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ΣNN quasibound states in He3 (K−, π∓) reactions at 600 MeV/c

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We theoretically demonstrate the inclusive and semiexclusive spectra in the $^3\text{He}(K^-, \pi^\mp)$ reactions at 600 MeV/c ($4\gamma'$) within a distorted-wave impulse approximation, using a coupled $(2N-\Lambda) + (2N-\Sigma)$ model with a spreading potential. We present the possible existence of the $\Sigma N$ quasibound state with $J^P = 1/2^+$, $T = 1$ near the $\Sigma$ threshold, predicted by a $2N-\Sigma$ folding-model potential derived from $YN$ g-matrices. The result shows that a signal of the $\frac{3}{2}^-\Sigma$ quasibound state is clearly confirmed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas a peak of the $\frac{1}{2}^-\Sigma$ quasibound state is rather reduced in the $\pi^+$ spectrum owing to the interference effects caused by the $\frac{3}{2}^-\Sigma$, $\frac{3}{2}^+\Lambda$ admixture in the $YN$ pair. The mechanism of $\Sigma$ production for these spectra and charge symmetry breaking effects are also discussed.

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1. INTRODUCTION

One of the most important subjects on strangeness nuclear physics is to understand properties of a $\Sigma$ hyperon in nuclei as well as $\Sigma N$ interaction [1]. The $\Sigma$ hyperon is expected to play an essential role in the description of the $\Lambda NN$ three-body force in hypernuclei [2], and the maximal mass and particle fraction of neutron stars or compact stars [3]. However, the $\Sigma N$ interaction has still been in quantitative ambiguities because the $\Sigma N$ scattering data are very limited [1,4].

In the 1990s, many efforts were made in $\Sigma$ hypernuclear studies on $s$- and $p$-shell nuclei using $(K^-, \pi^\mp)$ reactions at CERN, BNL, and KEK [5,6]. It has been known that there is no evidence of a $\Sigma$ nuclear state [7], except $\frac{3}{2}^-\Sigma$ which is established to be a quasibound (or unstable bound) state experimentally [8–10], as predicted in Ref. [11]. Saha et al. [12] reported that there is a strong repulsion in the real part with a sizable imaginary part of the $\Sigma$-nucleus potential analyzing nuclear $(\pi^-, K^+)$ spectra on C, Si, Ni, In, and Bi targets. This repulsion originates from the $\Sigma N \frac{3}{2}^-$, $I = 3/2$ channel that corresponds to a quark Pauli-forbidden state in baryon-baryon systems in the flavor SU(6) symmetry [13].

Several theoretical works [14–18] performed to investigate the $\Sigma NN$ systems that have total isospin $T = 0$, 1, 2 and total spin $S = 1/2$, 2/2. Garcilazo [14] showed that the $\Sigma^- nn$ system with $T = 2$ has no bound state in a Faddeev calculation with a separable $\Sigma N$ potential, and Statler and Gibson [15] confirmed it using the Jülich potential. Afman and Gibson [16] demonstrated that an enhancement in the $\Delta d$ cross section near the $\Sigma + N + N$ threshold is associated with a resonance pole of the $\Sigma NN$ states having $T = 0$, $S = 1/2$ in the scattering amplitude. Dover et al. [17] discussed the spin-isospin selectivity of the $\Sigma N \rightarrow \Lambda N$ conversion decay in the $\Sigma NN$ states, assuming that the isospin and spin of the $NN$ pair, $I_2$ and $S_2$, are good quantum numbers. Their results suggested that the $\Sigma NN$ state with $T = 0$, $S = 1/2$ is the best candidate to be bound and relatively long lived when the $NN$ state takes $I_2 = 0$, $S_2 = 1$ [5]. It should be noticed that the $NN$ states with $(I_2, S_2) = (1,0)$ and $(0,1)$ admix each other in the $\Sigma NN$ state. Indeed, Koike and Harada [18] performed three-body $\Sigma NN$ coupled-channel calculations, leading to the fact that there exist quasibound states of $T = 1$, $S = 1/2$ in the isospinplet ($\frac{3}{2}^-\Sigma$, $\frac{3}{2}^+\Sigma$, $\frac{1}{2}^-\Sigma$) where the $\Sigma N$ potential strongly admixes ($I_2, S_2) = (1,0)$ and $(0,1)$ states in the $NN$ pair. Recently, Garcilazo et al. [19] have shown that a narrow quasibound state with $\Gamma \sim 2.1$ MeV exists near the $\Sigma$ threshold in the $T = 1$, $S = 1/2$ channel in $\Sigma NN$ systems, using Faddeev $\Lambda NN$-$\Sigma NN$ calculations with $NN$ and $YN$ potentials derived from a chiral constituent quark model. Consequently, one naïvely expects that the $\Sigma NN$ quasibound state exists near the $\Sigma$ threshold whenever a modern $YN$ potential is used.

On the other hand, it has been recognized that there is no evidence of a narrow $\Sigma NN$ quasibound state ($\frac{3}{2}^-\Sigma$) below the $\Sigma$ threshold in the $^3\text{He}(K^-, \pi^\mp)$ reaction at BNL-E774 experiments [20]. These contradictory arguments are still not settled: Is there a quasibound state in $\Sigma NN$ systems?

In this paper, we theoretically demonstrate the inclusive and semiexclusive spectra in $^3\text{He}(K^-, \pi^\mp)$ reactions at 600 MeV/c ($4\gamma'$) within a distorted-wave impulse approximation (DWIA), using a coupled $(2N-\Lambda) + (2N-\Sigma)$ model with a spreading potential. We focus on the behavior of signals of the $\Sigma NN$ quasibound states observed as $\frac{3}{2}^-\Sigma$ and $\frac{3}{2}^+\Sigma$ in $\pi^-$ and $\pi^+$ spectra, respectively, in order to study the $\Sigma N$ interaction and also a mechanism of $\Sigma$ production for these spectra.

In a previous paper [21], we reported theoretical calculations of $^3\text{He}(K^-, \pi^\mp)$ spectra at 600 MeV/c, using the $2N-\Sigma$ folding-model potential with g-matrices derived from the NF5 potential [22] that simulates the Nijmegen model F [23]. The result suggested that there are quasibound states in the $\Sigma NN$,

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systems. However, the folding models using NF$_2$ were not able to systematically explain $\Lambda$ binding energies of $^3$H, $^4$He, $^4$He$^*$, and $^5$He. One of the prescriptions to solve this overbinding problem of s-shell $\Lambda$ hypernuclei [24] may be to consider the coherent $\Lambda N - \Sigma N$ coupling [2]. The D2 potential [2,25] is a central version of the $YN$ potential that simulates the Nijmegen model D [26], and it is fulfilled in the coherent $\Lambda N - \Sigma N$ coupling. In this paper, therefore, we use a slightly modified potential (D2$'$) that is adjusted to the binding energies of s-shell hypernuclei [27].

The outline of this paper is as follows. In Sec. II, we will briefly mention the DWIA framework for $^3$He$(K^- , \pi^+)$ reactions, employing the coupled $(2N - \Lambda) + (2N - \Sigma)$ model in a Green’s function technique [28,29]. In Sec. III, we will construct an effective $2N - Y$ potential within a microscopic folding model with $YN$ g-matrices, and will discuss the structure of the $\Sigma NN$ quasibound states, taking into account threshold effects due to the mass difference among $\Sigma$ hyperons and the Coulomb forces. A pole position for the $\Sigma NN$ quasibound state is obtained on the complex $E$ plane. In Sec. IV, we will demonstrate the calculated spectra of the $^3$He$(K^- , \pi^+)$ reactions at 600 MeV/$c$ to see a possible signal of the $\Sigma NN$ quasibound state, which might be observed in forthcoming experiments at J-PARC facilities [4]. In Sec. V, we will discuss the sensitivity of the $\pi^+$ spectra to a pole position of the $\Sigma NN$ quasibound state. We will consider the reason why the peak of a $\frac{3}{2}_-^-$ quasibound state populated in the $\pi^+$ spectrum is not observed, and compare the calculated $\pi^+$ spectrum with the data observed in BNL-E774 experiments. Summary and conclusion are given in Sec. VI. In the Appendix, we will report binding energies of $^3$H, $^4$He, $^4$He$^*$, and $^5$He obtained by the folding-model calculations with D2$'$, and interference effects for cross sections by $(K^-, \pi^+)$ reactions owing to the $^3S_1 - ^3S_0$ admixture of the NN pair.

II. CALCULATIONS

Let us consider a theoretical framework for $^3$He$(K^- , \pi^+)$ reactions. Here we treat hypernuclear final states classified as

$$K^-^3He \rightarrow \pi^- pp\Lambda, \quad \pi^- dn\Sigma^+, \quad \pi^- pn\Sigma^+, \quad \pi^- pp\Sigma^0,\quad \text{(1)}$$

for the $\pi^-$ spectrum, and those as

$$K^-^3He \rightarrow \pi^+ nn\Lambda, \quad \pi^+ dn\Sigma^-, \quad \pi^+ pn\Sigma^-, \quad \pi^+ nn\Sigma^0,\quad \text{(2)}$$

for the $\pi^+$ spectrum. It seems that an impulse approximation works well in $K^-$ capture at incident $K^-$ beam of $p_{K^-} = 600$ MeV/$c$ [30]. Figure 1 illustrates typical diagrams for physical processes for the $(K^-, \pi^+)$ reactions. We will calculate these spectra within the distorted-wave impulse approximation (DWIA) for the $(2N - \Lambda) + (2N - \Sigma)$ model with a spreading potential [31], as we mention in this section.

