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Scientific paper

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Abstract

The present paper introduces a non-linear seismic response estimation method for reinforced concrete (RC) structures based on random vibration theory. Equivalent linear single-degree-of-freedom systems that have complex stiffness are adopted in order to approximate the displacement-dependent hysteretic characteristics of RC structures. The proposed method uses the transfer functions of the equivalent linear systems, root mean square responses, and peak factors. In contrast with existing approximation methods using response spectra, the proposed method allows individual evaluation of the power spectral densities and duration times of input motions.

Earthquake responses estimated using the proposed method have been verified through comparisons with results of non-linear time history response analyses and two existing approximation methods. The proposed method has exhibited relatively high accuracy and has been found to be useful for response estimation of RC structures, considering frequency characteristics and duration effects separately.

1. Introduction

Estimation of lateral displacements is important in earthquake-resistant design for reinforced concrete (RC) structures when subjected to earthquake ground motions. In particular, approximation methods by which to estimate the maximum displacement demands without the need for full characterization of the input motion, such as time-history data of ground acceleration, are useful. As reported by Miranda and Ruiz-Garcia (2002), a number of approximation methods have been proposed. For instance, methods based on equivalent linearization (e.g., Jacobsen 1930; Caughey 1960; Rosenblueth and Herrera 1964; Jennings 1968; Gulkan and Sozen 1974; Iwan 1980) and methods based on a displacement modification factor (e.g., Veletsos and Newmark 1960; Newmark and Hall 1982) are well known. Many existing approximation methods use response spectra to estimate the maximum non-linear displacement demand.

In Japan, the capacity spectrum method (Freeman 1978) has been applied in the calculation of response and limit strength (CRLS) provided in the Building Standard Law Enforcement Order of Japan since 2000. (The Building Standard Law was translated into English by the Building Center of Japan 2011.) The capacity spectrum method in CRLS is an approximation method that uses response spectra with equivalent linear systems. The effectiveness of this method has been verified and improvements have been proposed in previous studies (e.g., studies considering higher-order modes by Kuramoto (2004, 2005)).

However, a number of problems remain to be addressed with respect to existing approximation methods using response spectra, including the capacity spectrum method in CRLS. For example, existing approximation methods usually have difficulty evaluating frequency characteristics and duration effects of input motions separately, because response spectra include both characteristics of frequency and time domain. For performance-based design that more adequately considers the damaging properties of strong ground motions, it is desirable that the frequency characteristics and duration effects of input motions be evaluated individually in estimating the maximum displacement demands. Moreover, simultaneous evaluation of the effects of site amplifications and soil-structure interactions together with the responses of upper RC structures remain challenges.

In the present study, in order to overcome these problems, an approximation method is proposed that estimates the maximum non-linear responses of RC structures when subjected to strong input ground motions without the use of response spectra. In contrast to existing approximation methods using response spectra, the proposed method permits the individual evaluation of the effects of the frequency characteristics and duration times as an important temporal characteristic of the input motions by applying random vibration theory with an equivalent linearization technique.

A more noteworthy feature of the proposed method is the use of the transfer functions of equivalent linear systems, which have complex stiffness, to approximate the displacement-dependent hysteretic characteristics of RC structures. Therefore, the proposed method is highly related to previous studies on the evaluation of the op-

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timum yield strength of hysteretic dampers incorporated into RC structures (Shirai *et al*. 2010, 2011b). In addition, the present study is part of an attempt to develop a comprehensive earthquake-resistant-design method, as shown in **Fig. 1**. The goal of the comprehensive design method is to easily and simultaneously evaluate the effects of site amplifications, soil-structure interactions, and the responses of RC upper structures.

The results of the present study are intended to be applied to low-rise or mid-rise RC buildings in which first-order modes are predominant, with a focus on the post-yield responses of single-degree-of-freedom (SDOF) systems. Evaluating the responses of high-rise or irregular buildings that are significantly affected by higher-order modes is beyond the scope of the present study. Although numerous studies have examined cumulative damage indices for RC structures (e.g., Gosain *et al*. 1977; Banon *et al*. 1981; Park and Ang 1985; Fajfar 1992), the present paper deals with non-cumulative damage indices, i.e., maximum response displacement ductility factors, but does not consider cumulative damage indices.

The present paper is a significant revision of a previous study (Shirai *et al*., 2011a), and the differences from the previous study are: [1] several parts of the method have been modified, [2] more verifications have been carried out, and [3] a quantitative comparison with existing methods has been conducted. In the present paper, Section 2 presents a flowchart of the proposed method, and describes the proposed method in detail. This section also presents configuration examples of the proposed method. Section 3 describes the estimation results obtained using the proposed method and verification by comparison using the results of non-linear time history response analyses. Moreover, two existing approximation methods that use response spectra are compared in this section.

