\( \Sigma NN \) quasibound states in \(^3\)He\((K^-,\pi^+)\) reactions at 600 MeV/c

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We theoretically demonstrate the inclusive and semiexclusive spectra in the \(^3\)He\((K^-,\pi^+)\) reactions at 600 MeV/c (\(4\)\(^+\)) within a distorted-wave impulse approximation, using a coupled \((2N - \Lambda) + (2\Sigma - \Xi)\) model with a spreading potential. We present the possible existence of the \(\Sigma NN\) quasibound state with \(J^P = 1/2^+\), \(T \approx 1\) near the \(\Sigma\) threshold, predicted by a \(2N - Y\) folding-model potential derived from \(YN\) g-matrices. The result shows that a signal of the \(^3\)He quasibound state is clearly confirmed near the \(\Sigma\) threshold in the \(\pi^-\) spectrum, whereas a peak of the \(^3\)He quasibound state is rather reduced in the \(\pi^+\) spectrum owing to the interference effects caused by the \(^3\)S\(_1\)-\(\Xi_0\) admixture in the \(NN\) pair. The mechanism of \(\Sigma\) production for these spectra and charge symmetry breaking effects are also discussed.

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I. INTRODUCTION

One of the most important subjects on strangeness nuclear physics is to understand properties of a \(\Sigma\) hyperon in nuclei as well as \(\Sigma N\) interaction [1]. The \(\Sigma\) hyperon is expected to play an essential role in the description of the \(\Delta NN\) three-body force in hypernuclei [2], and the maximal mass and particle fraction of neutron stars or compact stars [3]. However, the \(\Sigma N\) interaction has still been in quantitative ambiguities because the \(\Sigma N\) scattering data are very limited [1,4].

In the 1990s, many efforts were made in \(\Sigma\) hypernuclear studies on \(s\)- and \(p\)-shell nuclei using \((K^-,\pi^+)\) reactions at CERN, BNL, and KEK [5,6]. It has been known that there is no evidence of a \(\Sigma\) nuclear state [7], except \(^3\)He which is established to be a quasibound (or unstable bound) state experimentally [8–10], as predicted in Ref. [11]. Saha et al. [12] reported that there is a strong repulsion in the real part with a sizable imaginary part of the \(\Sigma\)-nucleus potential analyzing nuclear \((\pi^-,K^+)\) spectra on C, Si, Ni, In, and Bi targets. This repulsion originates from the \(\Sigma N\) \(^3\)S\(_0\), \(I = 3/2\) channel that corresponds to a quark Pauli-forbidden state in baryon-baryon systems in the flavor SU(6) symmetry [13].

Several theoretical works [14–18] performed to investigate the \(\Sigma NN\) systems that have total isospin \(T = 0, 1, 2\) and total spin \(S = 1/2, 3/2\). Garcilazo [14] showed that the \(\Sigma^- nn\) system with \(T = 2\) has no bound state in a Faddeev calculation with a separable \(\Sigma N\) potential, and Stadler and Gibson [15] confirmed it using the Jülich potential. Afman and Gibson [16] demonstrated that an enhancement in the \(\Lambda d\) cross section near the \(\Sigma + N + N\) threshold is associated with a resonance pole of the \(\Sigma NN\) states having \(T = 0, S = 1/2\) in the scattering amplitude. Dover et al. [17] discussed the spin-isospin selectivity of the \(\Sigma N \to \Lambda N\) conversion decay in the \(\Sigma NN\) states, assuming that the isospin and spin of the \(NN\) pair, \(I_2\) and \(S_2\), are good quantum numbers. Their results suggested that the \(\Sigma NN\) state with \(T = 0, S = 1/2\) is the best candidate to be bound and relatively long lived when the \(NN\) state takes \(I_2 = 0, S_2 = 1\ [5]\). It should be noticed that the \(NN\) states with \((I_2,S_2) = (1,0)\) and \((0,1)\) admix each other in the \(\Sigma NN\) state. Indeed, Koike and Harada [18] performed three-body \(\Sigma NN\) coupled-channel calculations, leading to the fact that there exist quasibound states of \(T = 1, S = 1/2\) in the isotriplet \((\frac{1}{2}He,\frac{3}{2}He,\frac{3}{2}n)\) where the \(\Sigma N\) potential strongly admixes \((I_2,S_2) = (1,0)\) and \((0,1)\) states in the \(NN\) pair. Recently, Garcilazo et al. [19] have shown that a narrow quasibound state with \(\Gamma \approx 2.1\) MeV exists near the \(\Sigma\) threshold in the \(3\)He\((\pi^-,\pi^+)\) reaction at BNL-E774 experiments [20]. These contradictory arguments are still not settled: Is there a quasibound state in \(\Sigma NN\) systems?

In this paper, we theoretically demonstrate the inclusive and semiexclusive spectra in \(^3\)He\((K^-,\pi^+)\) reactions at 600 MeV/c (\(4\)\(^+\)) within a distorted-wave impulse approximation (DWIA), using a coupled \((2N - \Lambda) + (2\Sigma - \Xi)\) model with a spreading potential. We focus on the behavior of signals of the \(\Sigma NN\) quasibound states observed in \(^3\)He and \(^3\)He in \(\pi^-\) and \(\pi^+\) spectra, respectively, in order to study the \(\Sigma N\) interaction and also a mechanism of \(\Sigma\) production for these spectra.

In a previous paper [21], we reported theoretical calculations of \(^3\)He\((K^-,\pi^+)\) spectra at 600 MeV/c, using the \(2N - Y\) folding-model potential with g-matrices derived from the NF5 potential [22] that simulates the Nijmegen model F [23]. The result suggested that there are quasibound states in the \(\Sigma NN\)
systems. However, the folding models using NF\(_3\) were not able to systematically explain A binding energies of \(3^2\)H, \(4^4\)He, \(4^4\)He*, and \(5^5\)He. One of the prescriptions to solve this overbinding problem of s-shell A hypernuclei [24] may be to consider the coherent \(\Lambda N - \Sigma N\) coupling [2]. The \(D2\) potential [2,25] is a central version of the \(YN\) potential that simulates the Nijmegen model D [26], and it is fulfilled in the coherent \(\Lambda N - \Sigma N\) coupling. In this paper, therefore, we use a slightly modified potential (\(D2'\)) that is adjusted to the binding energies of s-shell hypernuclei [27].

The outline of this paper is as follows. In Sec. II, we will briefly mention the DWIA framework for \(^3\)He(\(\bar{K}^-\),\(\pi^+\)) reactions, employing the coupled \((2N - \Lambda) + (2N - \Sigma)\) model in a Green’s function technique [28,29]. In Sec. III, we will construct an effective \(2N - Y\) potential within a microscopic folding model with \(YN\) g-matrices, and will discuss the structure of the \(\Sigma N N\) quasibound states, taking into account threshold effects due to the mass difference among \(\Sigma\) hyperons and the Coulomb forces. A pole position for the \(\Sigma NN\) quasibound state is obtained on the complex \(E\) plane. In Sec. IV, we will demonstrate the calculated spectra of the \(^3\)He(\(\bar{K}^-\),\(\pi^+\)) reactions at 600 MeV/c to see a possible signal of the \(\Sigma NN\) quasibound state, which might be observed in forthcoming experiments at J-PARC facilities [4]. In Sec. V, we will discuss the sensitivity of the \(\pi^+\) spectrum to a pole position of the \(\Sigma NN\) quasibound state. We will consider the reason why the peak of a \(\pi^+n\) quasibound state populated in the \(\pi^+\) spectrum is not observed, and compare the calculated \(\pi^+\) spectrum with the data observed in BNL-E774 experiments. Summary and conclusion are given in Sec. VI. In the Appendix, we will report binding energies of \(^3\)H, \(^4\)He, \(^4\)He*, and \(^5\)He obtained by the folding-model calculations with \(D2'\), and interference effects for cross sections by (\(K^-\),\(\pi^+\)) reactions owing to the \(3^1\)-\(3^0\) admixture of the \(NN\) pair.

**II. CALCULATIONS**

Let us consider a theoretical framework for \(^3\)He(\(\bar{K}^-\),\(\pi^+\)) reactions. Here we treat hypernuclear final states classified as

\[
K^-\text{He} \rightarrow \pi^- pp\Lambda,
\pi^- dp\Sigma^+,
\pi^- pn\Sigma^+,
\pi^- pp\Sigma^0,
\]

for the \(\pi^-\) spectrum, and those as

\[
K^-\text{He} \rightarrow \pi^+ nn\Lambda,
\pi^+ dp\Sigma^-,
\pi^+ pn\Sigma^-,
\pi^+ nn\Sigma^0,
\]

for the \(\pi^+\) spectrum. It seems that an impulse approximation works well in \(K^-\) capture at incident \(K^-\) beam of \(p_{K^-} = 600\) MeV/c [30]. Figure 1 illustrates typical diagrams for physical processes for the \((K^-,\pi^+)\) reactions. We will calculate these spectra within the distorted-wave impulse approximation (DWIA) for the \((2N - \Lambda) + (2N - \Sigma)\) model with a spreading potential [31], as we mention in this section.

**A. Model wave functions**

In our calculation, we consider a model wave function \(\Psi_A\) of the \(^3\)He ground state \((J^P = 1^+\), \(T = 1/2\)) as a target nucleus, in the \(LS\)-coupling scheme. It is given by

\[
|\Psi_A\rangle = \tilde{A}\left[\left|\phi_0^{(2N)} \otimes \phi_0^{(\Lambda)}\right|_{L_A} \otimes X^{A}_{I_{A},S_{A}}\right]^{M_A},
\]

\[
X^{A}_{I_{A},S_{A}} = \left[\left|\chi_{I_{A},S_{A}}^{(2N)} \otimes \chi_{I_{A}/2,1/2}^{(\Lambda)}\right|_{1/2,1/2}\right],
\]

where \(\tilde{A}\) is the antisymmetrized operator for nucleons, \(\phi_0^{(2N)}\) is the wave function of a \(2N\)-core subsystem, and \(\phi_0^{(\Lambda)}\) is the relative wave function between \(2N\) and \(\Lambda\) for a \(2N - N\) system in the \(^3\)He ground state. \(X^{A}_{I_{A},S_{A}}\) is the isospin-spin function for \(^3\)He, and \(\chi_{I_{A},S_{A}}^{(2N)}\) and \(\chi_{I_{A}/2,1/2}^{(\Lambda)}\) are the isospin-spin functions for \(2N\) (isospin \(I_{2N}\), spin \(S_{2N}\)) and \( \Lambda \) (\(I_{\Lambda} = 1/2\), \(S_{\Lambda} = 1/2\)), respectively.