A. Model wave functions

In our calculation, we consider a model wave function $\Psi_A$ of the $^3$He ground state ($J^P = 1/2^+$, $T = 1/2$) as a target nucleus, in the $LS$-coupling scheme. It is given by

$$|\Psi_A\rangle = \mathcal{A}[\phi_0^{(2N)} \otimes \phi_0^{(N)}]_{L_A} \otimes X^A_{I_A,S_A}]_{M_A}, \quad X^A_{I_A,S_A} = [X_{I_2,S_2} \otimes \chi_{1/2,1/2}]_{1/2,1/2}, \quad (3)$$

where $\mathcal{A}$ is the antisymmetrized operator for nucleons, $\phi_0^{(2N)}$ is the wave function of a $2N$-core subsystem, and $\phi_0^{(N)}$ is the relative wave function between $2N$ and $N$ for a $2N - N$ system in the $^3$He ground state. $X^A_{I_A,S_A}$ is the isospin-spin function for $^3$He, and $X_{I_2,S_2}$ and $\chi_{1/2,1/2}$ are the isospin-spin functions for $2N$ (isospin $I_2$, spin $S_2$) and $N$ ($I_N = 1/2, S_N = 1/2$), respectively. Here we obtain the wave function $\phi_0^{(N)}$ using the $2N - N$ potential $U^{(N)}$ that was derived from microscopic three-body calculations [11] with a central potential of Tamagaki’s C3G [32]. This potential can reproduce the experimental data of the binding energy of $B_N = 8.07$ MeV and the nuclear root-mean-square distance of $\langle R^2 \rangle^{1/2} = 2.64$ fm for the $^3$H + $p$ system in $^3$He [33].

For hypernuclear $YNY$ final states, we consider wave functions $\Psi_B$ for $2N - Y$ systems with $J^P$ on physical particle (charge) bases. The wave functions are written by

$$|\Psi_B\rangle = \sum_a [\phi_a^{(2N)} \otimes \phi_i^{(Y)}]_{L_B} \otimes X^B_{I_B,S_B}]_{M_B}, \quad X^B_{I_B,S_B} = [X_{I_2,S_2} \otimes \chi_{I_2,1/2}]_{I_B,S_B}, \quad (4)$$

where $\phi_i^{(Y)}$ is the relative wave function between $2N$ and $Y$ with the angular momentum $\ell_Y$, and $X^B_{I_B,S_B}$ is the isospin-spin
TABLE I. Hypernuclear final states in \((K^-\pi^+)\) reactions on a $^3$He target and the threshold mass of the 2N – Y particle channels.

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Channels</th>
<th>$M_0$ (MeV)</th>
<th>$\omega_0$ (MeV)</th>
<th>$\Delta M_0$ (MeV)</th>
<th>$I_2$</th>
<th>$S_2$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>((K^-\pi^-))</td>
<td>{pp}$\Lambda$</td>
<td>2992.3</td>
<td>183.8</td>
<td>0.0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>{pn}$\Sigma^+$</td>
<td>3065.0</td>
<td>256.6</td>
<td>72.8</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>{pn}$\Sigma^+$</td>
<td>3067.2</td>
<td>258.8</td>
<td>75.0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>{pp}$\Sigma^0$</td>
<td>3069.2</td>
<td>260.8</td>
<td>77.0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>((K^-\pi^+))</td>
<td>{nn}$\Lambda$</td>
<td>2994.8</td>
<td>186.4</td>
<td>0.0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>{pn}$\Sigma^+$</td>
<td>3073.0</td>
<td>264.6</td>
<td>78.3</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>{pn}$\Sigma^-$</td>
<td>3075.2</td>
<td>266.8</td>
<td>80.5</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>{nn}$\Sigma^0$</td>
<td>3071.6</td>
<td>263.2</td>
<td>77.0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

B. Distorted-wave impulse approximation

According to the DWIA [28, 35–37], the inclusive differential cross section for nuclear \((K^-\pi)\) reactions in the laboratory

\[
\frac{d^2\sigma}{dE_{\pi}d\Omega_{\pi}} = \beta \frac{1}{|J_A|} \sum_B |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \times \delta(E_{\pi} + E_B - E_K - E_A),
\]

with the strangeness-exchange external operator including zero-range $K^-N\rightarrow\pi Y$ interactions

\[
\hat{F} = \int \frac{d\mathbf{r}}{\Delta k} \chi^{-+}_{\pi}(\mathbf{p}_\pi, \mathbf{r}) \chi^{++}_{K}(\mathbf{p}_K, \mathbf{r})
\]

\[
\times \sum_{j=1}^{A} \int \frac{d\mathbf{r}_j}{\delta(\mathbf{r} - \mathbf{r}_j)} \hat{O}_j,
\]

where $|J| = 2J + 1$; $E_\pi$, $E_K$, $E_B$, and $E_A$ are energies of an outgoing $\pi^\pm$, an incoming $K^-$, the hypernuclear state, and the target nucleus, respectively. The baryon operator $\hat{O}_j$ can change the $j$th nucleon into a hyperon in the nucleus, and $\mathbf{r}$ is the relative coordinate between the mesons and the center of mass of the nucleus; $\hat{f}_{\pi}(\mathbf{r}, \omega)$ is the Fermi-averaged amplitude for the $K^-N\rightarrow\pi Y$ reaction in the nuclear medium on the laboratory frame, where $\omega_{KN}$ is the total energy of $K^- - N$ subsystems.

The momentum and energy transfer to the 2N – Y final state in these reactions is given by

\[
q = p_K - p_\pi, \quad \omega = E_K - E_\pi,
\]

where $p_K$ and $p_\pi$ ($E_K$, $E_\pi$) are the laboratory momenta (energies) of the incident $K^-$ and outgoing $\pi^\pm$ in the many-body $K^- + A \rightarrow \pi + X^0 B^+$ reaction, respectively. The kinematical factor $\beta$ in Eq. (7) [38, 39] expresses the translation from the two-body $K^- - N$ laboratory system to the $K^- - A$
laboratory system [40], which is given by
\[
\beta = \left( 1 + \frac{E_y^{(0)} p_y^{(0)} - p_K^{(0)} \cos \theta_{lab}}{E_y^{(0)} p_y^{(0)}} \right) p_y E_y^{(0)},
\]
where \(p_K^{(0)}\) and \(p_y^{(0)}\) are momenta of \(K^-\) and \(\pi\) (energies of \(\pi\) and hyperon) in the two-body \(K^- + N \rightarrow \pi + Y\) reaction, respectively.

The distorted waves \(\chi_{\pi}^{(-\lambda)}(p_y, r)\) and \(\chi_{\lambda}^{(+\lambda)}(p_y, r)\) in Eq. (8) express the outgoing \(\pi\) and incoming \(K^-\) ones, respectively [41]:
\[
\chi_{\pi}^{(-\lambda)}(p_y, r) = \sum_{\lambda} \sqrt{4\pi |\lambda|} j_{\lambda}(\theta_{lab}, r) Y_{\lambda}^{0}(\hat{r}),
\]
where \(j_{\lambda}\) is the radial distorted wave with the angular momentum \(\lambda\), and \(\theta_{lab}\) is the scattering angle to the forward direction in \((K^-, \pi)\) reactions. The computational procedure for the distorted waves is simplified with the help of the eikonal approximation [35,42] because the distortions for mesons are not so important in few-body systems.

According to the Green’s function method [28,29], we can rewrite a sum of the final states in Eq. (7) as
\[
\sum_{R} |\Psi_B\rangle \langle \Psi_B| \delta(E - E_B) = -\frac{1}{\pi} \text{Im} \hat{G}(E).
\]
Thus the inclusive differential cross section is written by
\[
\frac{d^2\sigma}{dE \, d\Omega_y^{(0)}} = \beta \frac{1}{|J_A|} \sum_{M_A} |S_{\sigma}|^2,
\]
where \(|S_{\sigma}|^2\) is the relative Green’s function for \(\alpha\alpha'\) channels satisfies the following multichannel coupled equation:
\[
\hat{G}_{\alpha\alpha'}(\omega) = \hat{G}_{\alpha\alpha'}^{(0)}(\omega) \delta_{\alpha\alpha'} + \hat{G}_{\alpha\alpha'}^{(0)}(\omega) \sum_{\gamma \gamma'} \hat{U}_{\gamma\gamma'} \hat{G}_{\gamma\gamma'}(\omega).
\]
Solving this coupled equation numerically, we obtain the complete Green’s function \(\hat{G}_{\alpha\alpha'}(\omega)\) [44]. Here we use partial waves of \(\hat{G}_{\alpha\alpha'}(\omega)\) with \(J_B\) as a function of the relative distance \(R\) between \(2N\) and \(Y\), and its explicit form is written as
\[
\hat{G}_{\alpha\alpha'}^{(0)}(\omega) = \sum_{LM} \Phi_{Y\alpha}(\hat{R}) |\phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')\rangle R R' \langle \phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')| \Phi_{Y\alpha}(\hat{R})^\dagger.
\]
and its explicit form is written as
\[
\hat{G}_{\alpha\alpha'}^{(0)}(\omega) = \sum_{LM} \Phi_{Y\alpha}(\hat{R}) |\phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')\rangle R R' \langle \phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')| \Phi_{Y\alpha}(\hat{R})^\dagger.
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\hat{G}_{\alpha\alpha'}^{(0)}(\omega) = \sum_{LM} \Phi_{Y\alpha}(\hat{R}) |\phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')\rangle R R' \langle \phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')| \Phi_{Y\alpha}(\hat{R})^\dagger.
\]

C. Coupled-channel Green’s functions

The Green’s function method [28,29] facilitates parametrizing complicated many-body effects in a simple and tractable way, keeping the proper aspects of quantum mechanical systems. This technique can well describe an unstable hadron nuclear system such as a \(\Sigma^+\), \(\Sigma^-\), or \(K^-\) nuclear state [29].

The complete Green’s function \(\hat{G}\) in Eq. (14) [43] provides all information concerning hyperon-nucleus dynamics as a function of the energy transfer \(\omega = E_B - E_A\), which is related to the energy \(E_y = E_B - m_y + M_C = -B_y\) measured from the \(Y\)-hyperon-nucleus threshold, where \(m_y\) and \(M_C\) are masses of the \(Y\) and the core nucleus, respectively. Here we will consider \(2N - Y\) states within coupled \((2N - \Lambda) + (2N - \Sigma)\) channels with a spreading potential [30].