2. Proposed response estimation method

2.1 Outline of the proposed method

In the present paper, a method of estimating non-linear responses of RC structures when subjected to strong seismic motions is proposed. The proposed method is based on random vibration theory and uses the equivalent linearization technique. The primary feature of the proposed method is to separately evaluate the effects of the frequency spectral characteristics and duration times of input motions.

A flowchart of the proposed method is shown in **Fig. 2** and a description is provided below. In Step 1, an input ground motion is configured. Specifically, as a frequency characteristic and an important temporal characteristic, the power spectral density $G_I(\omega)$ and duration time t_d, respectively, are configured individually. This is one of the advantages of the proposed method. In Step 2, the parameters of the non-linear SDOF system of the RC structure are configured. In Step 3, the maximum response displacement ductility factor μ (= δ/δ_y) of the system is assumed. Here, δ is the peak displacement in the stationary response, and δ_y is the yield displacement. In addition, the initial value of the assumed μ should be sufficiently small. In Step 4, the transfer function $|H_{eq}(\omega)|$ is calculated. Here, $|H_{eq}(\omega)|$ is the amplitude of the transfer function of the equivalent linear system that corresponds to the non-linear system at the assumed μ.

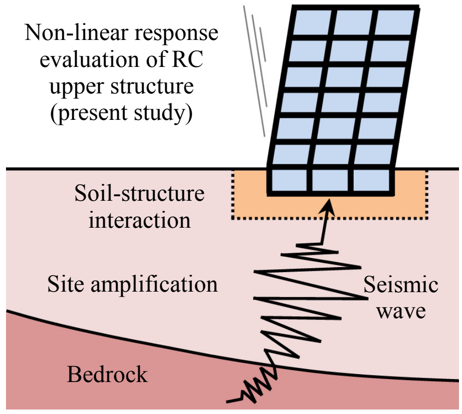

Fig. 1 Overview of research (development of comprehensive earthquake-resistant-design method).

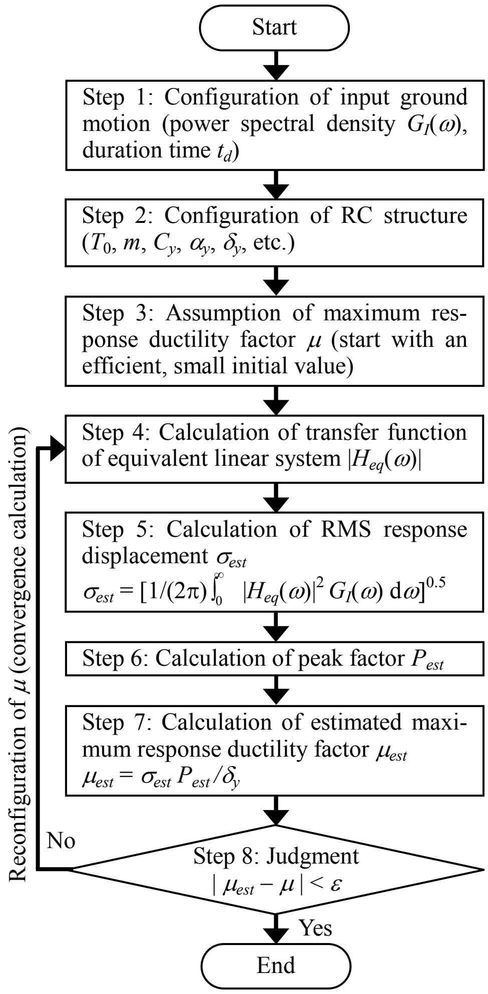

Fig. 2 Flowchart of the proposed method.

Note that $|H_{eq}(\omega)|$ is defined as the ratio of the complex amplitude of the response displacement to the input acceleration in the frequency domain. In Step 5, the root mean square (RMS) response displacement σ_{est} is calculated from $G_I(\omega)$ and $|H_{eq}(\omega)|$ in the frequency domain. In Step 6, the peak factor P_{est} is calculated based on, for example, t_d and the equivalent period T_{eq}. Therefore, the proposed method can reflect the effects of the duration times of input motions on response estimation. In Step 7, the estimated maximum response ductility factor μ_{est} is obtained by multiplying σ_{est} by P_{est} and dividing by δ_y. In Step 8, μ_{est} is evaluated by comparison with the assumed μ. If μ_{est} agrees with the assumed μ within an acceptable error ε, then μ_{est} is determined as the estimated maximum response ductility factor, and the estimation procedure is finished. Otherwise, the assumed μ is increased, and the procedure returns to Step 4 until the calculation converges.

2.2 Details and configuration examples of the proposed method

This section describes the proposed method in detail and also provides configuration examples.

(1) Input ground motions

In the present study, a total of 15 simulated earthquake waves (Waves L1 through L5, M1 through M5, and S1 through S5) are used as input motions.