We obtain the wave function \(\phi_0^{(\Lambda)}\) using the \(2N - N\) potential \(U^{(\Lambda)}\) that was derived from microscopic three-body calculations [11] with a central potential of Tamagaki’s C3G [32]. This potential can reproduce the experimental data of the binding energy of \(B_\Lambda = 8.07\) MeV and the nuclear root-mean-square distance of \((R^2)^{1/2} = 2.64\) fm for the \(^2\)H + \(p\) system in \(^3\)He [33].

For hypernuclear \(YN\) final states, we consider wave functions \(\Psi_B\) for \(2N - Y\) systems with \(J^P = \frac{1}{2}\) on physical particle (charge) bases. The wave functions are written by

\[
|\Psi_B\rangle = \sum_{\alpha} \left[\left|\phi_{\alpha}^{(2N)} \otimes \phi_{I_{\gamma}S_{\gamma}}^{(Y)}\right|_{L_B} \otimes X^{B}_{I_{\gamma},S_{\gamma}}\right]^{M_B},
\]

\[
X^{B}_{I_{\gamma},S_{\gamma}} = \left[\left|\chi_{I_{\gamma},S_{\gamma}}^{(2N)} \otimes \chi_{I_{\gamma}/2,1/2}^{(Y)}\right|_{1/2,1/2}\right],
\]

where \(\phi_{I_{\gamma}S_{\gamma}}^{(Y)}\) is the relative wave function between \(2N\) and \(Y\) with the angular momentum \(\ell_{\gamma}\), and \(X^{B}_{I_{\gamma},S_{\gamma}}\) is the isospin-spin
function for $2N - Y$ in the $\alpha$ channel; $\chi_{\alpha}^{(Y)}$ is the isospin-spin function for $Y$ (isospin $I_Y$, $S_Y = 1/2$). The channel indices $\alpha$ indicate the final states as listed in Table I, where $\{N_1 N_2\} = N_1 N_2 + N_2 N_1$ and $\{N_1 N_2\} = N_1 N_2 - N_2 N_1$ denote the 2N states with $S_0$, $I = 1$ and $S_1$, $I = 0$, respectively. Since a spin-flip process in $(K^-, \pi^\mp)$ reactions at low momentum regions, e.g., $p K^- = 600$ MeV/c may be negligibly small, as seen in Sec. II E, we consider only spin $S_0 = 1/2$ for the $2N - Y$ systems formed from $^3$He, omitting $S_3 = 3/2$. Therefore, we take the $2N - Y$ final states on $J_B = |L_B \pm 1/2|$ with $L_B = 0,1, \ldots , S_0 = 1/2$, where $J_B$ and $L_B$ are the total and orbital angular momenta, respectively.

We use the wave functions $\phi_{\alpha,i}^{(2N)}$, which are obtained with the C3G potential. The wave function $\phi_{p}^{(2N)}$ for the $\{pn\}$ ($S_0$, $I = 0$) state has a nuclear bound state with $B_N = 2.22$ MeV for $^2$H$(J^s = 1^+)$. Because there is no bound state in the $\{pp\}$ ($S_0$, $I = 1$) state, we use a continuum-discretized wave function $\phi_{\alpha,j}^{(2N)}$ which is obtained in the momentum bin method [34],

$$\tilde{\phi}_{\alpha,j}^{(2N)}(r) = \frac{1}{\Delta k} \int_{k_i}^{k_{i+1}} \phi_{\alpha,j}^{(2N)}(k,r) dk,$$

where $\Delta k = k_{i+1} - k_i$, and $r$ and $k$ are the radial coordinate and the momentum between two nucleons, respectively. The scattering wave function $\phi_{\alpha,j}^{(2N)}(k,r)$ satisfies the Schrödinger equation

$$(T_a + V_{a}^{(NN)}(r) - E_a)\phi_{\alpha,j}^{(2N)}(k,r) = 0$$

with the energy $E_a = k^2/2\mu > 0$, where $\mu$ is the reduced mass of the 2N system. This method is often used in continuum-discretized coupled-channel (CDCC) calculations [34], and it may work well in continuum dynamics involving $NN$ breakup processes. Here we used the only lowest discretized state with $k_0 = 0.0$ fm$^{-1}$ and $\Delta k = 0.20$ fm$^{-1}$ for the $\{pp\}$ state.

Figure 2 shows the wave functions $\phi_{\alpha}^{(2N)}$ of the $\{pn\}$ and $\{pp\}$ states, together with the intranuclear 2N wave function $\phi_0^{(2N)}$ in the $^3$He ground state, which is derived from three-body calculations with the C3G potential.

### B. Distorted-wave impulse approximation

According to the DWIA [28,35–37], the inclusive differential cross section for nuclear $(K^-, \pi^-)$ reactions in the laboratory frame is given by (in units $\hbar = c = 1$)

$$\frac{d^2\sigma}{dE_\pi d\Omega_\pi} = \beta \frac{1}{[J\Lambda]} \sum_{A,j} \sum_{B} |\langle \Psi_B | \tilde{F} | \Psi_A \rangle |^2 \times \delta(E_{\pi} + E_B - E_K - E_A)$$

with the strangeness-exchange external operator including zero-range $K^- N \rightarrow \pi Y$ interactions

$$\tilde{F} = \int dr \chi^{-\pi} (p_\pi, r) \chi^{(K)} (p_K, r)$$

$$\times \sum_{j=1}^{A} \int \tilde{f}_{\gamma j} (0, K_N) \delta (r - r_j) \hat{O}_j,$$

where $[J] = 2J + 1$; $E_\pi$, $E_K$, $E_B$, and $E_A$ are energies of an outgoing $\pi^\mp$, an incoming $K^-$, the hypernuclear state, and the target nucleus, respectively. The baryon operator $\hat{O}_j$ can change the $j$th nucleon into a hyperon in the nucleus, and $r$ is the relative coordinate between the mesons and the center of mass of the nucleus; $\tilde{f}_{\gamma j}$ is the Fermi-averaged amplitude for the $K^- N \rightarrow \pi + Y$ reaction in the nuclear medium on the laboratory frame, where $\omega_{K_N}$ is the total energy of $K^- - N$ subsystems. The momentum and energy transfer to the $2N - Y$ final state in these reactions is given by

$$q = p_K - p_\pi, \quad \omega = E_K - E_\pi,$$

where $p_K$ and $p_\pi$ ($E_K$, $E_\pi$) are the laboratory momenta (energies) of the incident $K^-$ and outgoing $\pi^\mp$ in the many-body $K^- + A \rightarrow \pi + \Lambda B^*$ reaction, respectively. The kinematical factor $\beta$ in Eq. (7) [38,39] expresses the translation from the two-body $K^- - N$ laboratory system to the $K^- - A$
laboratory system [40], which is given by
\begin{equation}
\beta = \left(1 + \frac{E_v^{(0)} E_{\pi}^{(0)} - p_{K}^{(0)} \cos \theta_{lab}}{E_v^{(0)} p_{\pi}^{(0)} E_{\pi}^{(0)}}\right) \frac{p_{\pi} E_{\pi}}{p_{\pi}^{(0)} E_{\pi}^{(0)}},
\end{equation}
where \(p_{K}^{(0)}\) and \(p_{\pi}^{(0)}\) (\(E_v^{(0)}\) and \(E_{\pi}^{(0)}\)) are momenta of \(K^-\) and \(\pi\) (energies of \(\pi\) and hyperon) in the two-body \(K^- + N \rightarrow \pi + Y\) reaction, respectively.

The distorted waves \(\chi_{K}^{(-)}(p_{\pi}, r)\) and \(\chi_{K}^{(+)}(p_{\pi}, r)\) in Eq. (8) express the outgoing \(\pi\) and incoming \(K^-\) ones, respectively [41]:
\begin{equation}
\chi_{K}^{(-)}(p_{\pi}, r) = \sum_{\lambda} \sqrt{4\pi/|\lambda|} j_{\lambda}(\theta_{lab}, r) Y_{\lambda}^{(r)},
\end{equation}
where \(j_{\lambda}(\theta_{lab}, r)\) is the radial distorted wave with the angular momentum \(\lambda\), and \(\theta_{lab}\) is the scattering angle to the forward direction in \((K^-, \pi)\) reactions. The computational procedure for the distorted waves is simplified with the help of the eikonal approximation [35,42] because the distortions for mesons are not so important in few-body systems.

According to the Green’s function method [28,29], we can rewrite a sum of the final states in Eq. (7) as:
\begin{equation}
\sum_{\lambda} |\Psi_R(\lambda)\rangle |\Psi_B(\lambda)\rangle \delta(E - E_B) = -\frac{1}{\pi} \text{Im} \hat{G}(E).
\end{equation}
Thus the inclusive differential cross section is written by
\begin{equation}
\frac{d^2\sigma}{dE d\Omega_{\pi}} = \beta \frac{1}{|J_A|} \sum_{M_A} S_\pi,
\end{equation}
where the strength function is given by
\begin{equation}
S_\pi = -\frac{1}{\pi} \text{Im} (F \hat{G}(E) F),
\end{equation}
where \(|F\rangle\) denotes the \(2N - Y\) doorway states excited initially by external field, which is defined as
\begin{equation}
|F\rangle = \tilde{F} |\Psi_A\rangle,
\end{equation}
involving the \(\tilde{f}(\gamma, \pi)\) amplitudes.