For \(2N - Y\) final states, the complete Green’s function in the \(P\) space is given by
\[
\hat{G}(\omega) = \frac{1}{\omega - \hat{H} + i\epsilon}.
\]

\(\hat{H}\) is the total Hamiltonian of the \(2N - Y\) system with \(\hat{H}|\Psi_B\rangle = E_B|\Psi_B\rangle\), and \(P\) is Feshbach’s projection operator for the model space we consider. Then we can calculate the complete Green’s function by solving the following equation:
\[
\hat{G}(\omega) = \hat{G}(\omega) + \hat{G}(\omega) \hat{U} \hat{G}(\omega),
\]
where \(\hat{G}(\omega)\) is the free Green’s function for the \(2N - Y\) system, and \(\hat{U}\) is the operator of a potential energy for the relative motion between \(2N\) and \(Y\). In order to extend it to a coupled-channel system, we introduce projection operators of \(P_a\) into the \(a\) channel in the \(P\) space, where \(P = \sum_a P_a\). In the case of \(P = P_a + P_a',\) for example, we obtain
\[
\hat{G}(\omega) = (P_a + P_a') \hat{G}(\omega) (P_a + P_a'),
\]
where we define \(\hat{G}_{\alpha\alpha'}(\omega)\) and \(\hat{G}_{\alpha\alpha'}(\omega)\) [44]. The complete Green’s function for \(\alpha\alpha'\) channels satisfies the following multichannel coupled equation:
\[
\hat{G}_{\alpha\alpha'}(\omega) = \hat{G}_{\alpha\alpha'}^{(0)}(\omega) \delta_{\alpha\alpha'} + \hat{G}_{\alpha\alpha'}^{(0)}(\omega) \sum_{\gamma \gamma'} \hat{U}_{\gamma\gamma'} \hat{G}_{\gamma\gamma'}(\omega)\]

Solving this coupled equation numerically, we obtain the complete Green’s function \(\hat{G}_{\alpha\alpha'}(\omega)\) [44]. Here we use partial waves of \(\hat{G}_{\alpha\alpha'}(\omega)\) with \(J_B\) as a function of the relative distance \(R\) between \(2N\) and \(Y\), and its explicit form is written as
\[
\hat{G}_{\alpha\alpha'}^{(0)}(\omega) = \sum_{LM} \Phi_{Y\alpha}(\hat{R}) |\phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')\rangle R R' \langle \phi_{\alpha\alpha'}^{(2N)}(\omega; R, R')| \Phi_{Y\alpha}(\hat{R})^\dagger.
\]
TABLE II. Isospin-spin spectroscopic factor $C_{Yα}$ for the $2N − Y$ channel.

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Channels</th>
<th>$C_{Yα}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(K^−, π^−)$</td>
<td>$</td>
<td>pp⟩\Lambda$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>pn⟩</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>pn⟩</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>pp⟩\Sigma^0$</td>
</tr>
<tr>
<td>$(K^−, π^+)$</td>
<td>$</td>
<td>nn⟩\Lambda$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>pn⟩</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>pn⟩</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>nn⟩</td>
</tr>
</tbody>
</table>

where the factor of $M_C/M_A$ denotes the recoil effects, leading to the effective momentum transfer of $(M_C/M_A)q$. The isospin-spin spectroscopic factor for the $2N − Y$ channel is obtained as

$$C_{Yα} = \langle X_{Y,S}^B \sum_{j=1}^A \hat{O}_j | X_{Y,S}^A \rangle. \quad (24)$$

In Table II, we show the values of $C_{Yα}$ for non-spin-flip processes, which are seemed to be dominant ones in nuclear $(K^−, π)$ reactions near 600 MeV/c.

D. The decomposition of the inclusive cross sections into components

The inclusive cross sections can be decomposed into partial cross sections corresponding to different physical processes [28,29,45], as classified in Table I. We obtain the decomposition of the strength function $S_π$ of Eq. (14) as

$$S_π = S^{(pp)\Lambda} + S^{(pn)|Σ^+⟩} + S^{(pn)|Σ^−⟩} + S^{(pp)|Σ^0⟩} + S^{(Conv)}$$

for the $π^−$ spectrum, and that as

$$S_π = S^{(pn)|Σ^+⟩} + S^{(pn)|Σ^−⟩} + S^{(nn)|Σ^0⟩} + S^{(Conv)}$$

for the $π^+$ spectrum. The partial strength functions are defined by

$$S_π^{(Conv)} = −\frac{1}{\pi} \langle F| \hat{G}^{(−)}(0)| \hat{G}(0)|F⟩,$$

and

$$\hat{G}^{(−)} = 1 + \hat{U} \hat{G} \hat{U}$$

and $\hat{G}$ is the Möller wave operator, and $W_{αα'}$ is a spreading (imaginary) potential for the complicated nuclear excited states from the $α'$ channel. It should be noticed that $\hat{G}_α W_{αα'} \hat{G}_{α'}$ denotes the spreading processes, which are predominantly caused by the $ΣN → ΛN$ conversion into complicated $N + N + Λ$ states because a produced $Σ$ subsequently interacts with a second nucleon, and its converted $Λ$ pair gains large energy from the mass difference $m_Σ − m_Λ \approx 70$ MeV. Indeed, the peak below the $Σ$ threshold is connected with the secondary processes

$$\frac{1}{2}\text{He} → p + p + Λ,$$

in the $π^−$ spectrum, or with those

$$\frac{1}{2}n → n + n + Λ,$$

in the $π^+$ spectrum. The decomposition near the $Σ$ threshold can help us to understand the structure of the $YNN$ quasibound state and its decay property.

E. Fermi-averaged amplitudes for $K^− + N → π + Y$ in nuclear medium

It is recognized that the spectral shape for DWIA is sensitive to the elementary $K^− + N → π + Y$ amplitudes of $f_{(Yπ)}$ in nuclear medium in Eq. (8) [37,46–48]. When we evaluate the nuclear $(K^−, π^±)$ cross sections with the $K^− + N → π + Y$ amplitudes, it is important to take into account the Fermi motion of a struck nucleon in nuclear medium [37]. This effect is considerably enhanced for the $π^−$ channel as discussed in Ref. [30,49]. We use the momentum distribution $ρ(p)$ of a struck nucleon in $^3$He, which is assumed as a simple harmonic oscillator with a size parameter $b_N = 1.31$ fm, leading to $(p^2)^{1/2} = 184$ MeV/c in the nucleus.

In Fig. 3, we show the Fermi-averaged laboratory cross sections of $K^− + n → π^− + Λ$, $K^− + p → π^− + Σ^+$, $K^− + n → π^+ + Σ^0$, and $K^− + p → π^+ + Σ^−$ reactions on nuclei, at detected $π$ angles $θ_{lab} = 0^°$ and $10^°$, as a function of the incident $K^−$ laboratory momentum $p_K$. $f_{(Yπ)}$ and $g_{(Yπ)}$ denote the non-spin-flip and spin-flip components of the Fermi-averaged amplitudes, respectively. The shape of the Fermi-averaged cross section sizably becomes broader, and its value is not so changed by a choice of the target, as discussed by Dover et al. [5,42,48]. Since the spin-flip cross section of $|g_{(Yπ)}|^2$ is negligibly small, we consider only the non-spin-flip process in the nuclear $(K^−, π^±)$ reaction.

Furthermore, it is noticed that the Fermi-averaged amplitudes $f_{(Yπ)}$ remain in ambiguities, e.g., the relative phase of $φ_{α}$ and $φ_{α'}$ for $Λπ$ ($Σπ I = 1$) to $Σπ I = 0$ channels, as discussed in Ref. [30]. Thus we assume $φ_{α} = +15.8^°$ and $φ_{α'} = +33.2^°$, which were determined by fitting the data overall in $^3$He$(K^−, π^±)$ reactions at $p_K = 600$ MeV/c [30], and we use them in our calculations.

III. MICROSCOPIC COUPLED-CHANNEL $2N − Y$ POTENTIALS

In order to describe the $ΣNN$ quasibound states, we construct a microscopic effective $2N − Y$ potential using the $YN$ $g$-matrices in folding-model calculations [52,58]. In Ref. [18], three-body coupled-channel calculations for $ΣNN$...
FIG. 3. Fermi-averaged cross sections for (a) $K^- + n \rightarrow \pi^- + \Lambda$, (b) $K^- + p \rightarrow \pi^- + \Sigma^+$, (c) $K^- + n \rightarrow \pi^- + \Sigma^0$, and (d) $K^- + p \rightarrow \pi^- + \Sigma^-$ reactions in nuclear medium. Solid and long-dashed curves denote non-spin-flip Fermi-averaged laboratory cross sections, $|f|^2$, for $\theta_{lab} = 0^\circ$ and $10^\circ$, respectively, and the dot-dashed curve denotes a spin-flip Fermi-averaged one, $|g|^2$, for $\theta_{lab} = 10^\circ$. The thin-solid and thin-dashed curves are for non-spin-flip laboratory elementary cross sections in free space at $\theta_{lab} = 0^\circ$ and $10^\circ$, respectively, and the thin-dot-dashed curve is for a spin-flip one at $\theta_{lab} = 10^\circ$. The elementary amplitudes are used by Gopal et al. [49].

systems suggested that the channel coupling plays an important role in making a bound state which has strong admixtures of $(I_2, S_2) = (0, 1)$ and $(1, 0)$ states in the $NN$ pair, e.g., $[pn]\Sigma^+$, $[pn]\Sigma^+$, and $[pp]\Sigma^0$ states admix each other in $^3\Lambda$He. Its origin is due to the $(\sigma_N \cdot \sigma_{\Sigma})(\tau_N \cdot t_{\Sigma})$ term in the $\Sigma N$ OBE potentials [5]. This nature is quite different from the weak-coupling state like $[pn] + \Lambda$ in $^2\Lambda$H, and it must be involved in the $2N - Y$ potential.

In folding-model calculations, the effective $2N - Y$ potential for $\alpha\alpha'$ channels is obtained as

$$U_{\alpha\alpha'}(R) = \int \rho_{\alpha\alpha'}(r)\bar{\sigma}_{\alpha\alpha'}(r_1) + \sigma_{\alpha\alpha'}(r_2)dr,$$  \hspace{1cm} (32)

where $r_1 = R + r/2$ and $r_2 = R - r/2$ is the relative coordinate between $N_1$ ($N_2$) and $Y$, as shown in Fig. 4. The nucleon or transition density for $\alpha\alpha'$ channels is given by

$$\rho_{\alpha\alpha'}(r) = \langle \phi^{(2N)}_{\alpha'} | \sum_i \delta(r - r_i) | \phi^{(2N)}_{\alpha} \rangle.$$  \hspace{1cm} (33)

In Table III, we show matrix elements of isospin-spin averaged potentials for $\alpha\alpha'$ channels, in which the g-matrices $\bar{\sigma}_{\alpha\alpha'}$ for $\Lambda N - \Lambda N$, $\Lambda N - \Sigma N$, and $\Sigma N - \Sigma N$ states can be obtained by solving the coupled Bethe-Goldstone equation with appropriate parameters of the starting energy $E_S$ and Fermi momentum $k_F$.