The velocity response spectra of the input motions are shown in **Fig. 3** (Waves L1, M1, and S1). Each input motion is fitted to the same target response acceleration spectrum of the Building Standard Law Enforcement Order of Japan (referred to as the Kokuji Spectrum in Japan). The target response spectrum has a pseudo-velocity constant periodic band when the period is greater than 0.64 s (damping ratio: 0.05). One-sided power spectral densities (PSDs) of the input motions are shown in **Fig. 4** (Waves L1, M1, and S1). In this figure, the PSDs are smoothed by a Parzen window with a bandwidth of 0.5 Hz.

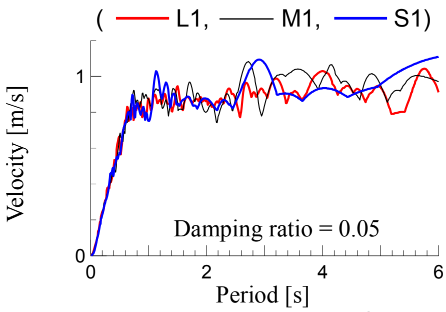

Fig. 3 Velocity response spectra of input motions.

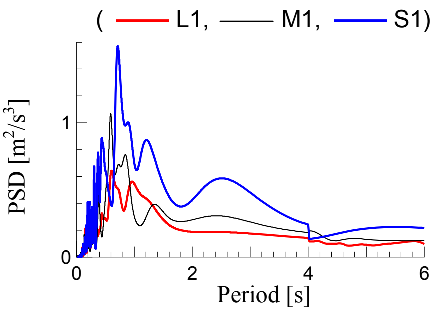

Fig. 4 Power spectral densities of input motions (one-sided PSD, smoothed by a Parzen window with a bandwidth of 0.5 Hz).

Table 1 Parameters of time envelope functions of input motions.

Wave	a_1 [s]	a_2 [s]	a_3
L1 through L5	6	126	0.04
M1 through M5	4	34	0.08
S1 through S5	2	8	0.24

Time envelope functions of the input motions and its parameters are shown in **Fig. 5** and **Table 1**. These envelope functions have been proposed by Amin and Ang (1968). Waves L1 through L5 have large values of t_d, Waves M1 through M5 have moderate values of t_d, and Waves S1 through S5 have small values of t_d. The phase angles for each wave are given by random numbers. The time history accelerations of the input motions are shown in **Fig. 6** (Waves L1, M1, and S1). In the present paper, t_d is calculated according to the definition of the time interval during which the central 90% of the contribution to the integral of the square of acceleration takes place (Trifunac and Brady 1975). For example, t_d = 121.1 s (Wave L1), 39.0 s (Wave M1), and 10.9 s (Wave S1).

In addition, as described in Section 3, the amplitude of each of the above input motions is multiplied by the amplification coefficient, A_c = 1.5.

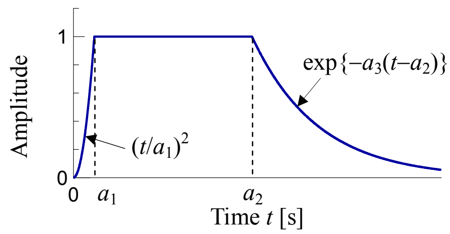

Fig. 5 Time envelope functions of input motions.

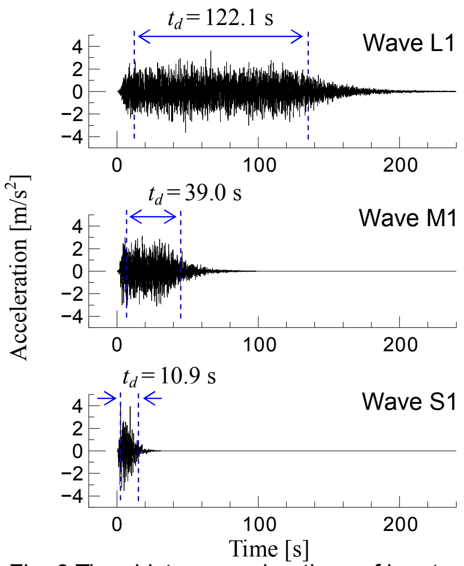

Fig. 6 Time history accelerations of input motions.

(2) RC structures

In the present paper, non-linear SDOF systems are used as models of RC structures. The systems have degrading tri-linear type restoring force characteristics, as shown in **Fig. 7**. However, these hysteretic models are non-degrading strength and stiffness models under cyclic loading. The structural properties of the models are shown in **Table 2**. The initial period T_0 (= $2\pi\ (m/k_0)^{0.5}$) is configured to be between 0.2 and 1.5 s. Here, m is the mass, and k_0 is the initial stiffness. Each yield shear force coefficient C_y is computed so that the exact maximum response ductility factor reaches 1.5, 2, 3, 4, 5, and 6 for each T_0 and each input motion by preliminary parametric non-linear time history response analysis.

