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Few-body systems, 54(7-10): 1205-1209

2013-08

http://hdl.handle.net/2115/56641

The final publication is available at link.springer.com.

article (author version)

fb20r2_harada.pdf

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Production spectra of the $\Sigma NN$ quasibound states in $^3\text{He}(K^-, \pi^\mp)$ reactions

Toru Harada · Yoshiharu Hirabayashi

Abstract We theoretically demonstrate the inclusive and semiexclusive spectra in the $^3\text{He}(K^-, \pi^\mp)$ reactions at 600 MeV/c ($4^\circ$) within a distorted-wave impulse approximation, using a coupled ($2N$-$\Lambda$)+($2N$-$\Sigma$) model with a spreading potential. It is shown that a signal of a $\frac{3}{2}$He quasibound state is clearly observed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of a $\frac{1}{2}$n quasibound state is relatively reduced in the $\pi^+$ spectrum. The mechanism of $\Sigma$ production for these spectra is discussed.

Keywords Sigma hypernuclei · Quasibound states · Production

1 Introduction

One of the most important subjects on strangeness nuclear physics is to understand properties of a $\Sigma$ hyperon in nuclei as well as the nature of $\Sigma N$ interaction, e.g., the $\Sigma^-$ hyperon is expected to play an essential role in the description of neutron stars [?]. Many efforts for $\Sigma$ hypernuclear studies on $s$- and $p$-shell nuclei have been carried out in $(K^-, \pi^\mp)$ reactions at CERN, BNL and KEK. However, it has been known that there is no observation of a $\Sigma$ nuclear state [?], except $^4$He, which is established to be a quasibound (or unstable bound) state experimentally [?,?], as predicted by Ref. [?]. Moreover, Saha et al. [?] reported that the $\Sigma$-nucleus potential has a strong repulsion in the real part with a sizable imaginary part, analyzing nuclear $(\pi^-, K^+)$ spectra on C, Si, Ni, In and Bi targets. This repulsion originates from the $\Sigma N$ $^3S_1$, $I=3/2$ channel that corresponds to a quark Pauli-forbidden state in the baryon-baryon system [?].

Presented at the 20th International IUPAP Conference on Few-Body Problems in Physics, 20 - 25 August, 2012, Fukuoka, Japan

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On the other hand, several theoretical predictions [3, 4, 5] have suggested a possible candidate of a $\Sigma NN$ quasibound state: Koike and Harada [3] found that there are $\Sigma NN$ quasibound states with $S=1/2$, $T=1$ (1/2$^-_K$, 1/2$^-_H$ and 1/2$^+_n$) due to the coupling through the $\Sigma N$ potential which strongly admixes 1$^+T_0$, $I=1$ and 3$^-T_1$, $I=0$ states in the $NN$ pair. Recently, Garcielo et al. [4] showed that a narrow $\Sigma NN$ quasibound state exists near $\Sigma$ threshold in the $S=1/2$, $T=1$ channel by $\Lambda NN$-$\Sigma NN$ Faddeev calculations. However, it has long been recognized that there is no evidence of a narrow structure for the $\Sigma NN$ quasibound state (1/2$^-_n$) below the $\Sigma$ threshold by the $^3$He($K^-$, $\pi^+$) reaction at BNL-E774 experiments [5]. These contradictory arguments are still not settled: Is there a quasibound state in $\Sigma NN$ systems?

In this article, we theoretically demonstrate the inclusive and semiexclusive spectra in $^3$He($K^-$, $\pi^+$) reactions at 600 MeV/c (4°) within a distorted-wave impulse approximation (DWIA), using a coupled (2$N$-$\Lambda$)+(2$N$-$\Sigma$) model with a spreading potential. Here we focus on behavior of a signal of the $\Sigma NN$ quasibound state in the $\pi^-$ and $\pi^+$ spectra in order to study the mechanism of $\Sigma$ production for these spectra.