Figure 5 shows the wave functions for $\rho_{\alpha\alpha'}$ of Eq. (33), as a function of the distance $r$ between nucleons. For the $\{pp\}\Lambda$ channel, here we used the $\{pp\}$ wave function $\phi_u^{(2N)}$ obtained by CDCC as the $NN$-pair nucleus, as given in Fig. 2, because the $\Lambda N$ interaction is very weak. For the $\Sigma N$ channels, on the other hand, we must consider nuclear contraction of the $NN$ pair because $\Sigma N$ potentials may induce $NN$-pair admixture between $^3\Sigma_1$ and $^1\Sigma_0$ states in $\Sigma NN$ systems [18]. In the $\{pn\}\Sigma^+$ channel, the wave function derived from three-body $\Sigma NN$ calculations is not so changed that in $^2\Lambda$H. In the $\{pp\}\Sigma^0$ or
The solid curve denote the potentials in the folding model, as a function of the relative distance $r$. The transition densities of $\alpha\alpha/\Sigma_1$ provide us the coherent structure of $\Sigma NN$ quasibound states in $^3\text{He}$, leading to the results that will confirm the concept of the coherent $\Lambda N - \Sigma N$ coupling in $\alpha\alpha$.

Moreover, it should be noticed that the imaginary part of $U_{\alpha\alpha}$ for the $\alpha\alpha$ channel is regarded as a spreading potential $W_{\alpha\alpha}$. This is significant to describe the complicated surrounding $\Lambda$ excited states with all the $2\Lambda$ breakup processes, because $\Sigma$ hypernuclear states can be connected with highly $\Lambda$ excited states via the strong $\Lambda N - \Sigma N$ coupling. In the folding model with $D''$, we can also reproduce the binding energy and width of $^3\text{He}$, in a comparison with the data, as shown in Appendix A.

Figure 6 displays the real and imaginary parts of the effective $2N - Y$ potential $\hat{U}_{\alpha\alpha}(R)$ for $^3\text{He}$ ($J^\pi = 1/2^+$) at $E_A = 70$ MeV that corresponds to the $\Sigma$ threshold region, as a function of the relative distance $R$ between $2N$ and $Y$. We find that the imaginary potentials $\{\rho_{\alpha\alpha}\}^\imath$, $\{\rho_{\alpha\alpha}\}^\imath - \{\rho_{\alpha\alpha}\}_0$, and $\{\rho_{\alpha\alpha}\}^\imath - \{\rho_{\alpha\alpha}\}_0$ are quite strong. This nature originates from the fact that the $\Sigma N$ potential has a strong isospin-spin dependence, as pointed out by Dover and Gal [40] and suggested by recent $YN$ potential models [53]. For the imaginary parts ($W_{\alpha\alpha}$), we also recognize that there is the spin-isospin selectivity [40] for $\Sigma N - \Lambda N$ conversion decays. It is very important to realize whether or not the quasibound state has a narrow width in $\Sigma NN$ systems. Strengths of $W_{\alpha\alpha}$ for diagonal $\{\rho_{\alpha\alpha}\}^\imath + \{\rho_{\alpha\alpha}\}_0$ and $\{\rho_{\alpha\alpha}\}^\imath - \{\rho_{\alpha\alpha}\}_0$ channels are 5.0 MeV and 13.2 MeV at the nuclear center, which are consistent with quenching factors $Q = 1/3$ and 1, respectively, as given in Table II of Ref. [40].

**IV. RESULTS**

**A. $\Sigma NN$ quasibound states**

In order to obtain eigenvalues for bound and resonance states simultaneously, we solve the multichannel equation for the $2N - Y$ systems by the complex scaling method [54,55]. Here we use the $2N - Y$ potential given in Fig. 6 and the Coulomb force. We find that a pole position for $^3\text{He}$ ($J^\pi = 1/2^+$, $L = 0$, $S = 1/2$) as a complex eigenvalue of the $2N - Y$ system, $\epsilon_{\Sigma(+)}^{(\text{pole})} = E_{\Sigma(+)} - i\Gamma_{\Sigma(+)}$ on the second Riemann sheet $\{\rho_{\alpha\alpha}\}^\imath$ that is identified by a set of four signs of $\text{Re}k_{\{\rho_{\alpha\alpha}\}^\imath}$, $\text{Im}k_{\{\rho_{\alpha\alpha}\}^\imath}$, $\text{Re}k_{\{\rho_{\alpha\alpha}\}^\imath}$, $\text{Im}k_{\{\rho_{\alpha\alpha}\}^\imath}$ on the complex $E$ plane. The

- **TABLE III.** Isospin-spin averaged matrix elements of $\bar{\rho}_{\alpha\alpha}$ for the $YN$ potential terms in $^3\text{He}$ and $^3\text{He}$, $\{\rho_{\alpha\alpha}\}$ denotes a $YN - YN$ potential for the isospin $I$ and spin $S$ state.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\alpha'$</th>
<th>$\bar{\rho}_{\alpha\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${pp}$ $\Delta$</td>
<td>${pp}$ $\Delta$</td>
<td>$\frac{3}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
<tr>
<td>${pn}$ $\Sigma^+$</td>
<td>${pn}$ $\Sigma^+$</td>
<td>$-\frac{1}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
<tr>
<td>${pn}$ $\Sigma^+$</td>
<td>${pn}$ $\Sigma^+$</td>
<td>$-\sqrt{\frac{3}{2}} (\hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta)$</td>
</tr>
<tr>
<td>${pp}$ $\Sigma^0$</td>
<td>${pp}$ $\Sigma^0$</td>
<td>$\frac{1}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
<tr>
<td>${pn}$ $\Sigma^+$</td>
<td>${pn}$ $\Sigma^+$</td>
<td>$\frac{3}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
<tr>
<td>${pn}$ $\Sigma^+$</td>
<td>${pn}$ $\Sigma^+$</td>
<td>$\frac{3}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
<tr>
<td>${nn}$ $\Lambda$</td>
<td>${nn}$ $\Lambda$</td>
<td>$\frac{3}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
<tr>
<td>${nn}$ $\Sigma^0$</td>
<td>${nn}$ $\Sigma^0$</td>
<td>$\frac{3}{2} \hat{g}<em>{1/2}^\alpha + \frac{1}{2} \hat{g}</em>{1/2}^\beta$</td>
</tr>
</tbody>
</table>

**FIG. 5.** Radial wave functions of the intranuclear $\Sigma N$ states in transition densities of $\rho_{\alpha\alpha}$, which are used to calculate the $2N - Y$ potentials in the folding model, as a function of the relative distance $r$. The solid curve denote the $\gamma\gamma$ wave functions in the $\Sigma NN$ quasibound state, which is obtained by three-body calculations [18]. The dash curves denote the $\gamma\gamma$ wave functions in free space.

**FIG. 6.** Real and imaginary parts of the effective $2N - Y$ potential $\hat{U}_{\alpha\alpha}(R)$ for $^3\text{He}$ ($J^\pi = 1/2^+$) at $E_A = 70$ MeV that corresponds to the $\Sigma$ threshold region, as a function of the relative distance $R$ between $2N$ and $Y$. We find that the coupling potentials $\{\rho_{\alpha\alpha}\}^\imath$, $\{\rho_{\alpha\alpha}\}^\imath - \{\rho_{\alpha\alpha}\}_0$, and $\{\rho_{\alpha\alpha}\}^\imath - \{\rho_{\alpha\alpha}\}_0$ are quite strong. This nature originates from the fact that the $\Sigma N$ potential has a strong isospin-spin dependence, as pointed out by Dover and Gal [40] and suggested by recent $YN$ potential models [53]. For the imaginary parts ($W_{\alpha\alpha}$), we also recognize that there is the spin-isospin selectivity [40] for $\Sigma N - \Lambda N$ conversion decays. It is very important to realize whether or not the quasibound state has a narrow width in $\Sigma NN$ systems. Strengths of $W_{\alpha\alpha}$ for diagonal $\{\rho_{\alpha\alpha}\}^\imath + \{\rho_{\alpha\alpha}\}_0$ and $\{\rho_{\alpha\alpha}\}^\imath - \{\rho_{\alpha\alpha}\}_0$ channels are 5.0 MeV and 13.2 MeV at the nuclear center, which are consistent with quenching factors $Q = 1/3$ and 1, respectively, as given in Table II of Ref. [40].
between 2

FIG. 6. (a) Real and (b) imaginary parts of the calculated effective $2N - Y$ potential $V_{\text{eff}}(R)$ for $^3\text{He}$ ($J^\pi = 1/2^+$) at $E_\Lambda = 70$ MeV in the folding-model potential, as a function of a relative distance $R$ between $2N$ and $Y$.

pole is located as

$$E_{\Sigma^+}^{(\text{pole})} (^3\text{He}) = +0.96 - i 4.5 \text{ MeV}, \quad (34)$$

where $E_{\Sigma^+}$ is measured from the $d + \Sigma^+$ threshold, as shown in Table IV. Its width becomes $\Gamma_{\Sigma} = 9.0 \text{ MeV}$. Since the pole lies in the second quadrant ($\text{Re} k_{\Sigma^+} < 0, \text{Im} k_{\Sigma^+} > 0$) on the complex $k_{\Sigma^+}$ plane, the wave function behaves as

$$\exp(ik_{\Sigma^+}R) = \exp(i\text{Re} k_{\Sigma^+}R) \exp(-\text{Im} k_{\Sigma^+}R) \to 0 \quad (35)$$
in the asymptotic region ($R \to \infty$). Hence this state is identified to be a quasibound (an unstable bound) state. In the $\Lambda$ region, we also confirm that there is no pole of $^3\Lambda\text{He}$ bound state below the $p + p + \Lambda$ threshold.