(3) Transfer functions

The proposed method uses the equivalent linearization technique as shown in **Fig. 8a**. In order to easily simulate the displacement-dependent hysteretic behaviors of RC structures, the complex stiffness models are used, as shown in **Fig. 8b**. Here, x is the response displacement, x_g is the input displacement, k and k' are the real and imaginary parts of the complex stiffness, β is the complex damping ratio (= $k'/(2k)$), and i is the imaginary unit (= $(-1)^{0.5}$). The equation of motion of the linear SDOF system shown in **Fig. 8b** is given as follows:

$$m\ddot{x} + (1+2\beta i)kx = -m\ddot{x}_g \quad (\omega > 0)\,, \tag{1}$$

where ω is the circular frequency.

The amplitude of the transfer function of the linear system $|H(\omega)|$ is given as follows:

$$|H(\omega)| = \sqrt{\frac{m^2}{(2\beta k)^2 + (m\omega^2 - k)^2}}\,, \tag{2}$$

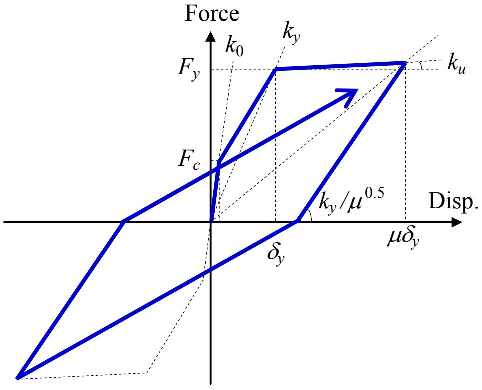

Fig. 7 Restoring force characteristics.

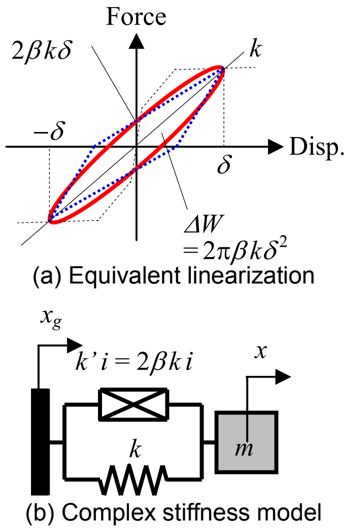

Fig. 8 Equivalent linear complex stiffness model.

Table 2 Structural properties of estimation and analysis models.

Parameter	Symbol	Set value
Mass	m	10^7 kg
Acceleration of gravity	g	9.80665 m/s^2
Initial period	T_0	between 0.2 and 1.5 s (at 0.02 intervals)
Initial stiffness	k_0	$= m(2\pi/T_0)^2$
Yield shear force coefficient	C_y	C_y is computed for each T_0 and each input motion so as to maximum response ductility factor reaches μ = 1.5, 2, 3, 4, 5, and 6 by non-linear time history analysis
Yield shear force	F_y	$= mgC_y$
Yield stiffness ratio	α_y	0.3
Secant stiffness at the yield point	k_y	$= \alpha_y k_0$
Yield displacement	δ_y	$= F_y/k_y$
Crack shear force coefficient	C_c	1/3
Crack shear force	F_c	$= C_c F_y$
Stiffness after yielding	k_u	= 0 (estimation methods (common)) $= k_0/100$ (non-linear time history analysis)
Viscous damping ratio	h	0.02 (non-linear time history analysis)
Initial complex damping ratio	β_0	0.02 (proposed estimation method)
Reduction coefficient of equivalent damping	γ	0.2 (proposed estimation method)
Exceedance probability	p_0	= 1−exp(−1) (proposed estimation method)
Acceptable error	ε	0.001 (proposed estimation method)
Amplitude coefficient of input motion	A_c	1.5

where $|H(\omega)|$ is defined as the ratio of the complex amplitude of the response displacement to the input acceleration in the frequency domain.

In order to approximate the non-linear response using the response of the equivalent linear system, the equivalent stiffness and the equivalent damping ratio are used. In the present paper, the amplitude of the transfer function of the equivalent linear system that corresponds to the non-linear system at the assumed μ, $|H_{eq}(\omega)|$, is defined as follows:

$$\left|H_{eq}(\omega)\right| = \sqrt{\frac{m^2}{\left(2\beta_{eq}k_{eq}\right)^2 + \left(m\omega^2 - k_{eq}\right)^2}}\ , \tag{3}$$

where k_{eq} is the equivalent stiffness, and β_{eq} is the equivalent complex damping ratio.

The equivalent stiffness k_{eq} is given as follows:

$$k_{eq} = \frac{k_0\alpha_y}{\eta^2\mu}\ , \tag{4}$$

where α_y is the yield stiffness ratio, and η is the modification factor considering the non-linear transient response given by Eq. (5) as a function of the assumed μ.

$$\eta = \sqrt{\frac{1+\mu}{2\mu}}\ . \tag{5}$$

Equation (5) was proposed by Nakamura and Kabeyasawa (1998) as a modified equation of the equivalent periods of energy responses of RC structures.

