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Production spectra of the ΣNN quasibound states in ^{3}He(K^-, $\pi^\mp$) reactions

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Abstract We theoretically demonstrate the inclusive and semiexclusive spectra in the ^{3}He(K^-, $\pi^\mp$) reactions at 600 MeV/c (4°) within a distorted-wave impulse approximation, using a coupled ($2N$-Λ)+($2N$-Σ) model with a spreading potential. It is shown that a signal of a $^3_\Sigma$He quasibound state is clearly observed near the Σ threshold in the π^- spectrum, whereas a peak of a $^3_\Sigma$n quasibound state is relatively reduced in the π^+ spectrum. The mechanism of Σ production for these spectra is discussed.

Keywords Sigma hypernuclei · Quasibound states · Production

1 Introduction

One of the most important subjects on strangeness nuclear physics is to understand properties of a Σ hyperon in nuclei as well as the nature of ΣN interaction, e.g., the Σ^- hyperon is expected to play an essential role in the description of neutron stars [?]. Many efforts for Σ hypernuclear studies on s- and p-shell nuclei have been carried out in (K^-, $\pi^\mp$) reactions at CERN, BNL and KEK. However, it has been known that there is no observation of a Σ nuclear state [?], except $^4_\Sigma$He, which is established to be a quasibound (or unstable bound) state experimentally [?,?], as predicted by Ref. [?]. Moreover, Saha *et al.* [?] reported that the Σ-nucleus potential has a strong repulsion in the real part with a sizable imaginary part, analyzing nuclear (π^-, K^+) spectra on C, Si, Ni, In and Bi targets. This repulsion originates from the ΣN $^3S_1$, I=3/2 channel that corresponds to a quark Pauli-forbidden state in the baryon-baryon system [?].

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Table 1 Hypernuclear final states in $(K^-,\pi^\mp)$ reactions on a ^{3}He target

Reactions	(a)	(b)	(c)	(d)
(K^-, π^-)	$pp\Lambda$	$d\Sigma^+$	$pn\Sigma^+$	$pp\Sigma^0$
(K^-, π^+)	$nn\Lambda$	$d\Sigma^-$	$pn\Sigma^-$	$nn\Sigma^0$

On the other hand, several theoretical predictions [**?**,**?**,**?**] have suggested a possible candidate of a ΣNN quasibound state: Koike and Harada [**?**] found that there are ΣNN quasibound states with S=1/2, T=1 ($^3_\Sigma$He, $^3_\Sigma$H and $^3_\Sigma$n) due to the coupling through the ΣN potential which strongly admixes $^1S_0$, I=1 and $^3S_1$, I=0 states in the NN pair. Recently, Garcilazo *et al.* [**?**] showed that a narrow ΣNN quasibound state exists near Σ threshold in the S=1/2, T=1 channel by ΛNN-ΣNN Faddeev calculations. However, it has long been recognized that there is no evidence of a narrow structure for the ΣNN quasibound state ($^3_\Sigma$n) below the Σ threshold by the ^{3}He(K^-, π^+) reaction at BNL-E774 experiments [**?**]. These contradictory arguments are still not settled: Is there a quasibound state in ΣNN systems?

In this article, we theoretically demonstrate the inclusive and semiexclusive spectra in ^{3}He(K^-, $\pi^\mp$) reactions at 600 MeV/c (4°) within a distorted-wave impulse approximation (DWIA), using a coupled ($2N$-Λ)+($2N$-Σ) model with a spreading potential. Here we focus on behavior of a signal of the ΣNN quasibound state in the π^- and π^+ spectra in order to study the mechanism of Σ production for these spectra.