For $^3\Sigma$ ($J^\pi = 1/2^+, L = 0, S = 1/2$), we find

$$E_{\Sigma^0}^{(\text{pole})} (^3\Sigma) = -0.58 - i 5.3 \text{ MeV}, \quad (36)$$

where $E_{\Sigma^0}$ is measured from the $n + n + \Sigma^0$ threshold, and $\Gamma_{\Sigma} = 10.5 \text{ MeV}$, as shown in Table IV.

In order to see the contributions of the $2N - Y$ components in these pole states, we calculate probabilities of isospin $T(I_S I_I)$ states for $^3\text{He}$ ($T_\Sigma = +1$) and $^3\Sigma$ ($T_\Sigma = -1$):

$$P_{Ti(I_S I_I)} = \left| \langle \Psi_{T_i(I_{S_i} I_{I_i})} | \Psi_{T_i(I_S I_I)}^{(\text{pole})} \rangle \right|^2, \quad (37)$$

where $\Psi_{T_i(I_S I_I)}^{(\text{pole})}$ is the isospin state defined in Eqs. (B1) and (B4). In Table V, we show values of $P_{Ti(I_S I_I)}$ together with values of probabilities on the $\Sigma$ charge bases. We find that values of a sum of $P_{T = 1}$ account for 99.6% and 97.9% in the $^3\Sigma$ and $^3\Sigma$ states, respectively, including $\{pp\}\Lambda$ states. Hence the total isospin $T = 1$ becomes an almost good quantum number.

B. $\pi^-$ spectrum

Figure 7 shows the calculated inclusive spectrum of the $^3\text{He}(K^-, \pi^-)$ reaction at 600 MeV/$c$ ($4^0$) from $\Lambda$ to $\Sigma$ regions, together with the $J^\pi = 1/2^+$ ($L = 0, S = 1/2$) component, which is predominantly connected with $\Sigma N \to \Lambda N$ conversion processes of $[^3\Sigma\text{He}] \to p + p + \Lambda$ decays. The $\Sigma$ hyperon produced in the real or virtual $^3\Sigma$ state subsequently interacts with a second nucleon, and it is converted to a $\Lambda$ via $\Sigma N \to \Lambda N$. 

### TABLE V. Probabilities of channel components of the pole states of $[^3\Sigma\text{He}^+]$ and $[^3\Sigma\text{n}^+]$ with $J^\pi = 1/2^+$ on complex $E$ plane. Calculated values are obtained by the complex scaling method.

<table>
<thead>
<tr>
<th>States</th>
<th>Components</th>
<th>Probabilities (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[^3\Sigma\text{He}^+]$</td>
<td>${pp}\Lambda$</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>${pp}\Sigma^+$</td>
<td>54.9</td>
</tr>
<tr>
<td></td>
<td>${pp}\Sigma^-$</td>
<td>24.7</td>
</tr>
<tr>
<td></td>
<td>${pp}^0$</td>
<td>18.3</td>
</tr>
<tr>
<td>$T = 1 (I_2 = 0, S_2 = 1)$</td>
<td>54.9</td>
<td></td>
</tr>
<tr>
<td>$T = 1 (I_2 = 1, S_2 = 0)$</td>
<td>42.5</td>
<td></td>
</tr>
<tr>
<td>$T = 2$</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>$[^3\Sigma\text{n}^+]$</td>
<td>${nn}\Lambda$</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>${nn}\Sigma^+$</td>
<td>39.5</td>
</tr>
<tr>
<td></td>
<td>${nn}\Sigma^-$</td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td>${nn}^0$</td>
<td>37.2</td>
</tr>
<tr>
<td>$T = 1 (I_2 = 0, S_2 = 1)$</td>
<td>39.5</td>
<td></td>
</tr>
<tr>
<td>$T = 1 (I_2 = 1, S_2 = 0)$</td>
<td>56.0</td>
<td></td>
</tr>
<tr>
<td>$T = 2$</td>
<td>2.10</td>
<td></td>
</tr>
</tbody>
</table>
\( \Lambda N \) conversion processes inducing 2N-nuclear breakup due to the mass difference \( m_\Sigma - m_\Lambda \approx 77 \) MeV. It is recognized that a clear peak just below the \( d + \Sigma^+ \) threshold in the \( \pi^- \) spectrum, which corresponds to the \( ^3\text{He} \) quasibound state with \( J^\pi = 1/2^+ \). It is located near 0.546 GeV in the Riemann sheet \( [---++] \) near the \( \Sigma \) threshold. Such a \( \Sigma N \rightarrow \Lambda N \) conversion spectrum with \( p + p + \Lambda \) may give evidence of the existence of the \( ^3\text{He} \) quasibound state.

In the \( \pi^- \) spectrum we obtain the decomposition of the inclusive spectrum into partial spectra of the \( \Lambda \), \( \pi^- \) continuum states. Figure 8 illustrates the contributions of the \( \Lambda \)-emitted processes of \( \{pp\} \Lambda \) and \( ^3\text{He} \rightarrow p + p + \Lambda \\) conversion near the \( \Sigma \) threshold, together with those of the \( \Sigma \)-emitted processes of \( \{pn\} \Sigma^+ \), \( \{pn\} \Sigma^+ \), and \( \{pp\} \Sigma^0 \). For the \( \Sigma \) continuum region, we find that the contribution of the \( \{pp\} \Sigma^0 \) component is larger than that of the \( \{pn\} \Sigma^+ \) component because the production amplitudes have \( |J_{(\Sigma^- \pi^-)}| \gg |J_{(\Sigma^+ \pi^-)}| \) near \( p_\Sigma = 600 \) MeV/c. Below the \( d + \Sigma^+ \) threshold, the \( ^3\text{He} \) quasibound state is predominantly populated via the \( \Sigma^0 \) components by \( J_{(\Sigma^- \pi^-)} \).

C. \( \pi^+ \) spectrum

The nuclear \( (K^-, \pi^+ ) \) reaction at forward direction of \( p_{K^+} = 600 \) MeV/c seems to be appropriate to search a bound state in the \( \Sigma \) bound region. The reasons were because (i) this reaction can populate only the \( \Sigma^- \) components in the final states by its double-charge exchange reaction, so that the contribution of a \( \Lambda \) hyperon is removed out from the \( \pi^+ \) spectrum, and (ii) it has a substitutional mechanism under the near-recoilless condition so as to produce the \( 3^1n \) quasibound state from \( ^3\text{He} \), as well as the \( ^3\text{He} \) quasibound state in the \( (K^-, \pi^- ) \) reaction. Therefore, we have naively expected that a signal of the corresponding peak can be clearly observed in the \( \pi^+ \) spectrum, rather than the \( \pi^- \) one.

Figure 9 shows the calculated inclusive spectrum of the \( ^3\text{He}(K^-, \pi^+ ) \) reaction at 600 MeV/c from \( \Lambda \) to \( \Sigma \) regions, together with the \( J^\pi = 1/2^+ \) component in \( ^3\text{He} \rightarrow n + n + \Lambda \) conversion processes, obtained by folding with a detector resolution of 2 MeV FWHM. Surprisingly, we find that although the \( ^3\text{He} \) quasibound state exists, the corresponding peak disappears in the inclusive \( \pi^+ \) spectrum, rather than that of the \( \Sigma \) continuum states.

Figure 10 illustrates the contributions of the \( ^3\text{He} \rightarrow n + n + \Lambda \) conversion, \( \{pn\} \Sigma^0 \), \( \{pn\} \Sigma^- \), and \( \{pn\} \Sigma^+ \) processes near the \( \Sigma \) threshold. Because these states can be populated via
Although the threshold energy difference between components that account for 49.5%, 48.25%, and 2.3%, respectively.

To see the CSB effects, we obtain a charge symmetric (CS) and probabilities of the \([\Sigma n]/\Sigma^0\) reaction at 600 MeV \(c/4\) near the \(\Sigma\) threshold, together with the components of \([pn]/\Sigma^0\), \([nn]/\Sigma^0\), and \([n\Sigma]/\Sigma^0\), and \([n\Sigma]/\Lambda\) conversion processes, obtained by folding with a detector resolution of 2 MeV FWHM.

FIG. 10. The decomposition of the calculated inclusive \(\pi^+\) spectrum of the \(^3\)He\((K^-,\pi^+)\) reaction at 600 MeV/c (4') near the \(\Sigma\) threshold, together with the components of \([pn]/\Sigma^-\), \([pn]/\Sigma^0\), \([nn]/\Sigma^0\), and \([n\Sigma]/\Sigma^0\), and \([n\Sigma]/\Lambda\) conversion processes, obtained by folding with a detector resolution of 2 MeV FWHM.

only \(\Sigma^-\) productions by \(\vec{F}(\Sigma^-\pi^+)\) in the \((K^-,\pi^+)\) reaction. We find that the \([pn]/\Sigma^-\) and \([pn]/\Sigma^0\) components predominantly occur in \(\Sigma\) continuum regions, and that a small \([nn]/\Sigma^0\) component can be populated via the quasibound state near the \(n + n + \Sigma^0\) threshold, followed by \([n\Sigma]/\Lambda\) conversion processes, obtained by folding with a detector resolution of 2 MeV FWHM.

V. DISCUSSION

A. Charge symmetry breaking

It is noticed that the \(^3\)He and \(^1\)n quasibound states belong to \(J^\pi = 1/2^+\) isoscalar states in \(\Sigma NN\) systems, whereas a value of \(E_{\Sigma}^{(pole)}\) for \(^3\)He slightly differs from that of \(E_{\Sigma}^{(pole)}\) for \(^1\)n, as seen in Table IV. This discrepancy comes from the \(\Sigma\) threshold energy difference and the Coulomb force, leading to the charge symmetry breaking (CSB) in the \(\Sigma NN\) systems. We study a dependence of these pole positions and configurations of the \(2N - Y\) quasibound states on CSB effects.