C. Coupled-channel Green’s functions

The Green’s function method [28,29] facilitates parametrizing complicated many-body effects in a simple and tractable way, keeping the proper aspects of quantum mechanical systems. This technique can well describe an unstable hadron system such as a \(\Sigma^-, \Xi^-, \) or \(K^-\) nuclear state [29]. The complete Green’s function \(\hat{G}\) in Eq. (14) [43] provides all information concerning hyperon-nucleus dynamics as a function of the energy transfer \(\omega = E_B - E_A\), which is related to the energy \(E_Y = E_B - (m_Y + M_C) = -B_Y\) measured from the \(Y^+\) core-nucleus threshold, where \(m_Y\) and \(M_C\) are masses of the \(Y\) and the core nucleus, respectively. Here we will consider \(2N - Y\) states within coupled \((2N - \Lambda) + (2N - \Sigma)\) channels with a spreading potential [30].

For \(2N - Y\) final states, the complete Green’s function in the \(P\) space is given by
\begin{equation}
\hat{G}(\omega) = \frac{1}{\omega - \hat{H} + i\epsilon} P,
\end{equation}
where \(\hat{H}\) is the total Hamiltonian of the \(2N - Y\) system with \(\hat{H}|\Psi_B(\lambda)\rangle = E_B|\Psi_B(\lambda)\rangle\), and \(P\) is the Feshbach’s projection operator for the model space we consider. Then we can calculate the complete Green’s function by solving the following equation:
\begin{equation}
\hat{G}(\omega) = \hat{G}(0) + \hat{G}(0) \hat{U} \hat{G}(\omega),
\end{equation}
where \(\hat{G}(0)\) is the free Green’s function for the \(2N - Y\) system, and \(\hat{U}\) is the operator of a potential energy for the relative motion between \(2N\) and \(Y\). In order to extend it to a coupled-channel system, we introduce projection operators of \(P_\alpha\) into the \(\alpha\) channel in the \(P\) space, where \(P = \sum_{\alpha} P_\alpha\). In the case of \(P = P_a + P_a'\), for example, we obtain
\begin{equation}
\hat{G}(\omega) = (P_a + P_a') \hat{G}(\omega)(P_a + P_a'),
\end{equation}
where we define \(\hat{G}_a(\omega) = P_a \hat{G}(\omega) P_a'\). The complete Green’s function for \(\alpha a'\) channels satisfies the following multichannel coupled equation:
\begin{equation}
\hat{G}_{a a'}(\omega) = \hat{G}_{a a'}^{(0)}(\omega) \delta_{a a'} + \hat{G}_{a a'}^{(0)}(\omega) \sum_\gamma \hat{U}_{a a'} \hat{G}_{\gamma a'}(\omega).
\end{equation}
Solving this coupled equation numerically, we obtain the complete Green’s function \(\hat{G}_{a a'}(\omega)\) [44]. Here we use partial waves of \(\hat{G}_{a a'}(\omega)\) with \(J_\beta\) as a function of the relative distance \(R\) between \(2N\) and \(Y\), and its explicit form is written as
\begin{equation}
G_{a a'}^{(J_\beta)}(\omega; R, R') = \sum_{LM} \Phi_{a a'}^{(2N)} |J_{\beta}^{(LM)}(\hat{R})\rangle |\phi_{a a'}^{(2N)}| J_{\beta}^{(LM)}(\hat{R})\rangle^\dagger
\end{equation}
with
\begin{equation}
\Phi_{a a'}^{(2N)} |J_{\beta}^{(LM)}(\hat{R})\rangle = [Y_{L A}^{(R)} \otimes X_{L A}^{(R)}] J_{\beta}^{(LM)}(\theta_{lab}, \omega),
\end{equation}
where \(g^{(2N)}_{a a'}(\omega; R, R')\) is the relative Green’s function for \(Ya'Ya'\) channels. The explicit form of the double-differential cross section of Eq. (13) is written as
\begin{equation}
\frac{d^2\sigma}{d\Omega_{\pi} dE_{\pi}} = \sum_{J_{a a'} YY} \mathcal{T}_{(Y, \pi)} |\tilde{f}_{a a'} YY| C_{a a'}^{(2N)} C_{a a' Y Y},
\end{equation}
where
\begin{equation}
I^{(a a') J_{\beta}}_{YY NN} (\theta_{lab}, \omega) = (-\frac{1}{\pi}) \text{Im} \int_0^\infty dR dR' R R' \phi_{(2N)}^{(N)}(R),
\end{equation}
\begin{equation}
\times \tilde{g}_{\lambda}^{(J_{\beta})} |(\theta_{lab})(\hat{R})\rangle |\phi_{a a'}^{(2N)}(\omega; R, R')\rangle
\end{equation}
\begin{equation}
\times \tilde{g}_{\lambda}^{(J_{\beta})} |(\theta_{lab})(\hat{R})\rangle |\phi_{a a'}^{(2N)}(\omega; R, R')\rangle
\end{equation}
\begin{equation}
\times \frac{1}{\omega - \hat{H} + i\epsilon} P.
\end{equation}

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where the factor of $M_C/M_A$ denotes the recoil effects, leading to the effective momentum transfer of $(M_C/M_A)q$. The isospin-spin spectroscopic factor for the $2N-Y$ channel is obtained as

$$C_{Y\alpha} = \langle X^B_{Y,S} \sum_{j=1}^{A} \hat{O}_j | X^A_{Y,S} \rangle. \quad (24)$$

In Table II, we show the values of $C_{Y\alpha}$ for non-spin-flip processes, which are seemed to be dominant ones in nuclear $(K^-, \pi)$ reactions near 600 MeV/c.

### D. The decomposition of the inclusive cross sections into components

The inclusive cross sections can be decomposed into partial cross sections corresponding to different physical processes [28,29,45], as classified in Table I. We obtain the decomposition of the strength function $S_\pi$ of Eq. (14) as

$$S_\pi^- = S_{\pi^-}^{(pp)\Lambda} + S_{\pi^-}^{(pp)\Sigma^+} + S_{\pi^-}^{(nn)\Sigma^+} + S_{\pi^-}^{(pp)\Sigma^0} + S_{\pi^-}^{(Conv)}$$

for the $\pi^-$ spectrum, and that as

$$S_\pi^+ = S_{\pi^+}^{(nn)\Sigma^-} + S_{\pi^+}^{(nn)\Sigma^+} + S_{\pi^+}^{(nn)\Sigma^0} + S_{\pi^+}^{(Conv)}$$

for the $\pi^+$ spectrum. The partial strength functions are defined by

$$S_\pi^0 = \frac{1}{\pi} \langle F | \hat{\Omega}^{(-)} | (\text{Im} G^0_\alpha) \hat{\Omega}^{(-)} | F \rangle,$$

$$S_{\pi^0}^{(Conv)} = \frac{1}{\pi} \sum_{\alpha'\alpha} \langle F | \hat{G}^\dagger_{\alpha'} W_{\alpha'\alpha} \hat{G}_{\alpha'} | F \rangle,$$

where we used the identity

$$\text{Im} \hat{G} = \hat{\Omega}^{(-)} (\text{Im} G^0_\alpha) \hat{\Omega}^{(-)} + \hat{G}^\dagger (\text{Im} \hat{U}) \hat{G},$$

and $\hat{\Omega}^{(-)} = 1 + \hat{U} \hat{G}$ is the Möller wave operator, and $W_{\alpha'\alpha}$ is a spreading (imaginary) potential for the complicated nuclear excited states from the $\alpha'$ channel. It should be noticed that $\hat{\Omega}^{(-)} W_{\alpha'\alpha} \hat{G}_{\alpha'}$ denotes the spreading processes, which are predominantly caused by the $\Sigma N \rightarrow \Lambda N$ conversion into complicated $N+N+\Lambda$ states because a produced $\Sigma$ subsequently interacts with a second nucleon, and its converted $\Lambda$ pair gains large energy from the mass difference $m_\Sigma - m_\Lambda \cong 70$ MeV. Indeed, the peak below the $\Sigma$ threshold is connected with the secondary processes

$$\frac{1}{2}\text{He} \rightarrow p + p + \Lambda,$$

in the $\pi^-$ spectrum, or with those

$$\frac{1}{2}\Sigma \rightarrow n + n + \Lambda,$$

in the $\pi^+$ spectrum. The decomposition near the $\Sigma$ threshold can help us to understand the structure of the $YNN$ quasibound state and its decay property.

### E. Fermi-averaged amplitudes for $K^- + N \rightarrow \pi + Y$ in nuclear medium

It is recognized that the spectral shape for DWIA is sensitive to the elementary $K^- + N \rightarrow \pi + Y$ amplitudes of $\overline{t}_{1}(\pi\pi)$ in nuclear medium in Eq. (8) [37,46–48]. When we evaluate the nuclear $(K^-, \pi^\pm)$ cross sections with the $K^- + N \rightarrow \pi + Y$ amplitudes, it is important to take into account the Fermi motion of a struck nucleon in nuclear medium [37]. This effect is considerably enhanced near narrow $\Lambda/\Sigma$ resonances because their widths are smaller than the Fermi-motion energy of the struck nucleon. According to the procedure by Rosenthal and Tabakin [46], we perform the Fermi-averaging of the $K^- + N \rightarrow \pi + Y$ scattering $T$ matrix obtained by Gopal et al. [49]. We use the momentum distribution $\rho(p)$ of a struck nucleon in $^3\text{He}$, which is assumed as a simple harmonic oscillator with a size parameter $b_N = 1.31 \text{ fm}$, leading to $(p^2)^{1/2} \cong 184 \text{ MeV/c}$ in the nucleus.

In Fig. 3, we show the Fermi-averaged laboratory cross sections of $K^- + n \rightarrow \pi^- + \Lambda$, $K^- + p \rightarrow \pi^- + \Sigma^+$, $K^- + n \rightarrow \pi^- + \Sigma^0$, and $K^- + p \rightarrow \pi^+ + \Sigma^-$ reactions on nuclei, at detected $\pi$ angles $\theta_{\text{lab}} = 0^\circ$ and $10^\circ$, as a function of the incident $K^-$ laboratory momentum $p_K$. $\overline{t}_{1}(\pi\pi)$ and $\overline{g}(\pi\pi)$ denote the non-spin-flip and spin-flip components of the Fermi-averaged amplitudes, respectively. The shape of the Fermi-averaged cross section sizably becomes broader, and its value is not so changed by a choice of the target, as discussed by Dover et al. [5,42,48]. Since the spin-flip cross section of $|\overline{g}(\pi\pi)|^2$ is negligibly small, we consider only the non-spin-flip process in the nuclear $(K^-, \pi^\pm)$ reaction.