2 Calculations

Now we consider hypernuclear final states in $(K^-,\pi^\mp)$ reactions on a ^{3}He target, as shown in Table **??**. The model wavefunctions of $2N$-Y systems are assumed to be written as

$$\Psi(^3_Y\mathrm{He}) = \phi(\{pp\})\varphi_\Lambda + \phi([pn])\varphi^{(t)}_{\Sigma^+} + \phi(\{pn\})\varphi^{(s)}_{\Sigma^+} + \phi(\{pp\})\varphi_{\Sigma^0}, \qquad (1)$$

for the π^- spectrum, and those as

$$\Psi(^3_Y\mathrm{n}) = \phi(\{nn\})\varphi_\Lambda + \phi([pn])\varphi^{(t)}_{\Sigma^-} + \phi(\{pn\})\varphi^{(s)}_{\Sigma^-} + \phi(\{nn\})\varphi_{\Sigma^0}, \qquad (2)$$

for the π^+ spectrum. Here $\phi(\{N_1N_2\})$ and $\phi([N_1N_2])$ denote the $2N$ wavefunctions with $^1S_0$, I=1 and $^3S_1$, I=0 state, respectively, and φ_Λ, $\varphi^{(t,s)}_{\Sigma^\pm}$ and φ_{Σ^0} denote relative wavefunctions between $2N$ and Y (=Λ, $\Sigma^\pm$ or Σ^0), respectively.

According to the KAT theory [**?**], we calculate the effective $2N$-Y potential which is derived from a two-body YN potential microscopically. The effective $2N$-Y potential is written by

$$\hat{U}_{cc'} = \langle\phi(c)|\hat{V}^{\mathrm{ex}}\hat{F}^{\mathrm{ex}}|\phi(c')\rangle, \qquad (3)$$

where $\hat{V}^{\mathrm{ex}}\hat{F}^{\mathrm{ex}}$ is an external operator which is constructed from the multiple-scattering operators with YN g-matrices and on/off-shell correlation functions in nuclei [**?**]. In order to estimate them, we solve the Bethe-Goldstone equation for the YN system in nuclear medium, taking appropriate values of Es and k_f parameters, so that we can reproduce the binding energies of $B^{\mathrm{exp}}_\Lambda(^3_\Lambda\mathrm{H})$= 0.13 MeV in experimental data

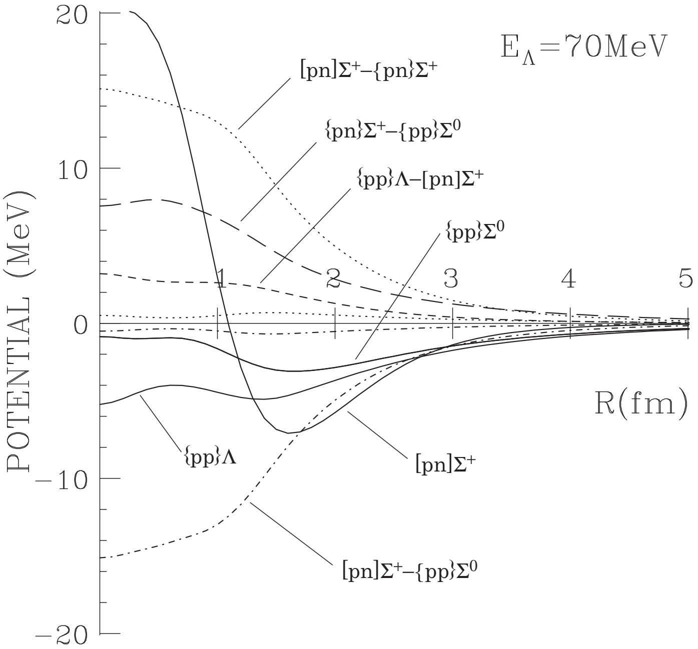

Fig. 1 Real parts of the effective $2N$-Y potential $\hat{U}_{cc'}(R)$ for ${}^{3}_{Y}$He ($J^{\pi}= 1/2^{+}$) at E_{Λ}=70 MeV which corresponds to the Σ threshold region, as a function of a relative distance R between $2N$ and Y.

and $B^{\rm cal}_{\Sigma}({}^{3}_{\Sigma}{\rm He})$ obtained in three-body calculations [?]. For a spreading (imaginary) potential that describes $2N$-breakup processes due to the $\Sigma N \to \Lambda N$ conversion, we determine the strength of its potential to reproduce the width of $\Gamma^{\rm cal}_{\Sigma}({}^{3}_{\Sigma}{\rm He})$ [?].

