Production spectra of the $\Sigma NN$ quasibound states in $^3$He($K^-$, $\pi^\mp$) reactions

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Abstract We theoretically demonstrate the inclusive and seminclusive spectra in the $^3$He($K^-$, $\pi^\mp$) reactions at 600 MeV/c ($4^\circ$) within a distorted-wave impulse approximation, using a coupled (2N-\Lambda)+(2N-\Sigma) model with a spreading potential. It is shown that a signal of a $\frac{3}{2}^-$He quasibound state is clearly observed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of a $\frac{1}{2}^-$n quasibound state is relatively reduced in the $\pi^+$ spectrum. The mechanism of $\Sigma$ production for these spectra is discussed.

Keywords Sigma hypernuclei · Quasibound states · Production

1 Introduction

One of the most important subjects on strangeness nuclear physics is to understand properties of a $\Sigma$ hyperon in nuclei as well as the nature of $\Sigma N$ interaction, e.g., the $\Sigma^-$ hyperon is expected to play an essential role in the description of neutron stars [?]. Many efforts for $\Sigma$ hypernuclear studies on s- and p-shell nuclei have been carried out in ($K^-$, $\pi^\mp$) reactions at CERN, BNL and KEK. However, it has been known that there is no observation of a $\Sigma$ nuclear state [?], except $\frac{3}{2}^-$He, which is established to be a quasibound (or unstable bound) state experimentally [?, ?], as predicted by Ref. [?]. Moreover, Saha et al. [?] reported that the $\Sigma$-nucleus potential has a strong repulsion in the real part with a sizable imaginary part, analyzing nuclear ($\pi^-$, $K^+$) spectra on C, Si, Ni, In and Bi targets. This repulsion originates from the $\Sigma N$ $^3S_1$, $I=3/2$ channel that corresponds to a quark Pauli-forbidden state in the baryon-baryon system [?].

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On the other hand, several theoretical predictions [7,8,9] have suggested a possible candidate of a $\Sigma NN$ quasibound state: Koike and Harada [7] found that there are $\Sigma NN$ quasibound states with $S=1/2$, $T=+1 (\frac{3}{2}, -\frac{1}{2})$ and $\frac{3}{2}, \frac{1}{2}$ due to the coupling through the $\Sigma N$ potential which strongly admixes $1S_0$, $I=1$ and $3S_1$, $I=0$ states in the $NN$ pair. Recently, Garciaiz et al. [7] showed that a narrow $\Sigma NN$ quasibound state exists near $\Sigma$ threshold in the $S=1/2$, $T=1$ channel by $\Lambda NN-\Sigma NN$ Faddeev calculations. However, it has long been recognized that there is no evidence of a narrow structure for the $\Sigma NN$ quasibound state ($\frac{3}{2}, \frac{1}{2}$) below the $\Sigma$ threshold by the $\Lambda$(He($K^-$, $\pi^\pm$) He) reaction at BNL-E774 experiments [7]. These contradictory arguments are still not settled: Is there a quasibound state in $\Sigma NN$ systems?

In this article, we theoretically demonstrate the inclusive and semiexclusive spectra in $^3$He($K^-$, $\pi^\mp$) reactions at 600 MeV/c ($4\pi$) within a distorted-wave impulse approximation (DWIA), using a coupled ($2N-\Lambda$)+($2N-\Sigma$) model with a spreading potential. Here we focus on behavior of a signal of the $\Sigma NN$ quasibound state in the $\pi^-$ and $\pi^+$ spectra in order to study the mechanism of $\Sigma$ production for these spectra.

2 Calculations

Now we consider hypernuclear final states in ($K^-$, $\pi^\mp$) reactions on a $^3$He target, as shown in Table ??.

\begin{table}[h]
\centering
\begin{tabular}{|l|l|l|l|l|}
\hline
Reactions & ($a$) & ($b$) & ($c$) & ($d$) \\
\hline
($K^-$, $\pi^-$) & $pp\Lambda$ & $d\Sigma^+$ & $pn\Sigma^+$ & $pp\Sigma^0$ \\
($K^-$, $\pi^+$) & $nn\Lambda$ & $d\Sigma^-$ & $pn\Sigma^-$ & $nn\Sigma^0$ \\
\hline
\end{tabular}
\caption{Hypernuclear final states in ($K^-$, $\pi^\mp$) reactions on a $^3$He target}
\end{table}

The model wavefunctions of $2N-Y$ systems are assumed to be written as

$$
\Psi(3^3\text{He}) = \phi(pp)\varphi_A + \phi(pn)\varphi_{\Sigma^+}^{(t)} + \phi(pp)\varphi_{\Sigma^+}^{(s)} + \phi((pn))\varphi_{\Sigma^0},
$$

for the $\pi^-$ spectrum, and those as

$$
\Psi(3^3\text{n}) = \phi(nn)\varphi_A + \phi(pn)\varphi_{\Sigma^-}^{(t)} + \phi((pn))\varphi_{\Sigma^-}^{(s)} + \phi((nn))\varphi_{\Sigma^0},
$$

for the $\pi^+$ spectrum. Here $\phi(N_1N_2)$ and $\phi(N_1N_2)$ denote the $2N$ wavefunctions with $1S_0$, $I=1$ and $3S_1$, $I=0$ state, respectively, and $\varphi_A$, $\varphi_{\Sigma^\pm}^{(t,s)}$ and $\varphi_{\Sigma^0}$ denote relative wavefunctions between $2N$ and $Y$ (=A, $\Sigma^\pm$ or $\Sigma^0$), respectively.