To see the CSB effects, we obtain a charge symmetric (CS) state, neglecting the Coulomb force and replacing masses of \(m_{\Sigma^0}\) and \(m_{\rho,n}\) by averaged masses of \(m_{\Sigma} = 1193.2\) MeV and \(m_N = 938.9\) MeV, respectively. We find

\[
E_{\Sigma}^{(pole)}(CS) = -0.23 - i 4.7 \text{ MeV}, \tag{38}
\]

and probabilities of the \([NN]/\Sigma\), \([NN]/\Sigma\), and \([NN]/\Lambda\) components that account for 49.5%, 48.25%, and 2.3%, respectively. Although the threshold energy difference between \(pn\) \(\Sigma^-\) and \(pp\) \(\Sigma^0\) (\(nn\) \(\Sigma^-\) and \(pn\) \(\Sigma^0\)) amounts to 2.0 (-3.6) MeV, we find that the energy levels for \(^3\)He \((\Sigma^-n)\) is +0.96 (-0.58) MeV, which is slightly different from -0.23 MeV obtained for CS. This confirms the fact that the \(\Sigma NN\) quasibound states have a \(T \approx 1\) good isospin (97%–99%). We obtain that the width for \(^3\)He amounts to 9.0 MeV, which is slightly smaller than 9.4 MeV for CS because the \(pp\) \(\Sigma^0\) threshold is located above the \(d + \Sigma^0\) one. Contrary to \(^3\)He, the width of \(^1\)n becomes broader up to 10.5 MeV because the \(nn\) \(\Sigma^0\) threshold is located below the \(d + \Sigma\) threshold. Figure 11 illustrates energy levels and widths of the \(\Sigma NN\) quasibound states for \(^3\)He and \(^1\)n near the \(\Sigma\) threshold, together with the probabilities of the \(2N - Y\) components in the \(\Sigma NN\) systems. We also confirmed that the CSB effects rarely have an influence on the shape and magnitude of the \(\pi^+\) spectra.

B. Interference effects between production amplitudes

It should be noticed that a production cross section near the \(\Sigma\) threshold is very sensitive not only to the pole position but also to the configuration of the wave function of the quasibound state. We consider difference of the \(\Sigma\) production mechanism between the \(\pi^-\) and \(\pi^+\) spectra in terms of interference among \(\Sigma\) production amplitudes. In order to understand the behavior of the \(\pi^+\) spectra, we evaluate interference effects among configurations of the \(NN\) core states in \(\Sigma\) production amplitude, because the \(2N - Y\) potential should admix \(^3\)S_1 and \(^1\)S_0 states in the \(NN\) pair [18], depending on the nature of the \(\Sigma N\) potential.

In the \(\pi^+\) spectrum, production amplitude for \(^3\)He near the \(\Sigma\) threshold is approximately written as

\[
\langle \Sigma^0/\pi^- | T | ^3\text{He}K^- \rangle \approx \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \bar{F}(\Sigma^0\pi^-) \bar{F}(\Sigma^0\pi^-) \right\} \langle \Psi_{(1)}^{21} | \Psi^{(3\text{He})} \rangle + \left\{ \frac{1}{2} \bar{F}(\Sigma^0\pi^-) + \frac{1}{2} \bar{F}(\Sigma^0\pi^-) \right\} \langle \Psi_{(1)}^{11} | \Psi^{(3\text{He})} \rangle \tag{39}
\]

on isospin bases, where \(\Psi_{(1)}^{11}, \Psi_{(1)}^{21}\) is the isospin state defined in Eq. (B1). The wave function of \(\Psi_{(1)}^{11}\) is regarded as that of a \(^3\)He ground state with \(J^\pi = 1/2^+\), \(T = 1\), and \(\Psi_{(1)}^{21}\) as a \(^3\)He* excited state. The relative phase between \(\bar{F}(\Sigma^0\pi^-)\) and \(\bar{F}(\Sigma^0\pi^-)\) is \(\phi(\Sigma^0/\Sigma^0) = +4.8^\circ\) at \(p_{K^-} = 600\) MeV/c, so the component of \(\Psi_{(1)}^{11}\) is relatively enhanced in the \(\pi^+\) spectrum.
\( f_{(\Sigma^- \rightarrow \pi^+)} \) play an important role in populating the component of \( \Psi_{1^{-1}} \).

In the \( \pi^+ \) spectrum, on the other hand, production amplitude for \( \frac{1}{\Sigma} n \) near the \( \Sigma \) threshold is approximately written as

\[
\left\langle \frac{1}{\Sigma} n \left| T \right| 3\text{He} K^- \right\rangle = f_{(\Sigma^- \rightarrow \pi^+)} \left\{ \frac{1}{2} \left( \langle \Psi_{1^{-1}} | \Psi(3\text{He}) \rangle^2 - \langle \Psi_{1^{-1}} | \Psi(3\text{He}) \rangle^2 \right) + \frac{2}{4} \left( \langle \Psi_{1^{-1}} | \Psi(3\text{He}) \rangle + \langle \Psi_{1^{-1}} | \Psi(3\text{He}) \rangle \right) \right\}
\]

(40)

on isospin bases. We find that a cross section for \( \Psi_{1^{-1}} \) as the \( \frac{1}{\Sigma} n \) ground state is relatively reduced by a factor \( \left[ (2\sqrt{3} - \sqrt{2})/4 \right]^2 \approx 0.51^2 \approx 0.26 \), whereas that for \( \Psi_{1^{+}} \) as a \( \frac{1}{\Sigma} n \) excited state is enhanced by a factor \( \left[ (2\sqrt{3} + \sqrt{2})/4 \right]^2 = 1.22^2 \approx 1.49 \). The interference effects are considered as dynamical ones caused by \( ^3S_1 - ^1S_0 \) admixture in the \( NN \) pair for the \( \Sigma NN \) systems. This mechanism originates from properties of the \( \Sigma N \) interaction, and it is inevitable whenever we consider the \( ^3\text{He}(K^-, \pi^+) \) reaction. In Appendix B, we discuss in detail the interference effects.

C. Dependence of the \( \pi^\mp \) spectra on \( \Sigma \) widths

Several three-body \( YNN \) calculations suggested that there is a quasibound state in \( \Sigma NN \) systems. However, the \( \Sigma \) width is unsettled because the \( \Sigma N \) potential still remains quantitative ambiguities. It seems that the shape and magnitude of the peak for the \( \Sigma NN \) quasibound state is very sensitive to a value of its width. We demonstrate behavior of the \( \pi^\mp \) spectrum near the \( \Sigma \) threshold in order to compare it with experimental observations. We introduce an artificial factor of \( f_W \) changing the strength of the spreading potential:

\[
W_{\alpha\alpha'} \rightarrow f_W \times W_{\alpha\alpha'}.
\]

(41)

Here let us consider several cases of various widths in the \( \pi^\mp \) spectrum as follows:

(i) Case A was obtained by \( f_W = 1.00 \), leading to a broad width of \( \Gamma_\Sigma \approx 9 \) MeV. This width was suggested by a Faddeev calculation for the \( \Lambda + d \rightarrow \Sigma + N + N \) scattering near the \( \Sigma \) threshold by Afnan and Gibson [16].

<table>
<thead>
<tr>
<th>Case</th>
<th>( f_W )</th>
<th>( E_{\Sigma^+} ) (MeV)</th>
<th>( E_{\Sigma^0} ) (MeV)</th>
<th>( \Gamma_\Sigma ) (MeV)</th>
<th>( k_{\Sigma^+} ) (fm(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>+0.96</td>
<td>-3.24</td>
<td>9.0</td>
<td>-0.322 + i0.260</td>
</tr>
<tr>
<td>B</td>
<td>0.75</td>
<td>-0.10</td>
<td>-4.30</td>
<td>6.9</td>
<td>-0.249 + i0.257</td>
</tr>
<tr>
<td>C</td>
<td>0.50</td>
<td>-0.94</td>
<td>-5.14</td>
<td>4.8</td>
<td>-0.176 + i0.257</td>
</tr>
<tr>
<td>D</td>
<td>0.25</td>
<td>-1.53</td>
<td>-5.73</td>
<td>2.8</td>
<td>-0.101 + i0.259</td>
</tr>
</tbody>
</table>

FIG. 12. Shape dependence of the calculated \( \pi^- \) spectrum in the \( ^3\text{He}(K^-, \pi^-) \) reaction near the \( \Sigma \) threshold at 600 MeV/c (4\(^\circ\)), when changing the spreading potential \( W_{\alpha\alpha'} \) artificially by \( f_W = (a) \) 1.00, (b) 0.75, (c) 0.50, and (d) 0.25, which correspond to widths of \( \Gamma_\Sigma = 9.0, 6.9, 4.9, \) and 2.9 MeV, respectively. These spectra are obtained by folding with a detector resolution of 2 MeV FWHM.
TABLE VII. Energies and widths of the $^3_2 n$ quasibound state with $J^p = 1/2^+$, $T \simeq 1$ on complex energy plane when the spreading potential $W_{\alpha\alpha}'$ is artificially changed.

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_W$</th>
<th>$E_{\Sigma^-}$ (MeV)</th>
<th>$E_{\Sigma^+}$ (MeV)</th>
<th>$\Gamma_{\Sigma^-}$ (MeV)</th>
<th>$k_{\Sigma^-}$ (fm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>-1.87</td>
<td>-0.58</td>
<td>10.5</td>
<td>-0.263 + i0.374</td>
</tr>
<tr>
<td>B</td>
<td>0.75</td>
<td>-2.79</td>
<td>-1.50</td>
<td>8.1</td>
<td>-0.200 + i0.380</td>
</tr>
<tr>
<td>C</td>
<td>0.50</td>
<td>-3.51</td>
<td>-2.22</td>
<td>5.7</td>
<td>-0.138 + i0.388</td>
</tr>
<tr>
<td>D</td>
<td>0.25</td>
<td>-4.02</td>
<td>-2.73</td>
<td>3.2</td>
<td>-0.077 + i0.396</td>
</tr>
</tbody>
</table>

(ii) Case B was obtained by $f_W = 0.75$, leading to $\Gamma_{\Sigma^-} \simeq 7$ MeV, of which width was predicted in three-body calculations within a SAP approximation by Koike and Harada [18].

(iii) Case C was obtained by $f_W = 0.50$, leading to $\Gamma_{\Sigma^-} \simeq 5$ MeV, which is equivalent to a half of the width of Case A.

(iv) Case D was obtained by $f_W = 0.25$, leading to a narrow $\Sigma^- n$ width of $\Gamma_{\Sigma^-} \simeq 2–3$ MeV in recent Faddeev $\Lambda NN - \Sigma NN$ calculations using $NN$ and $YN$ potentials derived from a chiral constituent quark models by Garcilazo et al. [19].