Figure ?? displays real parts of the effective $2N$-Y potential $\hat{U}_{cc'}(R)$ for ${}^{3}_{Y}$He(J^{π}= $1/2^{+}$) at E_{Λ}=70 MeV which corresponds to the Σ threshold region, as a function of a relative distance R between $2N$ and Y. Here we used the Nijmegen model F simulated (NF$_S$) for YN [?], which was often used in full few-body calculations of A= 2-6 hypernuclei [?]. We find that the coupling components of $\{pn\}\Sigma^{+}$-$\{pp\}\Sigma^{0}$, $[pn]\Sigma^{+}$-$\{pn\}\Sigma^{+}$ and $[pn]\Sigma^{+}$-$\{pp\}\Sigma^{0}$ are quite large. This nature originates from the fact that the ΣN potential has a strong spin-isospin dependence, as suggested by recent YN potential models [?].

Let us consider the production spectra of the ΣNN quasibound states in ^{3}He(K^{-}, $\pi^{\mp}$) reactions. The inclusive spectrum of the double-differential cross section within the DWIA [?] is rewritten as

$$\frac{d^2\sigma}{d\Omega_\pi dE_\pi} = \beta(-\frac{1}{\pi}){\rm Im}\sum_{c'c}\langle F_{c'}|\hat{G}(\omega)|F_c\rangle, \qquad (4)$$

where $\hat{G}(\omega)$ is the complete Green's function for the $2N$-Y system, and β is a kinematical factor for the translation from K^{-}-N to K^{-}-^{3}He systems. The production function is written by $F_c = \overline{f}_{\pi Y}(\chi^{(-)}_{\pi})^{*}\chi^{(+)}_{K^-}\langle\phi(c)|\Psi_A\rangle$, where $\overline{f}_{\pi Y}$ is a Fermi-averaged amplitude for $K^{-}N{\to}\pi Y$ in nuclear medium, which is obtained from the elementary amplitude by Gopal *et al.* [?], $\chi^{(-)}_{\pi}$ and $\chi^{(+)}_{K^-}$ are meson distorted waves obtained with

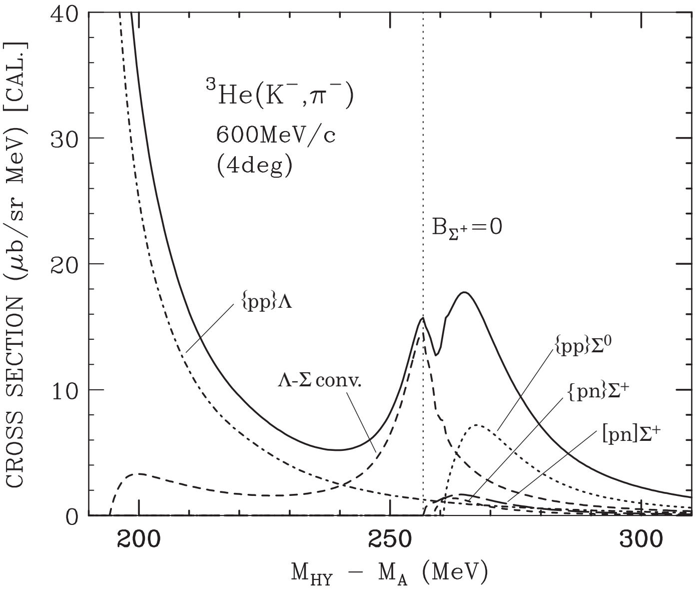

Fig. 2 Calculated spectrum of the ^{3}He(K^-, π^-) reaction at 600 MeV/c (4°) near the d+Σ^+ threshold, together with the contributions of $NN\Lambda$, $NN\Sigma$ and Λ-Σ conversion.

the help of the eikonal approximation, and $\langle \phi(c)|\Psi_A\rangle$ is a wave function for a struck nucleon in the ^{3}He target. The recoil effects are taken into account.