In reference to the previous studies (Gulkan and Sozen 1974; Shibata and Sozen 1976), the equivalent complex damping ratio β_{eq} is given as follows:

$$\beta_{eq} = \beta_0 + \gamma\left(1 - \frac{1}{\sqrt{\eta^2\mu}}\right)\ , \tag{6}$$

where β_0 is the initial complex damping ratio (= 0.02), and γ is the reduction coefficient of equivalent damping (= 0.2).

(4) RMS responses

The estimated RMS response displacement σ_{est} is given as follows:

$$\sigma_{est} = \sqrt{\frac{1}{2\pi}\int_0^\infty |\, H_{eq}(\omega)\,|^2\ G_I(\omega)\mathrm{d}\omega}\ . \tag{7}$$

Here, σ_{est} is calculated from the power spectral density of the input motion, $G_I(\omega)$, and the transfer function of the equivalent linear system, $|H_{eq}(\omega)|$, in the frequency domain.

According to Parseval's theorem, Eq. (7) gives the RMS of the time history response displacement indirectly. The RMS of the time history response is correlated with the maximum value of the time history response. Therefore, evaluating the RMS response in the frequency domain is useful. The RMS of transfer functions is particularly beneficial for solving optimum design problems for vibration control structures, such as in the previous study on the optimum yield strength of hysteretic dampers incorporated into RC structures (Shirai *et al.* 2011b).

(5) Peak factors

The peak factor is defined as the ratio of maximum response to the RMS response. Previous studies on peak factors have dealt primarily with linear response (e.g., Rosenblueth and Bustamante 1962; Ang 1974; Shibata 1981). In the present paper, a simple estimation equation is used to calculate the peak factor, P_{est}, as follows:

$$P_{est} = \sqrt{2\ln\left[\left\{1\Big/\ln\left(\frac{1}{1-p_0}\right)\right\}\left(\frac{2t_d}{T_{eq}}\right)\right]}\ , \tag{8}$$

where p_0 is the exceedance probability given as p_0 = 1−exp(−1), and T_{eq} is the equivalent period considering the non-linear transient response given by Eq. (9) as the function including η,

$$T_{eq} = \eta T_0\sqrt{\frac{\mu}{\alpha_y}} = T_0\sqrt{\frac{\eta^2\mu}{\alpha_y}}\ . \tag{9}$$

Equation (8) is an expansion of the equation described by Shibata (1981), which is based on a study by Ang (1974), to the non-linear transient response. As t_d increases, the value of P_{est} increases. The peak factor enables the proposed method to consider the effects of the duration times of input motions. Therefore, estimating peak factors with high accuracy is important.

(6) Estimated ductility factors and other configurations

The estimated maximum response ductility factor μ_{est} is obtained as follows:

$$\mu_{est} = \sigma_{est}P_{est}\big/\delta_y\ . \tag{10}$$

The acceptable error ε used in Step 8 is 0.001.

3. Estimation results and verifications of the proposed method

3.1 Non-linear time history analysis method

This section describes the method of non-linear time history response analyses, which are conducted to verify the adequacy of the proposed response estimation method. The non-linear SDOF systems and the degrading tri-linear type restoring force characteristics are used for the analytical models, as shown in **Fig. 7**. The parameters of the analytical models are configured as shown in **Table 2**, except for β_0, γ, p_0, and ε, which are only used for the proposed estimation method. The input motions for the time history analyses are the same as those for the estimation method. Viscous damping is provided as stiffness-proportional damping, and the viscous damping ratio h is 0.02 for secant stiffness at the yield point k_y. The Newmark-beta method (beta = 0.25) is