In Table VI, we obtain the calculated values of energies and widths of the $^3_2 \Lambda$ quasibound state on the complex $E$ plane. Figure 12 shows dependence of the shape and magnitude of the $\pi^-$ spectrum on the width. In Case A, we confirm that a peak of the $\pi^-$ spectrum is enhanced just below the $d + \Sigma^+$ threshold, as already seen in Fig. 8. In Case B, we also recognize that the peak can be observed as a candidate of the $\Sigma$ hypernuclear bound state, having a narrow width of $\Gamma_{\Sigma^-} \simeq 7$ MeV. This is equivalent to the value of $\langle \nu \sigma_{\Sigma^- n \rightarrow \Lambda n} \rangle_{\nu}$, where $\nu$ and $\sigma_{\Sigma^- n \rightarrow \Lambda n}$ are the velocity and total cross section data of $\Sigma^- n \rightarrow \Lambda n$ at low energies, respectively. In Cases C and D, we find that a peak of the quasibound state is more clearly observed below the $\Sigma$ threshold, so that it might be a very good candidate if the $\Sigma NN$ quasibound state has such a narrow width. Consequently, the $\pi^-$ spectrum near the $\Sigma$ threshold provides valuable information to understand the nature of $\Sigma N$ potentials and the structure of the $\Sigma NN$ quasibound state.

In Table VII, we obtain the calculated values of energies and widths of the $^3_2 n$ quasibound state on the complex $E$ plane. Figure 13 shows the shape and magnitude of the $\pi^+$ spectrum on the corresponding widths. In Cases A and B, we find that the shape of the spectrum is scarcely changed by a value of $f_W$. This originates from the fact that the contributions of the $\{pn\} \Sigma^-$ and $\{pn\} \Sigma^+$ components predominantly occur in continuum states, and the shape of continuum spectrum is insensibly influenced by the spreading potential $W_{\alpha\alpha}'$. For Case D, the quasibound state has a very narrow width of $\Gamma_{\Sigma^-} \simeq 3$ MeV, observed below the $n + n + \Sigma^0$ threshold. However, the cross section of the quasibound state is rather small, compared with the continuum one, because of a reduction mechanism caused by the interference effects in the $\pi^+$ spectrum, as discussed in Sec. VB.
D. Comparison with experimental data

It has been recognized that there is no evidence of a narrow structure for the $\Sigma NN$ quasibound state ($\Sigma_{\pi}$) below the $\Sigma$ threshold in the $^3\text{He}(K^-, \pi^+)$ reaction from E774 experiments at BNL [20]. Figure 14 shows the calculated inclusive $\pi^+$ spectrum at $p_{K^-} = 600$ MeV/c (4°), in order to be compared with the BNL-E774 data [20]. Here the spectrum was obtained by folding with a detector resolution of 5 MeV FWHM. We find that the calculated $\pi^+$ spectrum is in good agreement with the data. Because the shape of the spectrum is not so sensitive to its width, it seems that it is difficult to extract information on the width of the quasibound state from the data in the $\pi^+$ spectrum. Consequently, contradictory arguments against the existence of a $\Sigma NN$ bound state may be settled in our calculations.

VI. SUMMARY AND CONCLUSIONS

We have theoretically demonstrated the inclusive and semiexclusive spectra in the $^3\text{He}(K^-, \pi^+)$ reactions at 600 MeV/c (4°) within the DWIA, using the coupled $(2N - \Lambda) + (2N - \Sigma)$ model with the spreading potential. The effective $2N - Y$ potential derived from $YN$ $g$-matrices has strong isospin-spin dependence, and provides quasibound states ($\Sigma_{\pi}, \Sigma_{\pi}^*, \Sigma_{\pi}^\prime, \Sigma_{\pi}^\prime\prime, \Sigma_{\pi}^\prime\prime\prime, \Sigma_{\pi}^\prime\prime\prime\prime$) with $J^\pi = 1/2^+ (L = 0, S = 1/2), T \simeq 1$ near the $\Sigma$ threshold. The results are summarized as follows:

(i) The coupled-channel framework is essential for calculating the inclusive $\pi^-$ and $\pi^+$ spectra of the $^3\text{He}(K^-, \pi^+)$ reactions, in order to consider significant effects of interference between $K^- + N \rightarrow \pi + Y$ amplitudes and the threshold energy differences.

(ii) The effective $2N - Y$ potential is constructed by a folding-model potential with $YN$ $g$-matrices derived from the central $D^2$ potential, which is simulated to the Nijmegen model D. Such folding potentials can overcome serious overbinding problems in $s$-shell $\Lambda$ hypernuclei.

(iii) The calculated inclusive spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction shows a signal of the $\Sigma_{\pi}$ quasibound state with $J^\pi = 1/2^+$, $T = 1$ near the $\Sigma$ threshold, and its width has $\Gamma_{\Sigma} \simeq 9$ MeV.

(iv) The calculated inclusive spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction shows no peak of the $\Sigma_{\pi}$ quasibound state that is located near the $\Sigma$ threshold with $\Gamma_{\Sigma} = 10.5$ MeV, by interference effects caused by $\Sigma_{\pi}^\prime-\Sigma_{\pi}^\prime\prime$ admixture in the $NN$ pair for $\Sigma_{\pi}$ and the CSB effects. This spectrum is consistent with the BNL-E774 data.

In conclusion, we show that a signal of the $\Sigma_{\pi}$ quasibound state is clearly confirmed near the $\Sigma$ threshold in the $\pi^-$ spectrum, whereas the peak of the $\Sigma_{\pi}$ and $\pi^+$ spectra on $^3\text{He}$ targets provide valuable information on properties of $\Sigma NN$ quasibound states so as to study the $\Sigma N$ interaction.

ACKNOWLEDGMENTS

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APPENDIX A: BINDING ENERGIES OF $s$-SHELL $\Lambda$ HYPERNUCLEI IN FOLDING-MODEL CALCULATIONS

Let us consider $\Lambda$ binding energies of $\Lambda^*_3\text{He}, 4\Lambda^*_2\text{He}, 4\Lambda^*\text{He},$ and $5\Lambda^*\text{He}$ in folding models. We obtain $YN$ $g$-matrices in Eq. (32), solving the coupled Bethe-Goldstone equation

\[
\begin{pmatrix}
\Psi_A \\
\Psi_E
\end{pmatrix} = \begin{pmatrix}
\Phi_A \\
\Phi_E
\end{pmatrix} + \frac{Q}{e} \begin{pmatrix}
\nu \\
-\nu
\end{pmatrix} \begin{pmatrix}
\Psi_A \\
\Psi_E
\end{pmatrix},
\]

where $e$ and $Q$ are the energy denominator and Pauli exclusion operator, respectively [50,51], $\nu$ is a $YN$ potential including the $\Lambda N - \Sigma N$ coupling. The $D^2$ potential [2] is a central $YN$ potential that simulates the Nijmegen model D [23]. The $D^2$ potential we used in this paper is a modified version to reproduce the experimental value of $B_A(\Lambda^*_3\text{He})$, multiplying the strength of the long-range part in the $\Sigma N \Sigma_1$ by a factor (0.954) [27]. In Table VIII, we show the calculated results of $\Lambda$ binding energies of $s$-shell $\Lambda$ hypernuclei in the folding models. We obtain $B_\Lambda = 0.13, 2.4, 1.1,$ and 3.1 MeV for $\Lambda^*_3\text{He}, 4\Lambda^*_2\text{He}, 4\Lambda^*\text{He},$ and $5\Lambda^*\text{He}$, respectively. Here parameters of starting energies and Fermi momentum were taken to be ($E_x, k_F$) = (−2.2 MeV, 1.05 fm−1), (−8 MeV, 1.05 fm−1), (−8 MeV, 1.05 fm−1), (−28 MeV, 1.30 fm−1) for $\Lambda^*_3\text{He}, 4\Lambda^*_2\text{He}, 4\Lambda^*\text{He},$ and $5\Lambda^*\text{He}$, respectively. In order to compare them with the experimental data [56], we need to include rearrangement energies by $-\kappa_N U_{\text{arr}}$ [57,58] where we choose $\kappa_N = 0.06, 0.08, 0.08, and 0.115$ for $\Lambda^*_3\text{He}, 4\Lambda^*_2\text{He}, 4\Lambda^*\text{He},$ and $5\Lambda^*\text{He}$, respectively. We confirm that the calculated values of $B_\Lambda$ can reasonably...
TABLE VIII. Binding energies and $\Sigma$-mixing probabilities of $s$-shell $\Lambda$ hypernuclei in the folding-model potential calculations for g-matrices with the $YN$ $D^2$ potential, together with those obtained in Brueckner-Hartree-Fock [2] and SVM calculations [25,27]. Data are taken from Ref. [56].

<table>
<thead>
<tr>
<th></th>
<th>$^3\text{H}$ (1/2$^+$)</th>
<th></th>
<th>$^4\text{He}$ (0$^+$)</th>
<th></th>
<th>$^4\text{He}^*$ (1$^+$)</th>
<th></th>
<th>$^5\text{He}$ (1/2$^+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_\Lambda$ (MeV)</td>
<td>$P_\Sigma$ (%)</td>
<td>$B_\Lambda$ (MeV)</td>
<td>$P_\Sigma$ (%)</td>
<td>$B_\Lambda$ (MeV)</td>
<td>$P_\Sigma$ (%)</td>
<td>$B_\Lambda$ (MeV)</td>
<td>$P_\Sigma$ (%)</td>
</tr>
<tr>
<td>This work$^a$</td>
<td>with $-\kappa_N U$</td>
<td>0.13</td>
<td>0.07</td>
<td>2.4</td>
<td>2.0</td>
<td>1.1</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>w/o $-\kappa_N U$</td>
<td>0.26</td>
<td>0.10</td>
<td>3.0</td>
<td>2.1</td>
<td>1.5</td>
<td>0.03</td>
</tr>
<tr>
<td>BHF [2]</td>
<td></td>
<td>2.4</td>
<td>1.9</td>
<td>0.91</td>
<td>0.42</td>
<td>3.18</td>
<td>0.61</td>
</tr>
<tr>
<td>SVM [25,27]</td>
<td></td>
<td>0.056</td>
<td>0.14</td>
<td>2.23</td>
<td>1.85</td>
<td>0.13 $\pm$ 0.05</td>
<td>2.39 $\pm$ 0.03</td>
</tr>
</tbody>
</table>

$^a$Starting energies and Fermi momenta are used as $(E_\Sigma, k_\Sigma) = (2.4 \text{ MeV}, 1.05 \text{ fm}^{-1})$. $^b$Correction of rearrangement energies is taken from $-\kappa_N U$ [57,58], where we choose $\kappa_N = 0.06, 0.08, 0.08$, and 0.115 for $^3\text{H}$, $^4\text{He}$, $^4\text{He}^*$, and $^5\text{He}$, respectively.

reproduce the corresponding experimental data. This fact supports the importance of the coherent $\Lambda N - \Sigma N$ coupling in $s$-shell hypernuclei [2]. It should be noticed that our folding-model calculations provide to explain properties of the $s$-shell hypernuclei, whereas the values of $P_\Sigma$ for $^4\text{He}^*$ and $^5\text{He}$ should be in disagreement with those of SVM because no $D$-wave component is included in our model space.