The complete Green's function $\hat{G}(\omega)$ describes all information concerning (2N-Λ)+(2N-Σ) coupled-channel dynamics. We obtain it as a numerical solution of the multichannels radial coupled equations with the 2N-Y potential $\hat{U}$, which is written as

$$\hat{G}(\omega) = \hat{G}^{(0)}(\omega) + \hat{G}^{(0)}(\omega)\hat{U}\hat{G}(\omega), \tag{5}$$

where $\hat{G}^{(0)}(\omega)$ is a free Green's function. Therefore, we evaluate the inclusive π^- spectrum from Eq. (**??**), and also the semiexclusive spectra of (a)-(d) in Table **??** with the identity

$$\begin{aligned}\mathrm{Im}\hat{G}(\omega) = {} & \hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\hat{G}^{(0)}_{\Lambda}(\omega)\}\hat{\Omega}^{(-)} + \hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\hat{G}^{(0)}_{\Sigma^\pm}(\omega)\}\hat{\Omega}^{(-)} \\ & + \hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\hat{G}^{(0)}_{\Sigma^0}(\omega)\}\hat{\Omega}^{(-)} + \hat{G}(\omega)\{\mathrm{Im}\hat{U}\}\hat{G}(\omega),\end{aligned} \tag{6}$$

where $\hat{\Omega}^{(-)} = 1 + \hat{U}\hat{G}(\omega)$ is the Möller wave operator, and $\hat{G}^{(0)}_{Y}(\omega)$ denotes the free Green's function for the 2N-Y channel [**?**].

3 Results and discussion

Figures **??** shows the calculated spectrum of the ^{3}He(K^-, π^-) reaction at 600 MeV/c (4°) near the d+Σ^+ threshold, together with the components of $pp\Lambda$, $d\Sigma^+$, $pn\Sigma^+$, $pp\Sigma^0$ and Λ-Σ conversion, which will be carried out at forthcoming J-PARC facilities. It is recognized that a clear enhancement just below the d+Σ^+ threshold in the π^- spectrum is connected with dominance of the secondary process $[^3_\Sigma\mathrm{He}] \to pp\Lambda$, where

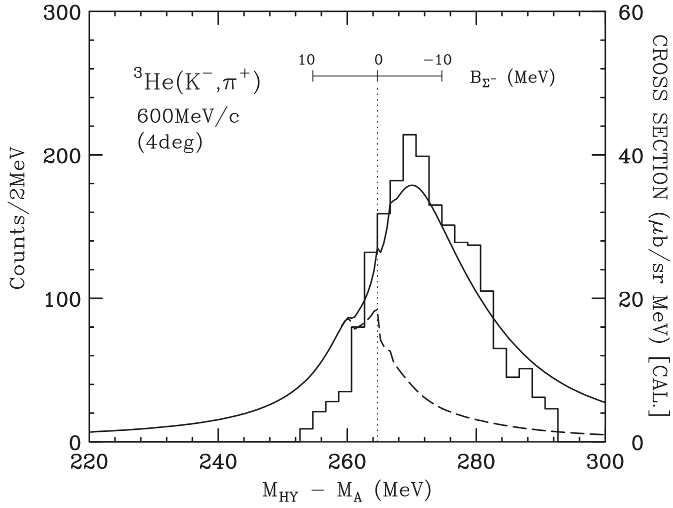

Fig. 3 Calculated spectrum of the ^{3}He(K^-, π^+) reaction at 600 MeV/c (4°) near the Σ threshold, together with the experimental data form BNL-E774 [?]. The dashed line denotes the contribution of the Λ-Σ conversion via the ${}^3_\Sigma$n quasibound state.

the produced Σ hyperon in the real or virtual ${}^3_\Sigma$He state subsequently interacts with a second nucleon, and it is converted to a Λ via the $\Sigma N \to \Lambda N$ processes inducing $2N$-nuclear breakup due to the mass difference $m_\Sigma - m_\Lambda \simeq 70$ MeV. We confirm that a pole of the quasibound state ${}^3_\Sigma$He with S=1/2, T= 1 resides on the second Riemann sheet in the Σ channel, and gives rise to a resonance in the Λ channel. The pole position corresponds to a complex eigenvalue of the $2N$-Y system on the complex energy plane. This complex eigenvalue represents

$$E^{(pole)}_{\Sigma^+}({}^3_\Sigma\mathrm{He}) = +1.2 - i\,3.1 \text{ MeV} \tag{7}$$

for NF$_S$, where the real part of $E^{(pole)}_{\Sigma^+}$ is measured from the d+Σ^+ threshold, and its width becomes Γ= 6.2 MeV.