According to the KAT theory [7], we calculate the effective $2N-Y$ potential which is derived from a two-body $YN$ potential microscopically. The effective $2N-Y$ potential is written by

$$
\hat{U}_{cc'} = \langle \phi(c) | \hat{V}_{\text{ex}} | \phi(c') \rangle,
$$

where $\hat{V}_{\text{ex}}$ is an external operator which is constructed from the multiple-scattering operators with $YN$ $g$-matrices and on/off-shell correlation functions in nuclei [7]. In order to estimate them, we solve the Bethe-Goldstone equation for the $YN$ system in nuclear medium, taking appropriate values of $E_s$ and $k_f$ parameters, so that we can reproduce the binding energies of $B_{\text{ex}}(\Lambda H) = 0.13$ MeV in experimental data.
Fig. 1 Real parts of the effective 2N-Y potential $\hat{U}_{cc}'(R)$ for $^3\text{He}$ ($J^\pi = 1/2^-$) at $E_A=70$ MeV which corresponds to the $\Sigma$ threshold region, as a function of a relative distance $R$ between $2N$ and $Y$.

and $B_{\Sigma}^{3\text{He}}(\frac{3}{2}^{-})$ obtained in three-body calculations [?]. For a spreading (imaginary) potential that describes $2N$-breakup processes due to the $\Sigma N \rightarrow \Lambda N$ conversion, we determine the strength of its potential to reproduce the width of $\Gamma^{\text{cal}}_\Sigma(\frac{3}{2}^+)$. [?]

Figure ?? displays real parts of the effective 2N-Y potential $\hat{U}_{cc}'(R)$ for $^3\text{He}(J^\pi = 1/2^+)$ at $E_A=70$ MeV which corresponds to the $\Sigma$ threshold region, as a function of a relative distance $R$ between $2N$ and $Y$. Here we used the Nijmegen model F simulated (NF$_S$) for $YN$ [?], which was often used in full few-body calculations of $A= 2-6$ hypernuclei [?]. We find that the coupling components of $\{ pn \} \Sigma^+ - \{ pp \} \Sigma^0$, $\{ pn \} \Sigma^+ - \{ pn \} \Sigma^+$ and $\{ pn \} \Sigma^+ - \{ pp \} \Sigma^0$ are quite large. This nature originates from the fact that the $\Sigma N$ potential has a strong spin-isospin dependence, as suggested by recent $YN$ potential models [?].

Let us consider the production spectra of the $\Sigma NN$ quasibound states in $^3\text{He}(K^-,\pi^\pm)$ reactions. The inclusive spectrum of the double-differential cross section within the DWIA [?] is rewritten as

$$\frac{d^2\sigma}{d\Omega dE_\pi} = \beta (-\frac{1}{\pi}) \text{Im} \sum_{c'c} \langle F_{c'} | \hat{G}(\omega) | F_c \rangle,$$

(4)

where $\hat{G}(\omega)$ is the complete Green’s function for the 2N-Y system, and $\beta$ is a kinematical factor for the translation from $K^-N$ to $K^-\text{He}$ systems. The production function is written by $F_c = F_{\pi Y}(\chi_{\pi}^{(-)} \chi_{K^-}^{(-)} \langle \phi|\Psi_{A} \rangle)$, where $F_{\pi Y}$ is a Fermi-averaged amplitude for $K^-N\rightarrow\pi Y$ in nuclear medium, which is obtained from the elementary amplitude by Gopal et al. [?], $\chi_{\pi}^{(-)}$ and $\chi_{K^-}^{(-)}$ are meson distorted waves obtained with
Fig. 2 Calculated spectrum of the $^3$He($K^-$, $\pi^-$) reaction at 600 MeV/c (4°) near the $d+\Sigma^+$ threshold, together with the contributions of $NN\Lambda$, $NN\Sigma$ and $\Lambda-\Sigma$ conversion.

3 Results and discussion

Figures ?? shows the calculated spectrum of the $^3$He($K^-$, $\pi^-$) reaction at 600 MeV/c (4°) near the $d+\Sigma^+$ threshold, together with the components of ppΛ, dΣ+, pnΣ+, ppΣ0 and $\Lambda-\Sigma$ conversion, which will be carried out at forthcoming J-PARC facilities. It is recognized that a clear enhancement just below the $d+\Sigma^+$ threshold in the $\pi^-$ spectrum is connected with dominance of the secondary process $^3\Sigma\text{He} \rightarrow pp\Lambda$, where...
the produced $\Sigma$ hyperon in the real or virtual $^3\text{He}$ state subsequently interacts with a second nucleon, and it is converted to a $\Lambda$ via the $\Sigma N \rightarrow \Lambda N$ processes inducing $2N$-nuclear breakup due to the mass difference $m_{\Sigma} - m_\Lambda \approx 70$ MeV. We confirm that a pole of the quasibound state $^3\text{He}$ with $S=1/2$, $T=1$ resides on the second Riemann sheet in the $\Sigma$ channel, and gives rise to a resonance in the $\Lambda$ channel. The pole position corresponds to a complex eigenvalue of the $2N-Y$ system on the complex energy plane. This complex eigenvalue represents