Furthermore, it is noticed that the Fermi-averaged amplitudes $\overline{t}_{1}(\pi\pi)$ remain in ambiguities, e.g., the relative phase of $\varphi_{\Lambda}$ ($\varphi_{\Sigma}$) for $\Lambda\pi$ ($\Sigma\pi$ $I = 1$) to $\Sigma\pi$ $I = 0$ channels, as discussed in Ref. [30]. Thus we assume $\varphi_{\Lambda} = +15.8^\circ$ and $\varphi_{\Sigma} = +33.2^\circ$, which were determined by fitting the data overall in $^3\text{He}(K^-, \pi^\pm)$ reactions at $p_K = 600 \text{ MeV/c}$ [30], and we use them in our calculations.

### III. MICROSCOPIC COUPLED-CHANNEL $2N-Y$ POTENTIALS

In order to describe the $\Sigma NN$ quasibound states, we construct a microscopic effective $2N-Y$ potential using the $YN$-g-matrices in folding-model calculations [52,58]. In Ref. [18], three-body coupled-channel calculations for $\Sigma NN$...
systems suggested that the channel coupling plays an important role in making a bound state which has strong admixtures of \((I_2, S_2) = (0, 1)\) and \((1, 0)\) states in the \(NN\) pair, e.g., \([pn]\Sigma^+\), \([pn]\Sigma^+\), \([pp]\Sigma^0\) states admix each other in \(3\Sigma_1\)He. Its origin is due to the \((\sigma_N \cdot \sigma_N)\tau_N \cdot t_N\) term in the \(\Sigma N\) OBE potentials [5]. This nature is quite different from the weak-coupling state like \([pn]\Lambda^0\) in \(3\Lambda\)H. In the \([pn]\) channel, the wave function derived from three-body \(\Sigma NN\) calculations is not so changed that in \(^2\)H. In the \([pp]\Sigma^0\) channel, the wave function derived from three-body \(\Sigma NN\) calculations is not so changed that in \(^2\)H. In the \([pp]\) channel, the wave function derived from three-body \(\Sigma NN\) calculations is not so changed that in \(^2\)H.

In folding-model calculations, the effective \(2N - Y\) potential for \(\alpha\alpha'\) channels is obtained as

\[
U_{\alpha\alpha'}(R) = \int \rho_{\alpha\alpha'}(r) (\bar{\sigma}_{\alpha\alpha'}(r_1) + \bar{\sigma}_{\alpha\alpha'}(r_2)) dr,
\]

where \(r_1 = R + r/2\) (\(r_2 = R - r/2\)) is the relative coordinate between \(N_1\) (\(N_2\)) and \(Y\), as shown in Fig. 4. The nucleon or transition density for \(\alpha\alpha'\) channels is given by

\[
\rho_{\alpha\alpha'}(r) = \langle \phi^{(2N)}_{\alpha} | \sum_i \delta(r - r_i) | \phi^{(2N)}_{\alpha'} \rangle.
\]

In Table III, we show matrix elements of isospin-spin averaged potentials for \(\alpha\alpha'\) channels, in which the \(g\)-matrices \(\bar{\sigma}_{\alpha\alpha'}\) for \(\Lambda N - \Lambda N\), \(\Lambda N - \Sigma N\), and \(\Sigma N - \Sigma N\) states can be obtained by solving the coupled Bethe-Goldstone equation with appropriate parameters of the starting energy \(E_S\) and Fermi momentum \(k_F\).

Figure 5 shows the wave functions for \(\rho_{\alpha\alpha'}\) of Eq. (33), as a function of the distance \(r\) between nucleons. For the \([pp]\Lambda\) channel, here we used the \([pp]\) wave function \(\phi^{(2N)}_{\alpha}\) obtained by CDCC as the \(NN\)-pair nucleus, as given in Fig. 2, because the \(\Lambda N\) interaction is very weak. For the \(\Sigma\) channels, on the other hand, we must consider nuclear contraction of the \(NN\) pair because \(\Sigma N\) potentials may induce \(NN\)-pair admixture between \(^3S_1\) and \(^1S_0\) states in \(\Sigma NN\) systems [18]. In the \([pn]\) channel, the wave function derived from three-body \(\Sigma NN\) calculations is not so changed that in \(^2\)H. In the \([pp]\Sigma^0\) or...
The solid curve denote the potentials in the folding model, as a function of the relative distance $r$ for the $YN$ potential terms in $^3\alpha\alpha$ and $\lambda J\Sigma$ denote a $YN - YN$ potential for the isospin $I$ and spin $S$ state.

$$\langle \Sigma \rangle_{\text{wave}}$$

Table III. Isospin-spin averaged matrix elements of $\langle \Sigma \rangle_{\text{wave}}$ for the $YN$ potential terms in $^3\alpha\alpha$ and $\lambda J\Sigma$ denote a $YN - YN$ potential for the isospin $I$ and spin $S$ state.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\alpha'$</th>
<th>$\langle \Sigma \rangle_{\text{wave}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(pp)\Lambda$</td>
<td>$(pp)\Lambda$</td>
<td>$+\frac{2}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2}$</td>
</tr>
<tr>
<td>$(pn)\Sigma^+$</td>
<td>$(pn)\Sigma^+$</td>
<td>$-\frac{2}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2}$</td>
</tr>
<tr>
<td>$(pn)\Sigma^+$</td>
<td>$(pn)\Sigma^+$</td>
<td>$-\left( \frac{1}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2} \right)$</td>
</tr>
<tr>
<td>$(pp)\Sigma^0$</td>
<td>$(pp)\Sigma^0$</td>
<td>$+ \left( \frac{1}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2} \right)$</td>
</tr>
<tr>
<td>$(pn)\Sigma^+$</td>
<td>$(pn)\Sigma^+$</td>
<td>$+ \frac{2}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2}$</td>
</tr>
<tr>
<td>$(pn)\Sigma^+$</td>
<td>$(pn)\Sigma^+$</td>
<td>$-\frac{2}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2}$</td>
</tr>
<tr>
<td>$(pp)\Sigma^0$</td>
<td>$(pp)\Sigma^0$</td>
<td>$+ \left( \frac{1}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2} \right)$</td>
</tr>
<tr>
<td>$(nn)\Lambda$</td>
<td>$(nn)\Lambda$</td>
<td>$+\frac{2}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2}$</td>
</tr>
<tr>
<td>$(pn)\Sigma^-$</td>
<td>$(pn)\Sigma^-$</td>
<td>$-\frac{2}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2}$</td>
</tr>
<tr>
<td>$(pn)\Sigma^-$</td>
<td>$(pn)\Sigma^-$</td>
<td>$+ \left( \frac{1}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2} \right)$</td>
</tr>
<tr>
<td>$(nn)\Sigma^0$</td>
<td>$(nn)\Sigma^0$</td>
<td>$-\left( \frac{1}{3} \frac{\Sigma_{1/2}}{2} + \frac{1}{3} \frac{\Sigma_{3/2}}{2} \right)$</td>
</tr>
</tbody>
</table>

Figure 5. Radial wave functions of the intranuclear $YN$ states in transition densities of $\rho_{\alpha\alpha'}$, which are used to calculate the $2N - Y$ potentials in the folding model, as a function of the relative distance $r$. The solid curve denote the $YN$ wave functions in the $\Sigma NN$ quasibound state, which is obtained by three-body calculations [18]. The dash curves denote the $YN$ wave functions in free space.

$\langle \Sigma \rangle_{\text{wave}}$ for the $\alpha\alpha'$ channel is regarded as a spreading potential $W_{\alpha\alpha'}$. This is significant to describe the complicated surrounding $\Lambda$ excited states with the $2N$ breakup processes, because $\Sigma$ hypernuclear states can be connected with highly $\Lambda$ excited states via the strong $\Lambda N - \Sigma N$ coupling. In the folding model with $D^2\omega$, we can also reproduce the binding energy and width of $^3\alpha\alpha$, and in a comparison with the data, as shown in Appendix A.

Figure 6 displays the real and imaginary parts of the effective $2N - Y$ potential $U_{\alpha\alpha'}(R)$ for $^3\alpha\alpha$ ($J^\pi = 1/2^+$) at $E_\chi = 70$ MeV which corresponds to the $\Sigma$ threshold region, as a function of the relative distance $R$ between $2N$ and $Y$. We find that the coupling potentials of $(pn)\Sigma^+ - (pp)\Sigma^0$, $(pn)\Sigma^+ - (pn)\Sigma^+$, and $(pn)\Sigma^+ - (pp)\Sigma^0$ are quite strong. This nature originates from the fact that the $\Sigma$ potential has a strong isospin-spin dependence, as pointed out by Dover and Gal [40] and suggested by recent $YN$ potential models [53]. For the imaginary parts ($W_{\alpha\alpha'}$), we also recognize that there is the spin-isospin selectivity [40] for $\Sigma N \rightarrow \Lambda N$ conversion decays. It is very important to realize whether or not the quasibound state has a narrow width in $\Sigma NN$ systems. Strengths of $W_{\alpha\alpha'}$ for diagonal $(pn)\Sigma^+$ and $(pp)\Sigma^0$ channels are $-5.0$ MeV and $-13.2$ MeV at the nuclear center, which are consistent with quenching factors $Q = 1/3$ and 1, respectively, as given in Table II of Ref. [40].