In order to describe properties of $\Sigma$ hypernuclei, we must consider the binding energy and width of a $^4\text{He}$ quasibound state with $J^\pi = 0^+$, $T = 1/2$. Let us calculate a pole position of $^4\text{He}$ which is located on the complex $E$ plane by the complex scaling method [54,55], when we use $(E_\Sigma, k_\Sigma) = (2.4 \text{ MeV}, 1.05 \text{ fm}^{-1})$ and $\kappa_N = 0.08$ as the parameters in folding-model calculations. In Table IX, we show the calculated result of the binding energy and width, in order to be compared with those of experimental data [9,10]. We find $B_{\Sigma^+} = 0.89 \text{ MeV}$ and $\Gamma_{\Sigma} = 11.8 \text{ MeV}$ with $YN$ g-matrices derived from the $D^2$ potential where $E_{\Sigma^+} = -B_{\Sigma^+} - i\frac{1}{2} \Gamma_{\Sigma}$ where $B_{\Sigma^+}$ is measured from the $^3\text{H} + \Sigma^+$ threshold.

TABLE IX. Binding energy and width of the $^4\text{He}$ quasibound state with $J^\pi = 0^+$, $T = 1/2$ in the folding-model potential calculations for g-matrices with the $YN$ $D^2$ potential, in a comparison with analysis of theoretical calculation [31] and experimental data taken from Refs. [9,10].

<table>
<thead>
<tr>
<th></th>
<th>$B_{\Sigma^+}$ (MeV)</th>
<th>$\Gamma_{\Sigma}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work$^a$</td>
<td></td>
<td>0.83</td>
</tr>
<tr>
<td>Harada [31]</td>
<td></td>
<td>1.1</td>
</tr>
<tr>
<td>Ota et al. [9]</td>
<td></td>
<td>2.8 $\pm$ 0.7</td>
</tr>
<tr>
<td>Nagae et al. [10]</td>
<td></td>
<td>4.4 $\pm$ 0.3 $\pm$ 1</td>
</tr>
</tbody>
</table>

$^a$Starting energy and Fermi momentum of $(E_\Sigma, k_\Sigma) = (2.4 \text{ MeV}, 1.05 \text{ fm}^{-1})$, and the rearrangement energy with $\kappa_N = 0.08$ are used.

APPENDIX B: PRODUCTION AMPLITUDES FOR $(K^-, \pi^\pi$) REACTIONS ON ISOSPIN $\Sigma NN$ STATES

In order to understand behavior of the $\pi^\pi$ spectra, we consider $\Sigma$ production amplitudes for isospin $\Sigma NN$ states in $^3\text{He}(K^-, \pi^\pi$) reactions. In the $\pi^\pi$ spectrum, we recognize that the $^4\text{He}$ quasibound state is populated via $\Gamma_{(\Sigma N)}$, $\Gamma_{(\Sigma^+ N)}$, and $\Gamma_{(\Sigma^+ \Sigma)}$, and this state is identified by a configuration of $\Psi_{(j;S_2)}$. We obtain

$$\Psi_{(s)}^{2,1} = \frac{1}{\sqrt{2}} |\{pp\}\Sigma^0_0 + \frac{1}{\sqrt{2}} |\{pn\}\Sigma^+_0 + \frac{1}{\sqrt{2}} |\{pn\}\Sigma^-_0 + \frac{1}{\sqrt{2}} |\{pp\}\Sigma^-_0,$$  \(\Psi_{(s)}^{1,1} = \frac{1}{\sqrt{2}} |\{pp\}\Sigma^0_0 - \frac{1}{\sqrt{2}} |\{pn\}\Sigma^+_0 - \frac{1}{\sqrt{2}} |\{pn\}\Sigma^-_0 + \frac{1}{\sqrt{2}} |\{pp\}\Sigma^-_0, \)  \(\Psi_{(t)}^{1,1} = |\{pn\}\Sigma^+_0, \)

where $s$ and $t$ denote spin-singlet ($j_2 = 1, S_2 = 0$) and spin-triplet ($j_2 = 0, S_2 = 1$) states, respectively, for the $2N$ pair in the $\Sigma NN$ systems. Therefore, we have total isospin $T = 1$ good states as

$$\Psi_{(s)}^{1,1} = a\Psi_{(s)}^{1,1} \pm b\Psi_{(t)}^{1,1},$$

owing to the strong admixture of $(J_2 = 1, S_2 = 0)$ and $(1,0)$ states in the $NN$ pair. Because the $2N$ -- $Y$ potential should admix $^3\Sigma_1$ and $^3\Sigma_0$ states in the $NN$ pair, depending on the nature of the $\Sigma N$ potential [18], interference effects of $\Sigma$ production amplitudes are important to make a shape of the $\pi^\pi$ spectrum. As seen in Sec. V A, when $a = b = 1/\sqrt{2}$ for simplicity we obtain $\Psi_{(s)}$ that corresponds to the ground state in $^4\text{He}$, and $\Psi_{(s)}^{1,1}$ to an excited state. If we approximately omit $\Lambda$ production amplitude of $\Gamma_{(\Lambda \pi^-)}$ in the $\Sigma$ threshold region, we obtain production amplitude for the $(K^-, \pi^-)$ reaction as

$$\langle \Sigma^+ \pi^- | T | ^3\text{He}K^- \rangle 
\simeq \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \Gamma_{(\Sigma^+ \Sigma)} - \Gamma_{(\Sigma^+ \pi^-)} \right\} \{\Psi_{(s)}^{2,1} |\Psi_{(\pi)}^{(3\text{He})}\}.$$
We find that interference effects between $\overline{f}_{(\Sigma^+\pi^-)}$ and $\overline{f}_{(\Sigma^0\pi^-)}$ play an important role in populating $\Psi_{(\pi^+)}^{(-1)}$ and $\Psi_{(\pi^+)}^{(+1)}$ within the $\Sigma NN$ states with $T = 1$. Considering the relative phase $\varphi(\Sigma^+/\Sigma^0) = +4.8^\circ$ between $\overline{f}_{(\Sigma^+\pi^-)}$ and $\overline{f}_{(\Sigma^0\pi^-)}$ at $p_{\pi^-} = 600$ MeV/c, we obtain that the component of $\Psi_{(\pi^+)}^{(-1)}$ that is identified as the $\frac{3}{2}^+ n$ quasibound state with $J^\pi = 1/2^+$, $T = 1$, is relatively enhanced in the $\pi^-$ spectrum, whereas the component of $\Psi_{(\pi^+)}^{(+1)}$ in $\Sigma$ continuum states is reduced.

For the $\pi^+$ spectrum, we find that the $\frac{3}{2}^+ n$ quasibound state is populated via only $\overline{f}_{(\Sigma^+\pi^-)}$. We obtain

$$\Psi_{(\pi^+)}^{(-1)} = \frac{1}{\sqrt{2}} |\langle pn \rangle \Sigma^+\rangle + \frac{1}{\sqrt{2}} |\langle nn \rangle \Sigma^0\rangle, $$
$$\Psi_{(\pi^+)}^{(+1)} = \frac{1}{\sqrt{2}} |\langle pn \rangle \Sigma^-\rangle - \frac{1}{\sqrt{2}} |\langle nn \rangle \Sigma^0\rangle, $$
$$\Psi_{(\pi^+)}^{(-1)} = |\langle pn \rangle \Sigma^-\rangle, $$

where the isospin $T = 1$ good states are written as

$$\Psi_{(\pi^+)}^{(-1)} = a\Psi_{(\pi^+)}^{(-1)} + b\Psi_{(\pi^+)}^{(+1)}. $$

If $a = b = 1/\sqrt{2}$, $\Psi_{(\pi^+)}^{(-1)}$ and $\Psi_{(\pi^+)}^{(+1)}$ are regarded as ground and excited states in $\frac{3}{2}^+ n$, respectively. Thus the production amplitude for the $(K^-, \pi^+)$ reaction is

$$\langle \frac{3}{2}^+ n \pi^+ | T \rangle | 3^+ HeK^- \rangle$$
$$\simeq \overline{f}_{(\Sigma^-\pi^+)} \left\{ \frac{1}{2} |\langle \psi_{(\pi^+)\Sigma^+} | 3^+ He\rangle| 2 \right.$$\n$$+ \frac{2\sqrt{3} - \sqrt{2}}{4} |\langle \Psi_{(\pi^+)}^{(-1)} | 3^+ He\rangle| \right.$$\n$$\left. + \frac{2\sqrt{3} + \sqrt{2}}{4} |\langle \Psi_{(\pi^+)}^{(+1)} | 3^+ He\rangle| \right\}. $$

We find that production amplitude for $\Psi_{(\pi^+)}^{(-1)}$ is relatively reduced by a factor $(2\sqrt{3} - \sqrt{2})/4 = 0.51$, whereas that for $\Psi_{(\pi^+)}^{(+1)}$ is enhanced by a factor $(2\sqrt{3} + \sqrt{2})/4 = 1.22$. This mechanism is inevitable whenever we consider the $3^+ He(K^-, \pi^+)$. 