$$E_{\Sigma^+}^{(pole)}(3 \text{He}) = +1.2 - i \ 3.1 \text{ MeV} \tag{7}$$

for $NF_{SS}$, where the real part of $E_{\Sigma^+}^{(pole)}$ is measured from the $d+\Sigma^-$ threshold, and its width becomes $\Gamma = 6.2$ MeV.

On the other hand, the ($K^-, \pi^+$) reaction on a nuclear target seems to be appropriate to search a bound state in the $\Sigma$ bound region. The reason is because (1) this reaction can only populate a $\Sigma^-$ configuration in final states by the double-charge exchange reaction, so that the contribution of a $\Lambda$ can be removed out from the $\pi^+$ spectrum, and (2) it has a substitutional mechanism under the near-recoilless condition so as to produce $\frac{1}{2}\Sigma$ which belongs to a $S=1/2$ isotriplet state from the $^3\text{He}$ target, as well as $\frac{3}{2}\Sigma$. Therefore, we often expect that a signal of the corresponding peak can be clearly observed in the $\pi^+$ spectrum, rather than the $\pi^-$ one.

Figure 3 shows the calculated spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction at 600 MeV/c ($4^\circ$) near the $\Sigma$ threshold, together with the experimental data form BNL-E774 \cite{7}. However, we find that no enhancement below the $d+\Sigma^-$ threshold is observed in the $\pi^+$ spectrum although there exists a quasibound state in $\frac{1}{2}\Sigma$. The shape of the calculated spectrum seems to agree with that of the E774 data \cite{7}.

In order to understand the behavior of the $\pi^+$ spectrum, we discuss interference effects among configurations of the $NN$ core states in the $\Sigma$ production amplitude, because the $2N-Y$ potential should admix $^1S_0$ and $^3S_1$ states in the $NN$ pair \cite{7}.

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**Fig. 3** Calculated spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction at 600 MeV/c ($4^\circ$) near the $\Sigma$ threshold, together with the experimental data form BNL-E774 \cite{7}. The dashed line denotes the contribution of the $\Lambda-\Sigma$ conversion via the $^3\Sigma$ quasibound state.
depending on the nature of the $\Sigma N$ potential. We get the production amplitude as

$$
\langle (\Sigma NN)\pi^+|T|^{3}\text{He}K^- \rangle
\approx T_n+\Sigma^{-}(1/2)(T=2|^{3}\text{He}) + \frac{2\sqrt{3} - \sqrt{2}}{4} (|^{3}\Sigma n|^{3}\text{He}) + \frac{2\sqrt{3} + \sqrt{2}}{4} (|^{3}\Sigma n^*|^{3}\text{He}),
$$

where $|^{3}\Sigma n⟩ = α|T=1(\pi^-)⟩ + β|T=1(\pi^+)⟩$ as a ground state of $(\Sigma NN)^0$. Here we assumed $α = -β = 1/\sqrt{2}$ for simplicity [7]. We find that a cross section for $^{3}\Sigma n$ as a ground state is relatively reduced by a factor $(2\sqrt{3} - \sqrt{2})/4 ≈ 0.51$, whereas that for $^{3}\Sigma n^*$ as an excited state is enhanced by a factor $(2\sqrt{3} + \sqrt{2})/4 ≈ 1.22$. This mechanism is inevitable whenever we consider the $^{3}\text{He}(K^-, \pi^+)$ reaction, and it gives a similar spectrum to the E774 data, as seen in Fig. ??.

4 Summary

We theoretically have demonstrated the inclusive and semiexclusive spectra in the $^{3}\text{He}(K^-, \pi^+)$ reactions at 600 MeV/c ($4°$) within the DWIA, using the coupled (2N-A)+(2N-$\Sigma$) model with the spreading potential. The effective 2N-$Y$ potential derived from the KAT theory has a strong spin-isospin dependence, and gives us quasibound states with $S=1/2$, $T=1 (\frac{1}{2}\text{He}, \frac{1}{2}\text{H}, \frac{1}{2}\text{n})$. Our result shows that a signal of the $\frac{1}{2}\text{He}$ quasibound state is clearly observed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of the $\frac{1}{2}\text{n}$ quasibound state is relatively reduced in the $\pi^+$ spectrum because of the admixture of the $^{3}\Sigma_0$ and $^{3}\Sigma_1$ states in the $NN$ pair, as seen in the BNL-E774 data. We believe that the $\pi^-$ and $\pi^+$ spectra on the $^{3}\text{He}$ target provide valuable information on properties of $\Sigma NN$ quasibound states so as to study $\Sigma N$ interaction. This investigation is in progress.

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