On the other hand, the (K^-, π^+) reaction on a nuclear target seems to be appropriate to search a bound state in the Σ bound region. The reason is because (1) this reaction can only populate a Σ^- configuration in final states by the double-charge exchange reaction, so that the contribution of a Λ can be removed out from the π^+ spectrum, and (2) it has a substitutional mechanism under the near-recoilless condition so as to produce ${}^3_\Sigma$n which belongs to a S=1/2 isotriplet state from the ^{3}He target, as well as ${}^3_\Sigma$He. Therefore, we often expect that a signal of the corresponding peak can be clearly observed in the π^+ spectrum, rather than the π^- one.

Figure **??** shows the calculated spectrum of the ^{3}He(K^-, π^+) reaction at 600 MeV/c (4°), together with the experimental data form BNL-E774 [?]. However, we find that no enhancement below the d+Σ^- threshold is observed in the π^+ spectrum although there exists a quasibound state in ${}^3_\Sigma$n. The shape of the calculated spectrum seems to agree with that of the E774 data [?].

In order to understand the behavior of the π^+ spectrum, we discuss interference effects among configurations of the NN core states in the Σ production amplitude, because the $2N$-Y potential should admix $^1S_0$ and $^3S_1$ states in the NN pair [?], depending on the nature of the ΣN potential. We get the production amplitude as

$$\langle(\Sigma NN)^0\pi^+|T|^3\mathrm{He}K^-\rangle$$
$$\simeq \overline{f}_{\pi^+\Sigma^-}\Big(\frac{1}{2}\langle T=2|^3\mathrm{He}\rangle + \frac{2\sqrt{3}-\sqrt{2}}{4}\langle{}^3_\Sigma\mathrm{n}|^3\mathrm{He}\rangle + \frac{2\sqrt{3}+\sqrt{2}}{4}\langle{}^3_\Sigma\mathrm{n}^*|^3\mathrm{He}\rangle\Big), \quad (8)$$

where $|{}^3_\Sigma\mathrm{n}\rangle = \alpha|T=1^{(s)}\rangle + \beta|T=1^{(t)}\rangle$ as a ground state of $(\Sigma NN)^0$. Here we assumed $\alpha= -\beta= 1/\sqrt{2}$ for simplicity [?]. We find that a cross section for ${}^3_\Sigma$n as a ground state is relatively reduced by a factor $(2\sqrt{3}-\sqrt{2})/4=$ 0.51, whereas that for ${}^3_\Sigma\mathrm{n}^*$ as an excited state is enhanced by a factor $(2\sqrt{3}+\sqrt{2})/4=$ 1.22. This mechanism is inevitable whenever we consider the ${}^3\mathrm{He}(K^-,\pi^+)$ reaction, and it gives a similar spectrum to the E774 data, as seen in Fig. **??**.

4 Summary

We theoretically have demonstrated the inclusive and semiexclusive spectra in the ${}^3\mathrm{He}(K^-, \pi^\mp)$ reactions at 600 MeV/c (4°) within the DWIA, using the coupled $(2N$-$\Lambda)+(2N$-$\Sigma)$ model with the spreading potential. The effective $2N$-Y potential derived from the KAT theory has a strong spin-isospin dependence, and gives us quasibound states with S=1/2, T=1 (${}^3_\Sigma$He, ${}^3_\Sigma$H, ${}^3_\Sigma$n). Our result shows that a signal of the ${}^3_\Sigma$He quasibound state is clearly observed near the Σ threshold in the π^- spectrum, whereas a peak of the ${}^3_\Sigma$n quasibound state is relatively reduced in the π^+ spectrum because of the admixture of the ${}^1S_0$ and ${}^3S_1$ states in the NN pair, as seen in the BNL-E774 data. We believe that the π^- and π^+ spectra on the ^{3}He target provide valuable information on properties of ΣNN quasibound states so as to study ΣN interaction. This investigation is in progress.

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