### 2 Calculations

Now we consider hypernuclear final states in ($K^-$, $\pi^\mp$) reactions on a $^3$He target, as shown in Table ???. The model wavefunctions of $2N$-$Y$ systems are assumed to be written as

$$
\Psi(^3\text{He}) = \phi((pp)) \varphi_A + \phi((pn)) \varphi_{\Sigma^+}^{(t)} + \phi((pn)) \varphi_{\Sigma^+}^{(s)} + \phi((pp)) \varphi_{\Sigma^0},
$$

for the $\pi^-$ spectrum, and those as

$$
\Psi(^3\text{He}) = \phi((nn)) \varphi_A + \phi((pm)) \varphi_{\Sigma^-}^{(t)} + \phi((pn)) \varphi_{\Sigma^-}^{(s)} + \phi((nn)) \varphi_{\Sigma^0},
$$

for the $\pi^+$ spectrum. Here $\phi((N_1N_2))$ and $\phi([N_1N_2])$ denote the $2N$ wavefunctions with 1$^+T_0$, $I=1$ and 3$^-T_1$, $I=0$ state, respectively, and $\varphi_A$, $\varphi_{\Sigma^\mp}$ and $\varphi_{\Sigma^0}$ denote relative wavefunctions between $2N$ and $Y$ ($=\Lambda$, $\Sigma^\pm$ or $\Sigma^0$), respectively.

According to the KAT theory [3], we calculate the effective $2N$-$Y$ potential which is derived from a two-body $YN$ potential microscopically. The effective $2N$-$Y$ potential is written by

$$
\tilde{U}_{cc'} = \langle \phi(c)|\tilde{V}^{\text{ex}}\tilde{F}^{\text{ex}}|\phi(c') \rangle,
$$

where $\tilde{V}^{\text{ex}}\tilde{F}^{\text{ex}}$ is an external operator which is constructed from the multiple-scattering operators with $YN$ $g$-matrices and on/off-shell correlation functions in nuclei [3]. In order to estimate them, we solve the Bethe-Goldstone equation for the $YN$ system in nuclear medium, taking appropriate values of $Es$ and $k_f$ parameters, so that we can reproduce the binding energies of $B^{\text{ex}}_{\Lambda}(\Lambda H) = 0.13$ MeV in experimental data.

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**Table 1** Hypernuclear final states in ($K^-$, $\pi^\mp$) reactions on a $^3$He target

<table>
<thead>
<tr>
<th>Reactions</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>($K^-$, $\pi^-$)</td>
<td>$pp\Lambda$</td>
<td>$d\Sigma^+$</td>
<td>$pn\Sigma^+$</td>
<td>$pp\Sigma^0$</td>
</tr>
<tr>
<td>($K^-$, $\pi^+$)</td>
<td>$nn\Lambda$</td>
<td>$d\Sigma^−$</td>
<td>$pn\Sigma^−$</td>
<td>$nn\Sigma^0$</td>
</tr>
</tbody>
</table>
and $B_{\Sigma}^{3\text{He}}(\Sigma N)$ obtained in three-body calculations [?]. For a spreading (imaginary) potential that describes $2N$-breakup processes due to the $\Sigma N \rightarrow \Lambda N$ conversion, we determine the strength of its potential to reproduce the width of $\Gamma^{\text{cal}}(\Sigma N)$ [?].

Figure ?? displays real parts of the effective $2N-Y$ potential $\hat{U}_{cc}(R)$ for $^3\text{He}(J^\pi = 1/2^+)$ at $E_A=70$ MeV which corresponds to the $\Sigma$ threshold region, as a function of a relative distance $R$ between $2N$ and $Y$. Here we used the Nijmegen model F simulated (NF) for $YN$ [?], which was often used in full few-body calculations of $A=2$-6 hypernuclei [?]. We find that the coupling components of $\{pn\} \Sigma^+ - \{pp\} \Sigma^0$, $\{pn\} \Sigma^+ - \{pp\} \Sigma^0$ and $\{pn\} \Sigma^+ - \{pp\} \Sigma^0$ are quite large. This nature originates from the fact that the $\Sigma N$ potential has a strong spin-isospin dependence, as suggested by recent $YN$ potential models [?].