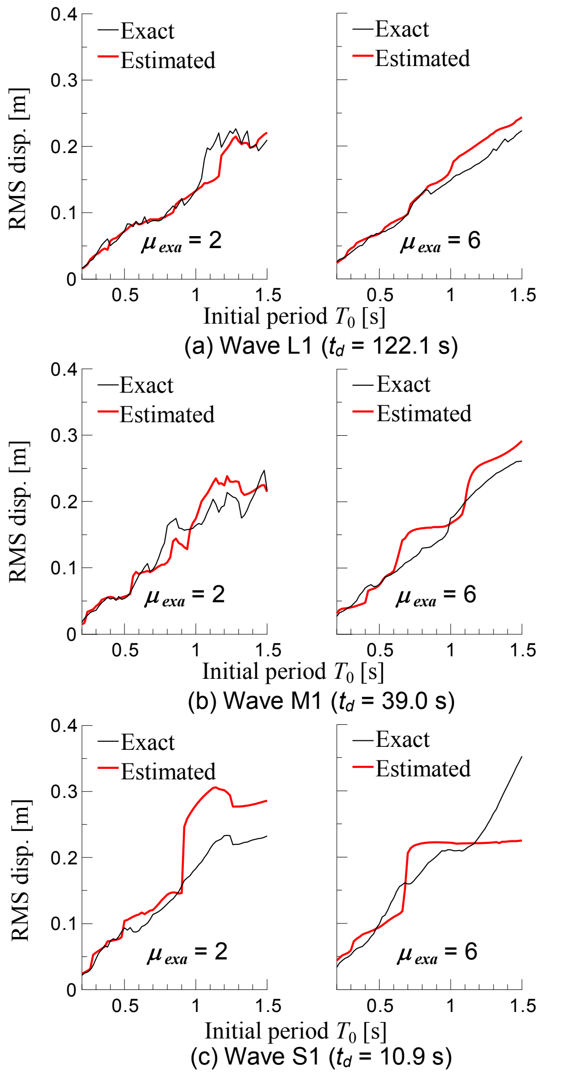

Fig. 9 RMS response displacement (proposed method).

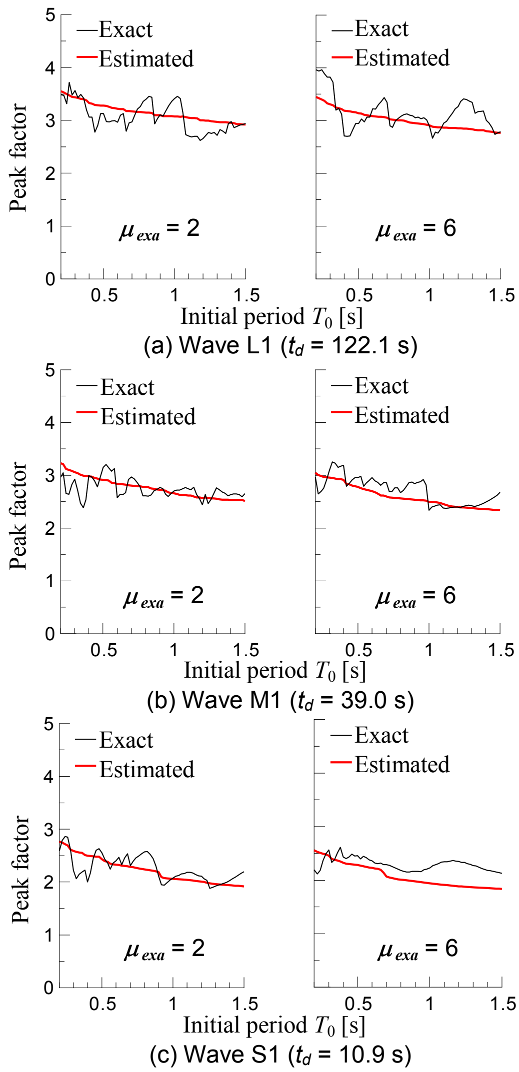

Fig. 10 Peak factor (proposed method).

used for numerical integration, and the time interval in computation is 0.001 s. Based on the time history analyses, the exact ductility factor μ_{exa}, the exact RMS response displacement σ_{exa}, and the exact peak factor P_{exa} are calculated using Eqs. (11) through (13),

$$\mu_{exa} = |x_{exa}(t)|_{\max} / \delta_y , \tag{11}$$

$$\sigma_{exa} = \sqrt{\frac{1}{t_d}\int_{t_1}^{t_2} |x_{exa}(t)|^2 \, dt} , \tag{12}$$

$$P_{exa} = \frac{|x_{exa}(t)|_{\max}}{\sigma_{exa}} , \tag{13}$$

where $|x_{exa}(t)|_{\max}$ is the exact maximum response displacement, and t_1 and t_2 are the starting time and ending time, respectively, of t_d.

3.2 Comparisons of the proposed method and time history analyses

This section describes the verification of the adequacy of the proposed estimation method. The estimation results have been compared with the results of the time history analyses for the case in which the input motions multiplied by the amplification coefficient A_c = 1.5 and structural properties described in Section 2 are used. The estimated RMS response displacement σ_{est}, estimated peak factor P_{est}, and estimated maximum response ductility factor μ_{est} are calculated according to the flowchart shown in **Fig. 2** and using Eqs. (3) through (10). On the other hand, the exact RMS σ_{exa}, exact peak factor P_{exa}, and exact ductility factor μ_{exa} are obtained based on the results of the non-linear time history response analyses described in Section 3.1 and using Eqs. (11) through (13).

Figure 9 compares the exact RMS response σ_{exa} and the estimate RMS response σ_{est} (Waves L1, M1, and S1, and exact maximum response ductility factor μ_{exa} = 2 and 6) for the case in which the horizontal axes show the initial period T_0 of the systems. This indicates that σ_{exa} (exact) decreases as the duration time t_d of the input motion becomes longer, and the tendency of σ_{exa} (exact) is generally approximated by σ_{est} (estimation).