IV. RESULTS

A. $\Sigma NN$ quasibound states

In order to obtain eigenvalues for bound and resonance states simultaneously, we solve the multichannel equation for the $2N - Y$ systems by the complex scaling method [54,55]. Here we use the $2N - Y$ potential given in Fig. 6 and the Coulomb force. We find that a pole position for $^3\alpha\alpha$ ($J^\pi = 1/2^+$, $L = 0$, $S = 1/2$) as a complex eigenvalue of the $2N - Y$ system, $E_{\Sigma^+} = E_{\Sigma^+} - i \Gamma_{\Sigma^+}$ on the second Riemann sheet $[\ldots + + +]$ that is identified by a set of four signs of $\text{Im} k_{(pn)\Sigma^+}$, $\text{Im} k_{(pn)\Sigma^+}$, $\text{Im} k_{(pn)\Sigma^+}$, $\text{Im} k_{(pn)\Sigma^+}$ on the complex $E$ plane. The
pole is located as
\[ E_{\Sigma^+}^{(pole)}(^{3}_{\Sigma}He) = +0.96 - i 4.5 \text{ MeV}, \]  
where \( E_{\Sigma^+} \) is measured from the \( d + \Sigma^+ \) threshold, as shown in Table IV. Its width becomes \( \Gamma_{\Sigma} = 9.0 \text{ MeV}. \) Since the pole lies in the second quadrant (\( \text{Re} k/\Sigma_1 < 0, \text{Im} k/\Sigma_1 > 0 \)) on the complex \( k/\Sigma_1 \) plane, the wave function behaves as
\[ \exp(ik_{\Sigma^+} R) = \exp(ik_{\Sigma^+} R) \exp(-\text{Im} k_{\Sigma^+} R) \rightarrow 0 \]  
in the asymptotic region (\( R \rightarrow \infty \)). Hence this state is identified to be a quasibound (an unstable bound) state. In the \( \Lambda \) region, we also confirm that there is no pole of a \( ^{3}_{\Lambda}\text{He} \) bound state below the \( p + p + \Lambda \) threshold.

For \( ^{3}_{\Sigma}n \) (\( J^p = 1/2^+, L = 0, S = 1/2 \)), we find
\[ E_{\Sigma^0}^{(pole)}(^{3}_{\Sigma}n) = -0.58 - i 5.3 \text{ MeV}, \]

where \( E_{\Sigma^0} \) is measured from the \( n + n + \Sigma^0 \) threshold, and \( \Gamma_{\Sigma} = 10.5 \text{ MeV}, \) as shown in Table IV.

In order to see the contributions of the \( 2N - Y \) components in these pole states, we calculate probabilities of isospin \( T(I_2S_2) \) states for \( ^{3}_{\Sigma}He \) \((T_2 = +1)\) and \( ^{3}_{\Sigma}n \) \((T_2 = -1)\):
\[ P_{T(I_2S_2)} = \left| \langle \Psi^{T,I}_{(I_2S_2)} | \Psi^{(pole)}_{Y} \rangle \right|^2, \]  
\( (37) \)

where \( \Psi^{T,I}_{(I_2S_2)} \) is the isospin state defined in Eqs. (B1) and (B4). In Table V, we show values of \( P_{T=1} \) together with values of probabilities on the \( \Sigma \) charge bases. We find that values of a sum of \( P_{T=1} \) account for 99.6% and 97.9% in the \( ^{3}_{\Sigma}He \) and \( ^{3}_{\Sigma}n \) states, respectively, including \( \{pp\} \Lambda \) states. Hence the total isospin \( T = 1 \) becomes an almost good quantum number.

**TABLE IV.** Energies and widths of \( 2N - Y \) systems on complex \( E \) plane.

<table>
<thead>
<tr>
<th>( J^p ) ( T )</th>
<th>( E_{\Lambda} )</th>
<th>( E_{\Sigma^+} )</th>
<th>( E_{\Sigma^0} )</th>
<th>( \Gamma_{\Sigma} )</th>
<th>( k_{\Sigma^+} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{3}_{\Sigma}He )</td>
<td>(( \frac{3}{2}^+ ), 1)</td>
<td>+73.7</td>
<td>+0.96</td>
<td>-32.4</td>
<td>9.0</td>
</tr>
<tr>
<td>( ^{3}_{\Sigma}n )</td>
<td>(( \frac{1}{2}^- ), 1)</td>
<td>+76.4</td>
<td>-1.87</td>
<td>-0.58</td>
<td>10.5</td>
</tr>
</tbody>
</table>

\( ^{3}_{\Sigma}He \) is measured from the \( 2H + \Sigma^+ \) threshold, and \( ^{3}_{\Sigma}n \) from the \( 2H + \Sigma^- \) threshold.

**B. \( \pi^- \) spectrum**

Figure 7 shows the calculated inclusive spectrum of the \( ^{3}_{\Sigma}He(K^-,\pi^-) \) reaction at 600 MeV/c \((4^+) \) from \( \Lambda \) to \( \Sigma \) regions, together with the \( J^p = 1/2^+ \) \((L = 0, S = 1/2 \) component\), which is predominantly connected with \( \Sigma N \rightarrow \Lambda N \) conversion processes of \( ^{3}_{\Sigma}He \rightarrow p + p + \Lambda \) decays. The \( \Sigma \) hyperon produced in the real or virtual \( ^{3}_{\Sigma}He \) state subsequently interacts with a second nucleon, and it is converted to a \( \Lambda \) via \( \Sigma N \rightarrow \Lambda N \) conversion process.

**TABLE V.** Probabilities of channel components of the pole states of \( ^{3}_{\Sigma}He \) and \( ^{3}_{\Sigma}n \) with \( J^p = 1/2^+ \) on complex \( E \) plane. Calculated values are obtained by the complex scaling method.

<table>
<thead>
<tr>
<th>States</th>
<th>Components</th>
<th>Probabilities (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{3}_{\Sigma}He )</td>
<td>( {pp}\Lambda )</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>( {pp} \Sigma^+ )</td>
<td>54.9</td>
</tr>
<tr>
<td></td>
<td>( {pp} \Sigma^0 )</td>
<td>48.7</td>
</tr>
<tr>
<td></td>
<td>( {nn} \Sigma^0 )</td>
<td>18.3</td>
</tr>
<tr>
<td></td>
<td>( T = 1 ) ((I_2 = 0, S_2 = 1) )</td>
<td>54.9</td>
</tr>
<tr>
<td></td>
<td>( T = 1 ) ((I_2 = 1, S_2 = 0) )</td>
<td>42.5</td>
</tr>
<tr>
<td></td>
<td>( T = 2 )</td>
<td>0.45</td>
</tr>
<tr>
<td>( ^{3}_{\Sigma}n )</td>
<td>( {nn} \Lambda )</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>( {nn} \Sigma^- )</td>
<td>39.5</td>
</tr>
<tr>
<td></td>
<td>( {nn} \Sigma^- )</td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td>( {nn} \Sigma^- )</td>
<td>37.2</td>
</tr>
<tr>
<td></td>
<td>( T = 1 ) ((I_2 = 0, S_2 = 1) )</td>
<td>39.5</td>
</tr>
<tr>
<td></td>
<td>( T = 1 ) ((I_2 = 1, S_2 = 0) )</td>
<td>56.0</td>
</tr>
<tr>
<td></td>
<td>( T = 2 )</td>
<td>2.10</td>
</tr>
</tbody>
</table>
ΛN conversion processes inducing 2N-nuclear breakup due to the mass difference $m_\Sigma - m_\Lambda \approx 77$ MeV. It is recognized that a clear peak just below the $d + \Sigma^+$ threshold in the $\pi^-$ spectrum, which corresponds to the $^3\text{He}(\pi^-)\Sigma^0\rightarrow p + p + \Lambda$, with a detector resolution of 2 MeV FWHM.

In the $\pi^-$ spectrum we obtain the decomposition of the inclusive spectrum into partial spectra of the $(pp)\Lambda$, $[pn]\Sigma^+$, $[pn]\Sigma^+$ and $(pp)\Sigma^0$ components, as given in Eq. (25). Figure 8 illustrates the contributions of the $\Lambda$-emitted processes of $(pp)\Lambda$ and $[^3\text{He}]\rightarrow p + p + \Lambda$ conversion near the $\Sigma$ threshold, together with those of the $\Sigma$-emitted processes of $[pn]\Sigma^+$, $[pn]\Sigma^+$, and $(pp)\Sigma^0$. For the $\Sigma$ continuum region, we find that the contribution of the $(pp)\Sigma^0$ component is larger than that of the $[pn]\Sigma^+$ component because the production amplitudes have $|J(\Sigma^+\tau)| \gg |J(\Sigma^+\tau)|$ near $p_{K^-} = 600$ MeV/c [5]. Below the $d + \Sigma^+$ threshold, the $^3\text{He}$ quasibound state is predominantly populated via the $\Sigma^0$ components by $J(\Sigma^+\tau)$.

C. $\pi^+$ spectrum

The nuclear ($K^-$, $\pi^+$) reaction at forward direction of $p_{K^-} = 600$ MeV/c seems to be appropriate to search a bound state in the $\Sigma$ bound region. The reasons were because (i) this reaction can populate only the $\Sigma^-$ components in the final states by its double-charge exchange reaction, so that the contribution of a $\Sigma$ hyperon is removed out from the $\pi^+$ spectrum, and (ii) it has a substitutional mechanism under the near-recoilless condition so as to produce the $^3\text{He}$ quasibound state from $^3\text{He}$, as well as the $^3\text{He}$ quasibound state in the ($K^-$, $\pi^-$) reaction. Therefore, we have naively expected that a signal of the corresponding peak can be clearly observed in the $\pi^+$ spectrum, rather than the $\pi^-$ one.

Figure 9 shows the calculated inclusive spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction at $600$ MeV/c from $\Lambda$ to $\Sigma$ regions, together with the $J^z = 1/2^+$ ($L = 0$, $S = 1/2$) component in $[^3\text{He}]\rightarrow n + n + \Lambda$ conversion processes, obtained by folding with a detector resolution of 2 MeV FWHM. Surprisingly, we find that although the $[^3\text{He}]\rightarrow n + n + \Lambda$ conversion state from $^3\text{He}$, as well as the $[^3\text{He}]\rightarrow n + n + \Lambda$ quasibound state near the $\Sigma$ threshold. Because these states can be populated via.

![FIG. 7. Calculated inclusive $\pi^-$ spectrum of the $^3\text{He}(K^-, \pi^-)$ reaction at 600 MeV/c (4°). The solid curve denotes the total spectrum, and the dashed curve denotes the $J^z = 1/2^+$ ($L = 0$, $S = 1/2$) component caused by conversion decay processes as $[^3\text{He}]\rightarrow p + p + \Lambda$, with a detector resolution of 2 MeV FWHM.](image1)

![FIG. 9. Calculated inclusive $\pi^+$ spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction at 600 MeV/c (4°). The solid curve denotes the total spectrum, and the dashed curve denotes the $J^z = 1/2^+$ ($L = 0$, $S = 1/2$) component caused by conversion decay processes as $[^3\text{He}]\rightarrow n + n + \Lambda$, with a detector resolution of 2 MeV FWHM.](image2)
Although the threshold energy difference between components that account for 49.5%, 48.25%, and 2.3%, respectively.

