_1$ states in the $NN$ pair, as seen in the BNL-E774 data. We believe that the $\pi^-$ and $\pi^+$ spectra on the $3^\text{He}$ target provide valuable information on properties of $\Sigma NN$ quasibound states so as to study $\Sigma N$ interaction. This investigation is in progress.

Acknowledgements The authors would like to thank T. Fukuda and Y. Akaishi for many valuable discussions. This work was supported by Grants-in-Aid for Scientific Research (C) (No. 22540294).

References