Figure 10 compares the exact peak factor P_{exa} and estimated peak factor P_{est} (Waves L1, M1, and S1, and μ_{exa}

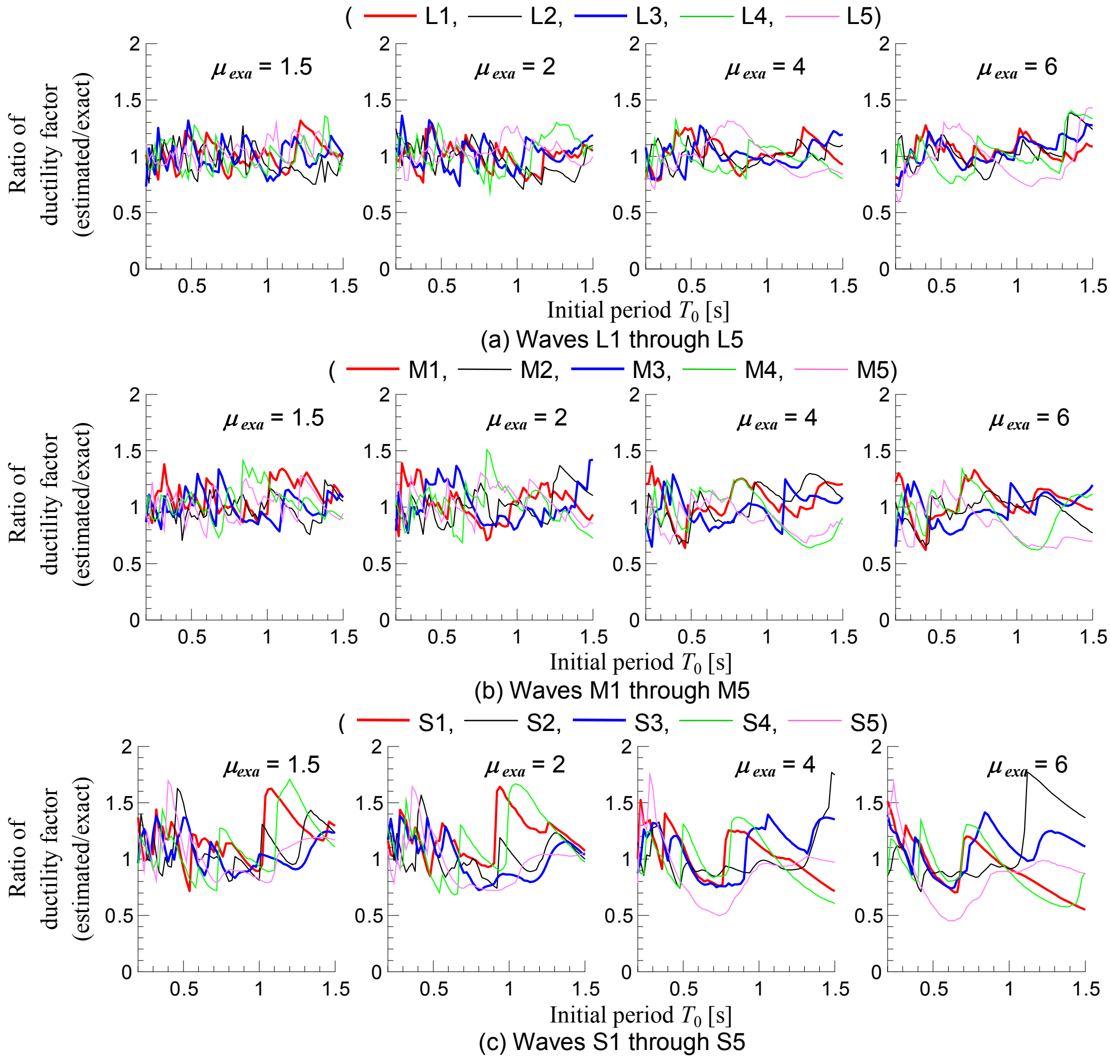

Fig. 11 Ratio of estimated ductility factor μ_{est} to exact ductility factor μ_{exa} (proposed method).

= 2 and 6), for the case in which the horizontal axis corresponds to T_0, as in **Fig. 9**. This indicates that P_{exa} (exact) increases as t_d becomes longer and T_0 becomes shorter, and P_{est} (estimation) approximately reflects the behavior of P_{exa} (exact).

Figure 11 shows the ratio of the estimated maximum response ductility factor μ_{est} to the exact ductility factor μ_{exa}, i.e., μ_{est}/μ_{exa} vs. T_0 (all 15 waves, and μ_{exa} = 1.5, 2, 4, and 6). Based on this figure, the estimated results obtained by the proposed method approximately agree with the exact results of the non-linear time history response analyses. Although there is room for improvement, it has been demonstrated that the proposed method is effective for the estimation of the maximum response of RC structures considering the frequency characteristics and the duration effects of the input ground motions separately.