This confirms the fact that the energy levels for \( _3^3\text{He} \) slightly differ from that of \( _3^3\text{n} \) near the \( \Sigma \) threshold, together with the components of \( \{3\pi\}^{-} \), \( \{pn\}^{-} \), and \( \{nn\}^{0} \), and \( \{1_{\Sigma}n\} \rightarrow n + n + \Lambda \) conversion processes, obtained by folding with a detector resolution of 2 MeV FWHM.

only \( \Sigma^{-} \) productions by \( \Sigma^{-} \) in the \( \Sigma^{-} + \pi^{+} \) reaction. We find that the \( \{pn\}^{+} \) and \( \{nn\}^{0} \) components predominantly occur in \( \Sigma^{+} \) continuum regions, and that a small \( \{nn\}^{0} \) component can be populated via the quasibound state near the \( n + n + \Sigma^{-} \) threshold, followed by \( \{1_{\Sigma}n\} \rightarrow n + n + \Sigma^{-} \).

**V. DISCUSSION**

**A. Charge symmetry breaking**

It is noticed that the \( _3^3\text{He} \) and \( _3^3\text{n} \) quasi-bound states belong to \( J^{\pi} = 1/2^{+} \) isodoublet states in \( \Sigma NN \) systems, whereas a value of \( E_{\Sigma}^{(pole)} \) for \( _3^3\text{He} \) slightly differs from that of \( E_{\Sigma}^{(pole)} \) for \( _3^3\text{n} \), as seen in Table IV. This discrepancy comes from the \( \Sigma \) threshold energy difference and the Coulomb force, leading to the charge symmetry breaking (CSB) in the \( \Sigma NN \) systems. We study a dependence of these pole positions and configurations of the \( 2N-Y \) quasi-bound states on CSB effects.

To see the CSB effects, we obtain a charge symmetric (CS) state, neglecting the Coulomb force and replacing masses of \( m_{\Sigma^{+0}} \) and \( m_{p,n} \) by averaged masses of \( \bar{m}_{\Sigma} = 1193.2 \text{ MeV} \) and \( \bar{m}_{N} = 938.9 \text{ MeV} \), respectively. We find

\[
E_{\Sigma}^{(pole)}(CS) = -0.23 - i 4.7 \text{ MeV},
\]

and probabilities of the \( \{NN\}^{0} \), \( \{NN\}^{+} \), and \( \{NN\}^{-} \) components that account for 49.5%, 48.25%, and 2.3%, respectively. Although the threshold energy difference between \( pn \Sigma^{+} \) and \( pp \Sigma^{0} \) (\( nn \Sigma^{-} \) and \( pn \Sigma^{0} \)) amounts to 2.0 (\( -3.6 \) MeV), we find that the energy levels for \( _3^3\text{He} \) and \( _3^3\text{n} \) is +0.96 (\( -0.58 \) MeV), which is slightly different from -0.23 MeV obtained for CS. This confirms the fact that the \( \Sigma NN \) quasi-bound states have a \( T \approx 1 \) good isospin (97%-99%). We obtain that the width for \( _3^3\text{He} \) amounts to 9.0 MeV, which is slightly smaller than 9.4 MeV for CS because the \( pp \Sigma^{0} \) threshold is located above the \( d + \Sigma^{-} \) one. Contrary to \( _3^3\text{He} \), the width of \( _3^3\text{n} \) becomes broader up to 10.5 MeV because the \( nn \Sigma^{0} \) threshold is located below the \( d + \Sigma^{-} \) threshold. Figure 11 illustrates energy levels and widths of the \( \Sigma NN \) quasi-bound states for \( _3^3\text{He} \) and \( _3^3\text{n} \) near the \( \Sigma \) threshold, together with the probabilities of the \( 2N-Y \) components in the \( \Sigma NN \) systems. We also confirmed that the CSB effects rarely have an influence on the shape and magnitude of the \( \pi^{+} \) spectra.

**B. Interference effects between production amplitudes**

It should be noticed that a production cross section near the \( \Sigma \) threshold is very sensitive not only to the pole position but also to the configuration of the wave function of the quasi-bound state. We consider difference of the \( \Sigma \) production mechanism between the \( \pi^{-} \) and \( \pi^{+} \) spectra in terms of interference among \( \Sigma \) production amplitudes. In order to understand the behavior of the \( \pi^{+} \) spectra, we evaluate interference effects among configurations of the \( NN \) core states in \( \Sigma \) production amplitude, because the \( 2N-Y \) potential should admix \( ^{3}S_{1} \) and \( ^{1}S_{0} \) states in the \( NN \) pair [18], depending on the nature of the \( \Sigma N \) potential.

In the \( \pi^{+} \) spectrum, production amplitude for \( _3^3\text{He} \) near the \( \Sigma \) threshold is approximately written as

\[
\langle _3^3\text{He} | \pi^{+} | T(3\Sigma K^{-}) \rangle \sim \frac{1}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} \sum_{\Sigma^{-}} \sum_{\Sigma^{0}} \langle \Psi_{(s)}^{2} | \Psi_{(s)}^{(3\text{He})} \rangle \right] + \left[ \frac{1}{\sqrt{2}} \sum_{\Sigma^{+}} \sum_{\Sigma^{0}} \langle \Psi_{(+)}^{1} | \Psi_{(+)}^{(3\text{He})} \rangle \right]
\]

and

\[
\langle _3^3\text{n} | \pi^{+} | T(3\Sigma K^{-}) \rangle \sim \frac{1}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} \sum_{\Sigma^{-}} \sum_{\Sigma^{0}} \langle \Psi_{(s)}^{2} | \Psi_{(s)}^{(3\text{n})} \rangle \right] + \left[ \frac{1}{\sqrt{2}} \sum_{\Sigma^{+}} \sum_{\Sigma^{0}} \langle \Psi_{(+)}^{1} | \Psi_{(+)}^{(3\text{n})} \rangle \right]
\]

on isospin bases, where \( \Psi_{(s)}^{2} \) is the isospin state defined in Eq. (B1). The wave function of \( \Psi_{(+)}^{1} \) is regarded as that of a \( _3^3\text{He} \) ground state with \( J^{\pi} = 1/2^{+} \), \( T = 1 \), and \( \Psi_{(+)}^{1} \) as a \( _3^3\text{He}^{*} \) excited state. The relative phase between \( \sum_{\Sigma^{-}} \sum_{\Sigma^{0}} \langle \Psi_{(s)}^{2} | \Psi_{(s)}^{(3\text{He})} \rangle \) and \( \sum_{\Sigma^{+}} \sum_{\Sigma^{0}} \langle \Psi_{(+)}^{1} | \Psi_{(+)}^{(3\text{n})} \rangle \) is \( 180^\circ \) at \( \rho K^{-} = 600 \text{ MeV/c} \), so the component of \( \Psi_{(+)}^{1} \) is relatively enhanced in the \( \pi^{+} \) spectrum.
\( \mathcal{J}_x \mathcal{J}_y \) play an important role in populating the component of \( \Psi_1^{(+)} \).

In the \( \pi^+ \) spectrum, on the other hand, production amplitude for \( \frac{1}{2} n \) near the \( \Sigma \) threshold is approximately written as

\[
\langle \frac{1}{2} n \pi^+ | T | ^3\text{He} K^- \rangle 
\approx \mathcal{J}_x \mathcal{J}_y \left\{ \frac{1}{2} \langle \Psi_1^{(-)} | \Psi(\pi^3\text{He}) \rangle + \frac{2\sqrt{3} - \sqrt{2}}{4} \langle \Psi_1^{(-)} | \Psi(\pi^3\text{He}) \rangle + \frac{2\sqrt{3} + \sqrt{2}}{4} \langle \Psi_1^{(+)} | \Psi(\pi^3\text{He}) \rangle \right\}
\]

(40)
on isospin bases. We find that a cross section for \( \Psi_1^{(+)} \) as the \( \frac{1}{2} n \) ground state is relatively reduced by a factor \((2\sqrt{3} - \sqrt{2})/4)^2 = 0.51^2 \approx 0.26\), whereas that for \( \Psi_1^{(-)} \) as a \( \frac{3}{2} n \) excited state is enhanced by a factor \((2\sqrt{3} + \sqrt{2})/4)^2 = 1.22^2 \approx 1.49\). The interference effects are considered as dynamical ones caused by \( ^3\Sigma_1 - ^1\Sigma_0 \) admixture in the \( NN \) pair for the \( \Sigma NN \) systems. This mechanism originates from properties of the \( \Sigma N \) interaction, and it is inevitable whenever we consider the \( ^3\text{He}(K^-, \pi^+) \) reaction. In Appendix B, we discuss in detail the interference effects.

C. Dependence of the \( \pi^\mp \) spectra on \( \Sigma \) widths

Several three-body \( YNN \) calculations suggested that there is a quasibound state in \( \Sigma NN \) systems. However, the \( \Sigma \) width is unsettled because the \( \Sigma N \) potential still remains quantitative ambiguities. It seems that the shape and magnitude of the peak for the \( \Sigma NN \) quasibound state is very sensitive to a value of its width. We demonstrate behavior of the \( \pi^\mp \) spectrum near the \( \Sigma \) threshold in order to compare it with experimental observations. We introduce an artificial factor of \( f_W \) changing the strength of the spreading potential:

\[
W_{\alpha\alpha'} \rightarrow f_W \times W_{\alpha\alpha'}.
\]

(41)

Here let us consider several cases of various widths in the \( \pi^\mp \) spectrum as follows:

(i) Case A was obtained by \( f_W = 1.00 \), leading to a broad width of \( \Gamma_\Sigma \approx 9 \) MeV. This width was suggested by a Faddeev calculation for the \( \Lambda + d \rightarrow \Sigma + N + N \) scattering near the \( \Sigma \) threshold by Afnan and Gibson [16].