Let us consider the production spectra of the $\Sigma NN$ quasibound states in $^3\text{He}(K^-, \pi^+)$ reactions. The inclusive spectrum of the double-differential cross section within the DWIA [?] is rewritten as

$$\frac{d^2\sigma}{d\Omega dE}\left|_{\text{inclusive}}\right. = \beta \left(-\frac{1}{\pi}\right) \text{Im} \sum_{\omega} (F_{\omega} \langle \hat{G}(\omega) | F_{\omega} \rangle),$$

where $\hat{G}(\omega)$ is the complete Green’s function for the $2N-Y$ system, and $\beta$ is a kinematical factor for the translation from $K^-N$ to $K^-\text{He}$ systems. The production function is written by $F_{\omega} = \mathcal{T}_{\pi Y}(\chi^{(\omega)}_{K^-}) \chi^{(\omega)}_{\pi} \langle \phi(c) | \Psi_A \rangle$, where $\mathcal{T}_{\pi Y}$ is a Fermi-averaged amplitude for $K^-N \rightarrow \pi Y$ in nuclear medium, which is obtained from the elementary amplitude by Gopal et al. [?]; $\chi^{(\pm)}_{K^-}$ are meson distorted waves obtained with
the help of the eikonal approximation, and \( \langle \phi(c)|\Psi_A \rangle \) is a wave function for a struck nucleon in the \(^3\)He target. The recoil effects are taken into account.

The complete Green’s function \( \hat{G}(\omega) \) describes all information concerning \((2N-A)+(2N-\Sigma)\) coupled-channel dynamics. We obtain it as a numerical solution of the multichannel radial coupled equations with the \(2N-Y\) potential \( \hat{U} \), which is written as

\[
\hat{G}(\omega) = \hat{G}^{(0)}(\omega) + \hat{G}^{(0)}(\omega) \hat{U} \hat{G}(\omega),
\]

where \( \hat{G}^{(0)}(\omega) \) is a free Green’s function. Therefore, we evaluate the inclusive \(\pi^-\) spectrum from Eq. (??), and also the semiexclusive spectra of \((a)-(d)\) in Table ?? with the identity

\[
\text{Im} \hat{G}(\omega) = \hat{\Omega}^{(-)} \dagger \{ \text{Im} \hat{G}^{(0)}_A(\omega) \} \hat{\Omega}^{(-)} + \hat{\Omega}^{(-)} \dagger \{ \text{Im} \hat{G}^{(0)}_{2N}(\omega) \} \hat{\Omega}^{(-)} + \hat{\Omega}^{(-)} \dagger \{ \text{Im} \hat{U} \} \hat{\Omega}^{(-)} + \hat{\Omega}^{(-)} \dagger \hat{G}(\omega) \{ \text{Im} \hat{U} \} \hat{G}(\omega),
\]

where \( \hat{\Omega}^{(-)} = 1 + \hat{U} \hat{G}(\omega) \) is the Möller wave operator, and \( \hat{G}^{(0)}_Y(\omega) \) denotes the free Green’s function for the \(2N-Y\) channel [?].

### 3 Results and discussion

Figures ?? shows the calculated spectrum of the \(^3\)He\((K^-, \pi^-)\) reaction at 600 MeV/c \((4^\circ)\) near the \(d+\Sigma^+\) threshold, together with the components of \(pp\Lambda\), \(d\Sigma^+\), \(pn\Sigma^+\), \(pp\Sigma^0\) and \(\Lambda-\Sigma\) conversion, which will be carried out at forthcoming J-PARC facilities. It is recognized that a clear enhancement just below the \(d+\Sigma^+\) threshold in the \(\pi^-\) spectrum is connected with dominance of the secondary process \[^3\Sigma\)He \(\rightarrow pp\Lambda\), where
Fig. 3 Calculated spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction at 600 MeV/c ($4^\circ$) near the $\Sigma$ threshold, together with the experimental data form BNL-E774 [?]. The dashed line denotes the contribution of the $\Lambda-\Sigma$ conversion via the $\frac{3}{2}^-$ states.

The produced $\Sigma$ hyperon in the real or virtual $^3\text{He}$ state subsequently interacts with a second nucleon, and it is converted to a $\Lambda$ via the $\Sigma N \rightarrow \Lambda N$ processes inducing 2N-nuclear breakup due to the mass difference $m_{\Sigma} - m_{\Lambda} \approx 70 \text{ MeV}$. We confirm that a pole of the quasibound state $^3\text{He}$ with $S = 1/2$, $T = 1$ resides on the second Riemann sheet in the $\Sigma$ channel, and gives rise to a resonance in the $\Lambda$ channel. The pole position corresponds to a complex eigenvalue of the 2N-$Y$ system on the complex energy plane. This complex eigenvalue represents

$$E_{\Sigma^+}^{(pole)}(^3\text{He}) = +1.2 - i 3.1 \text{ MeV}$$

for $\text{NF}_S$, where the real part of $E_{\Sigma^+}^{(pole)}$ is measured from the $d+\Sigma^+$ threshold, and its width becomes $\Gamma = 6.2 \text{ MeV}$.