<table>
<thead>
<tr>
<th>Case</th>
<th>( f_W )</th>
<th>( E_\Sigma^+ ) (MeV)</th>
<th>( E_\Sigma^- ) (MeV)</th>
<th>( \Gamma_\Sigma ) (MeV)</th>
<th>( k_{\Sigma^+} ) (fm(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>+0.96</td>
<td>-3.24</td>
<td>9.0</td>
<td>-0.322 + i0.260</td>
</tr>
<tr>
<td>B</td>
<td>0.75</td>
<td>-0.10</td>
<td>-4.30</td>
<td>6.9</td>
<td>-0.249 + i0.257</td>
</tr>
<tr>
<td>C</td>
<td>0.50</td>
<td>-0.94</td>
<td>-5.14</td>
<td>4.8</td>
<td>-0.176 + i0.257</td>
</tr>
<tr>
<td>D</td>
<td>0.25</td>
<td>-1.53</td>
<td>-5.73</td>
<td>2.8</td>
<td>-0.101 + i0.259</td>
</tr>
</tbody>
</table>

TABLE VI. Energies and widths of the \( \frac{1}{2}\text{He} \) quasibound state with \( J^P = 1/2^+ \), \( T \approx 1 \) on complex energy plane when the spreading potential \( W_{\alpha\alpha'} \) is artificially changed.

FIG. 12. Shape dependence of the calculated \( \pi^- \) spectrum in the \( ^3\text{He}(K^-, \pi^-) \) reaction near the \( \Sigma \) threshold at 600 MeV/c (4'), when changing the spreading potential \( W_{\alpha\alpha'} \) artificially by \( f_W = (a) 1.00 \), (b) 0.75, (c) 0.50, and (d) 0.25, which correspond to widths of \( \Gamma_\Sigma = 9.0, 6.9, 4.9, \) and \( 2.9 \) MeV, respectively. These spectra are obtained by folding with a detector resolution of 2 MeV FWHM.
TABLE VII. Energies and widths of the $^{3}_n$ quasibound state with $J^p = 1/2^+$, $T \simeq 1$ on complex energy plane when the spreading potential $W_{\alpha\alpha}'$ is artificially changed.

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_W$</th>
<th>$E_{\Sigma^-}$ (MeV)</th>
<th>$E_{\Sigma^0}$ (MeV)</th>
<th>$\Gamma_{\Sigma^-}$ (MeV)</th>
<th>$k_{\Sigma^-}$ (fm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>-1.87</td>
<td>-0.58</td>
<td>10.5</td>
<td>-0.263 + i0.374</td>
</tr>
<tr>
<td>B</td>
<td>0.75</td>
<td>-2.79</td>
<td>-1.50</td>
<td>8.1</td>
<td>-0.200 + i0.380</td>
</tr>
<tr>
<td>C</td>
<td>0.50</td>
<td>-3.51</td>
<td>-2.22</td>
<td>5.7</td>
<td>-0.138 + i0.388</td>
</tr>
<tr>
<td>D</td>
<td>0.25</td>
<td>-4.02</td>
<td>-2.73</td>
<td>3.2</td>
<td>-0.077 + i0.396</td>
</tr>
</tbody>
</table>

(ii) Case B was obtained by $f_W = 0.75$, leading to $\Gamma_{\Sigma^-} \simeq 7$ MeV, of which width was predicted in three-body calculations within a SAP approximation by Koike and Harada [18].

(iii) Case C was obtained by $f_W = 0.50$, leading to $\Gamma_{\Sigma^-} \simeq 5$ MeV, which is equivalent to a half of the width of Case A.

(iv) Case D was obtained by $f_W = 0.25$, leading to a narrow $\Sigma$ width of $\Gamma_{\Sigma^-} \simeq 2–3$ MeV in recent Faddeev $\Lambda NN$ calculations using $NN$ and $YN$ potentials derived from a chiral constituent quark models by Garcilazo et al. [19].

In Table VI, we obtain the calculated values of energies and widths of the $^{3}_n$He quasibound state on the complex $E$ plane. Figure 12 shows dependence of the shape and magnitude of the $\pi^-$ spectrum on the width. In Case A, we confirm that a peak of the $\pi^-$ spectrum is enhanced just below the $d + \Sigma^+$ threshold, as already seen in Fig. 8. In Case B, we also recognize that the peak can be observed as a candidate of the $\Sigma$ hypernuclear bound state, having a narrow width of $\Gamma_{\Sigma^-} \simeq 7$ MeV. This is equivalent to the value of $\langle v\sigma_{\Sigma^-p\to\Lambda n}\rangle_{av}$, where $v$ and $\sigma_{\Sigma^-p\to\Lambda n}$ are the velocity and total cross section data of $\Sigma^-p\to\Lambda n$ at low energies, respectively. In Cases C and D, we find that a peak of the quasibound state is more clearly observed below the $\Sigma$ threshold, so that it might be a very good candidate if the $\Sigma NN$ quasibound state has such a narrow width. Consequently, the $\pi^-$ spectrum near the $\Sigma$ threshold provides valuable information to understand the nature of $\Sigma NN$ potentials and the structure of the $\Sigma NN$ quasibound state.

In Table VII, we obtain the calculated values of energies and widths of the $^{3}_n$ quasibound state on the complex $E$ plane. Figure 13 shows the shape and magnitude of the $\pi^+$ spectrum on the corresponding widths. In Cases A and B, we find that the shape of the spectrum is scarcely changed by a value of $f_W$. This originates from the fact that the contributions of the $\langle pn\rangle\Sigma^-$ and $\langle pn\rangle\Sigma^0$ components predominantly occur in continuum states, and the shape of continuum spectrum is insensibly influenced by the spreading potential $W_{\alpha\alpha}'$. For Case D, the quasibound state has a very narrow width of $\Gamma_{\Sigma^-} \simeq 3$ MeV, observed below the $n + n + \Sigma^0$ threshold. However, the cross section of the quasibound state is rather small, compared with the continuum one, because of a reduction mechanism caused by the interference effects in the $\pi^+$ spectrum, as discussed in Sec. VA.

FIG. 13. Shape dependence of the calculated $\pi^+$ spectrum in the $^{3}_n^{\text{He}}(K^-, \pi^+)$ reaction near the $\Sigma$ threshold at 600 MeV/c (4'), when changing the spreading potential $W_{\alpha\alpha}'$ artificially by $f_W = (a) 1.00$, (b) 0.75, (c) 0.50, and (d) 0.25, which correspond to widths of $\Gamma_{\Sigma^-} = 10.5, 8.1, 5.7$, and 3.2 MeV, respectively. These spectra are obtained by folding with a detector resolution of 2 MeV FWHM.
our calculations. The effective $2N - Y$ potential is constructed by a folding-model potential with $YN$ $g$-matrices derived from the central $D2'$ potential, which is simulated to the Nijmegen model D. Such folding potentials can overcome serious overbinding problems in $s$-shell $\Lambda$ hypernuclei.

(ii) The calculated inclusive spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction shows a signal of the $\frac{1}{2}^+$ quasibound state with $J^\pi = 1/2^+$, $T = 1$ near the $\Sigma$ threshold, and its width has $\Gamma_\Sigma \approx 9$ MeV.

(iii) The calculated inclusive spectrum of the $^3\text{He}(K^-, \pi^+)$ reaction shows no peak of the $\frac{1}{2}^-$ quasibound state that is located near the $\Sigma$ threshold with $\Gamma_\Sigma = 10.5$ MeV, by interference effects caused by $\frac{3}{2}^1 - \frac{3}{2}^0$ admixture in the $NN$ pair for $\frac{1}{2}^-$ and the CSB effects. This spectrum is consistent with the BNL-E774 data.

In conclusion, we show that a signal of the $\frac{3}{2}^+$ quasibound state is clearly confirmed near the $\Sigma$ threshold in the $\pi^+$ spectrum, whereas the peak of the $\frac{1}{2}^-$ quasibound state is relatively reduced in the $\pi^-$ spectrum. We believe that the $\pi^-$ and $\pi^+$ spectra on $^3\text{He}$ targets provide valuable information on properties of $\Sigma NN$ quasibound states so as to study the $\Sigma N$ interaction.

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APPENDIX A: BINDING ENERGIES OF $s$-SHELL $\Lambda$ HYPERNUCLEI IN FOLDING-MODEL CALCULATIONS

Let us consider $\Lambda$ binding energies of $^3\Lambda\text{H}, ^4\Lambda\text{He}, ^4\Lambda\text{He}^*$, and $^5\Lambda\text{He}$ in folding models. We obtain $YN$ $g$-matrices in Eq. (32), solving the coupled Bethe-Goldstone equation

\[ \left[ \Psi_{\Sigma N} \right] = \left[ \Phi_{\Lambda} \right] + \frac{Q}{\varepsilon} v \left[ \Psi_{\Sigma N} \right], \]

where $\varepsilon$ and $Q$ are the energy denominator and Pauli exclusion operator, respectively [50,51], $v$ is a $YN$ potential including the $\Lambda N - \Sigma N$ coupling. The $D2'$ potential [2] is a central $YN$ potential that simulates the Nijmegen model D [23]. The $D2'$ potential we used in this paper is a modified version to reproduce the experimental value of $B_{\Lambda N}(^3\Lambda\text{H})$, multiplying the strength of the long-range part in the $\Sigma N$ $\Sigma_1$ by a factor (0.954) [27]. In Table VIII, we show the calculated results of $\Lambda$ binding energies of $s$-shell $\Lambda$ hypernuclei in the folding models. We obtain $B_{\Lambda N} = 0.13, 2.4, 1.1$, and $3.1$ MeV for $^3\Lambda\text{H}, ^4\Lambda\text{He}, ^4\Lambda\text{He}^*$, and $^5\Lambda\text{He}$, respectively. Here parameters of starting energies and Fermi momentum were taken to be $(E_F, k_F) = (-2.2, 0.65, (-8, 0.65), (-8, 1.05), (-8, 1.05), \text{fm}^{-1}$), and $(-28, 1.30)$ for $^3\Lambda\text{H}, ^4\Lambda\text{He}, ^4\Lambda\text{He}^*$, and $^5\Lambda\text{He}$, respectively. In order to compare them with the experimental data [56], we need to include rearrangement energies by $-\kappa N L_{\text{cap}}$ [57,58] where we choose $\kappa_N = 0.06, 0.08, 0.08$, and $0.115$ for $^3\Lambda\text{H}, ^4\Lambda\text{He}, ^4\Lambda\text{He}^*$, and $^5\Lambda\text{He}$, respectively. We confirm that the calculated values of $B_{\Lambda N}$ can reasonably
TABLE VIII. Binding energies and Σ-mixing probabilities of s-shell Λ hypernuclei in the folding-model potential calculations for g-matrices with the YN D2’ potential, together with those obtained in Brueckner-Hartree-Fock [2] and SVM calculations [25,27]. Data are taken from Ref. [56].

<table>
<thead>
<tr>
<th></th>
<th>³H (1/2⁺)</th>
<th>³He (0⁺)</th>
<th>³He⁺ (1⁺)</th>
<th>³He (1/2⁺)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B_Σ (MeV)</td>
<td>P_Σ (%)</td>
<td>B_Σ (MeV)</td>
<td>P_Σ (%)</td>
</tr>
<tr>
<td>This worka</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with −κ_N Ub</td>
<td>0.13</td>
<td>0.07</td>
<td>2.4</td>
<td>2.0</td>
</tr>
<tr>
<td>w/o −κ_N U</td>
<td>0.26</td>
<td>0.10</td>
<td>3.0</td>
<td>2.1</td>
</tr>
<tr>
<td>BHF [2]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SVM [25,27]</td>
<td>0.056</td>
<td>0.14</td>
<td>2.23</td>
<td>1.85</td>
</tr>
<tr>
<td>Expt. [56]</td>
<td>0.13 ± 0.05</td>
<td>2.39 ± 0.03</td>
<td>1.24 ± 0.04</td>
<td>3.12 ± 0.02</td>
</tr>
</tbody>
</table>

aStarting energies and Fermi momenta are used as (F_S, k_F) = (−2 MeV, 1.05 fm⁻¹), (−8 MeV, 1.05 fm⁻¹), (−8 MeV, 1.05 fm⁻¹), and (−28 MeV, 1.30 fm⁻¹) for ³H, ³He, ³He⁺, and ³He, respectively.
bCorrection of rearrangement energies is taken from −κ_N U [57,58], where we choose κ_N = 0.06, 0.08, 0.08, and 0.115 for ³H, ³He, ³He⁺, and ³He, respectively.

reproduce the corresponding experimental data. This fact supports the importance of the coherent ΛN − ΣN coupling in s-shell hypernuclei [2]. It should be noticed that our folding-model calculations provide to explain properties of the s-shell hypernuclei, whereas the values of P_Σ for ³He⁺ and ³He should be in disagreement with those of SVM because no D-wave component is included in our model space.

In order to describe properties of Σ hypernuclei, we must consider the binding energy and width of a ³He quasibound state with J^π = 0⁺, T = 1/2. Let us calculate a pole position of ³He which is located on the complex E plane by the complex scaling method [54,55], when we use (F_S, k_F) = (−8 MeV, 1.05 fm⁻¹) and κ_N = 0.08 as the parameters in folding-model calculations. In Table IX, we show the calculated result of the binding energy and width, in order to be compared with those of experimental data [9,10]. We find B_Σ⁺ = 0.89 MeV and Γ_Σ = 11.8 MeV with YN g-matrices derived from the D2’ potential where E_Σ⁺ = −B_Σ⁺ − iΓ_Σ/2, where B_Σ⁺ is measured from the ³H + Σ⁺ threshold.

TABLE IX. Binding energy and width of the ³He quasibound state with J^π = 0⁺, T = 1/2 in the folding-model potential calculations for g-matrices with the YN D2’ potential, in a comparison with analysis of theoretical calculation [31] and experimental data taken from Refs. [9,10].

<table>
<thead>
<tr>
<th></th>
<th>B_Σ⁺</th>
<th>Γ_Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MeV)</td>
<td>(MeV)</td>
</tr>
<tr>
<td>This worka</td>
<td>0.83</td>
<td>11.8</td>
</tr>
<tr>
<td>Harada [31]</td>
<td>1.1</td>
<td>12.4</td>
</tr>
<tr>
<td>Ota et al. [9]</td>
<td>2.8 ± 0.7</td>
<td>12.1 ± 1.2</td>
</tr>
<tr>
<td>Nagae et al. [10]</td>
<td>4.4 ± 0.3</td>
<td>7.0 ± 0.7</td>
</tr>
</tbody>
</table>

aStarting energy and Fermi momentum of (F_S, k_F) = (−8 MeV, 1.05 fm⁻¹), and the rearrangement energy with κ_N = 0.08 are used.
bE_Σ⁺ = −B_Σ⁺ − iΓ_Σ/2 determined on complex E plane, where B_Σ⁺ is measured from the ³H + Σ⁺ threshold.

APPENDIX B: PRODUCTION AMPLITUDES FOR (K⁻, π⁺) REACTIONS ON ISOSPIN ΣNN STATES

In order to understand behavior of the π⁺ spectrum, we consider Σ production amplitudes for isospin ΣNN states in ³He(K⁻, π⁺) reactions. In the π⁺ spectrum, we recognize that the ³He quasibound state is populated via J(Λπ⁻), J(Σ⁻π⁻), and J(Σ⁺π⁻), and this state is identified by a configuration of Ψ^T,Tz. We obtain

Ψ^T,Tz^{2,1}_± = \frac{1}{\sqrt{2}}[(pp)\Sigma_0]^{\pm} + \frac{1}{\sqrt{2}}[(pn)\Sigma^+]^{\pm},

Ψ^T,Tz^{1,1}_± = \frac{1}{\sqrt{2}}[(pp)\Sigma_0]^{±} - \frac{1}{\sqrt{2}}[(pn)\Sigma^+]^{±},

(\text{B1})

where s and t denote spin-singlet (I_z = 0, S_z = 0) and spin-triplet (I_z = 0, S_z = 1) states, respectively, for the ΣN pair in the ΣNN systems. Therefore, we have total isospin T = 1 good states as

Ψ^{1,1}_± = aΨ^{1,1}_± ± bΨ^{1,1}_±,

(\text{B2})

owing to the strong admixture of (I_z = 0, S_z = 0) and (1,0) states in the NN pair. Because the 2N − Y potential should admit ⁵S_1 and ⁵D_0 states in the NN pair, depending on the nature of the ΣN potential [18], interference effects of Σ production amplitudes are important to make a shape of the π⁺ spectrum. As seen in Sec. VA, when a = b = 1/√2 for simplicity we obtain Ψ_± that corresponds to the ground state in ³He, and Ψ^{1,1}_± to an excited state. If we approximately omit Λ production amplitude of J(Λπ⁻) in the Σ threshold region, we obtain production amplitude for the (K⁻, π⁺) reaction as

\langle ³He | T(³HeK⁻) \rangle ≃ \frac{1}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} J(Σ⁺π⁻) - J(Σ⁻π⁻) \right] \{Ψ^{2,1}_± |Ψ(³He)\}.
We find that interference effects between \( \mathcal{T}_{\Sigma^+\pi^-} \) and \( \mathcal{T}_{\Sigma^-\pi^+} \) play an important role in populating \( \Psi_{(i)}^{(-1)} \) and \( \Psi_{(i)}^{(+1)} \) within the \( \Sigma NN \) states with \( T = 1 \). Considering the relative phase \( \varphi(\Sigma^+/\Sigma^0) = +4.8^\circ \) between \( \mathcal{T}_{\Sigma^+\pi^-} \) and \( \mathcal{T}_{\Sigma^-\pi^+} \) at \( p_{K^-} = 600 \text{ MeV}/c \), we obtain that the component of \( \Psi_{(i)}^{(-1)} \) that is identified as the \(^3\text{He} \) quasibound state with \( J^p = 1/2^-, T = 1 \), is relatively enhanced in the \( \pi^- \) spectrum, whereas the component of \( \Psi_{(i)}^{(+1)} \) in \( \Sigma \) continuum states is reduced.

For the \( \pi^+ \) spectrum, we find that the \( \frac{3}{2}^- n \) quasibound state is populated via only \( \mathcal{T}_{\Sigma^-\pi^+} \). We obtain

\[
\Psi_{(i)}^{(-1)} = \frac{1}{\sqrt{2}} [(pn) \Sigma^+] + \frac{1}{\sqrt{2}} [(nn) \Sigma^0],
\]

\[
\Psi_{(i)}^{(+1)} = \frac{1}{\sqrt{2}} [(pn) \Sigma^-] - \frac{1}{\sqrt{2}} [(nn) \Sigma^0],
\]

\[
\Psi_{(i)}^{(+1)} = [(pn) \Sigma^-],
\]

where the isospin \( T = 1 \) good states are written as

\[
\Psi_{(i)}^{(1,-)} = a \Psi_{(i)}^{(-1)} \pm b \Psi_{(i)}^{(+1)}.
\]

If \( a = b = 1/\sqrt{2} \), \( \Psi_{(i)}^{(-1)} \) and \( \Psi_{(i)}^{(+1)} \) are regarded as ground and excited states in \( \sqrt{3}n \), respectively. Thus the production amplitude for the \((K^-, \pi^+)\) reaction is

\[
\langle \sqrt{3}n \pi^+ | T | ^3\text{He}K^- \rangle \\
\simeq \mathcal{T}_{\Sigma^-\pi^+} \left\{ \frac{1}{2} ( \Psi_{(i)}^{(-1)} | \Psi(\sqrt{3} \text{He}) ) + \frac{2\sqrt{3} - \sqrt{2}}{4} ( \Psi_{(i)}^{(-1)} | \Psi(\sqrt{3} \text{He}) ) \right\}.
\]

We find that production amplitude for \( \Psi_{(i)}^{(-1)} \) is relatively reduced by a factor \( (2\sqrt{3} - \sqrt{2})/4 = 0.51 \), whereas that for \( \Psi_{(i)}^{(-1)} \) is enhanced by a factor \( (2\sqrt{3} + \sqrt{2})/4 = 1.22 \). This mechanism is inevitable whenever we consider the \(^3\text{He}(K^-, \pi^+)\) reaction.