On the other hand, the $(K^-, \pi^+)$ reaction on a nuclear target seems to be appropriate to search a bound state in the $\Sigma$ bound region. The reason is because (1) this reaction can only populate a $\Sigma^-$ configuration in final states by the double-charge exchange reaction, so that the contribution of a $\Lambda$ can be removed out from the $\pi^+$ spectrum, and (2) it has a substitutional mechanism under the near-recoilless condition so as to produce $\frac{3}{2}^-$ states which belong to a $S = 1/2$ isotriplet state from the $^3\text{He}$ target, as well as $^3\text{He}$. Therefore, we often expect that a signal of the corresponding peak can be clearly observed in the $\pi^+$ spectrum, rather than the $\pi^-$ one.

Figure ?? shows the calculated spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction at 600 MeV/c ($4^\circ$), together with the experimental data form BNL-E774 [?]. However, we find that no enhancement below the $d+\Sigma^-$ threshold is observed in the $\pi^+$ spectrum although there exists a quasibound state in $\frac{3}{2}^-$. The shape of the calculated spectrum seems to agree with that of the E774 data [?].

In order to understand the behavior of the $\pi^+$ spectrum, we discuss interference effects among configurations of the $NN$ core states in the $\Sigma$ production amplitude, because the 2N-$Y$ potential should admix $^1S_0$ and $^3S_1$ states in the $NN$ pair [?],
depending on the nature of the $\Sigma N$ potential. We get the production amplitude as

$$\langle (\Sigma NN)^0 | \pi^+ | \text{HeK}^\pm \rangle \approx \frac{1}{2}\langle T = 1 | \pi^- | \text{He} \rangle \pm \frac{2\sqrt{3} - \sqrt{2}}{4} \langle \frac{3}{2}^+ \text{He} | \pi^- | \text{He} \rangle \pm \frac{2\sqrt{3} + \sqrt{2}}{4} \langle \frac{3}{2}^- \text{He} | \pi^- | \text{He} \rangle,$$

where $\langle \frac{3}{2}^+ \text{He} | \pi^- | \text{He} \rangle = \alpha | T = 1 \rangle + \beta | T = 1 \rangle$ as a ground state of $(\Sigma NN)^0$. Here we assumed $\alpha = 1/\sqrt{2}$ for simplicity [?]. We find that a cross section for $\frac{3}{2}^- \text{He}$ as a ground state is relatively reduced by a factor $(2\sqrt{3} - \sqrt{2})/4 = 0.51$, whereas that for $\frac{3}{2}^- \text{He}$ as an excited state is enhanced by a factor $(2\sqrt{3} + \sqrt{2})/4 = 1.22$. This mechanism is inevitable whenever we consider the $\text{He}(K^-, \pi^+) \text{ reaction}$, and it gives a similar spectrum to the E774 data, as seen in Fig. ??.

4 Summary

We theoretically have demonstrated the inclusive and semiexclusive spectra in the $\text{He}(K^-, \pi^+) \text{ reactions}$ at 600 MeV/c (4$^-$) within the DWIA, using the coupled $(2N-A)+(2N-\Sigma)$ model with the spreading potential. The effective $2N-Y$ potential derived from the KAT theory has a strong spin-isospin dependence, and gives us quasibound states with $S=1/2$, $T=1$ $(\frac{3}{2}^+ \text{He}, \frac{3}{2}^- \text{He}, \frac{1}{2}^+ \text{He})$. Our result shows that a signal of the $\frac{3}{2}^- \text{He}$ quasibound state is clearly observed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of the $\frac{3}{2}^- \text{He}$ quasibound state is relatively reduced in the $\pi^+$ spectrum because of the admixture of the $^3S_0$ and $^3S_1$ states in the $NN$ pair, as seen in the BNL-E774 data. We believe that the $\pi^-$ and $\pi^+$ spectra on the $\text{He}$ target provide valuable information on properties of $\Sigma NN$ quasibound states so as to study $\Sigma N$ interaction. This investigation is in progress.

Acknowledgements The authors would like to thank T. Fukuda and Y. Akaishi for many valuable discussions. This work was supported by Grants-in-Aid for Scientific Research (C) (No. 22540294).

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