3.3 Comparison to existing approximation methods

This section describes the comparison of the estimation results obtained by the proposed method and existing approximation methods. As examples of existing methods, two estimations using equivalent linearization and response spectra based on the capacity spectrum method in CRLS are carried out. A description of the two existing methods is provided below.

First, the equivalent period, T_{eq}, is given as follows:

$$T_{eq} = T_0 \sqrt{\frac{\mu}{\alpha_y}} \,. \tag{14}$$

Second, the equivalent viscous damping ratio, h_{eq}, is given as follows (two cases):

$$h_{eq} = 0.05 + 0.25\left(1 - \frac{1}{\sqrt{\mu}}\right) , \tag{15a}$$

$$h_{eq} = 0.05 + 0.20\left(1 - \frac{1}{\sqrt{\mu}}\right) . \tag{15b}$$

Here, Eq. (15a) is often used for the capacity spectrum method in CRLS, and Eq. (15b) is based on the empirical equation proposed by Gulkan and Sozen (1974).

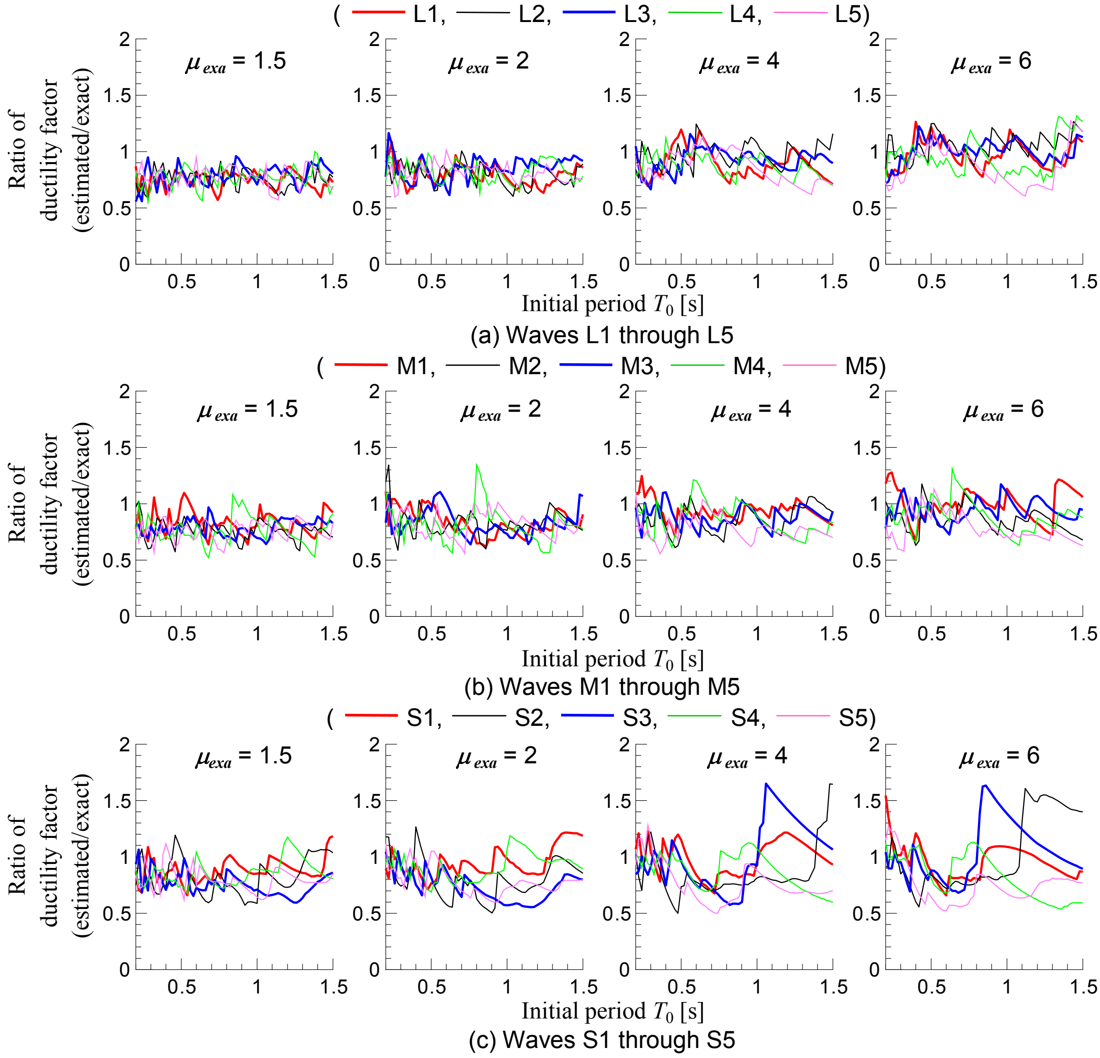

Fig. 12 Ratio of estimated ductility factor μ_{est} to exact ductility factor μ_{exa} (existing method using Eq. (15a)).

Then, the response acceleration of the equivalent system, $S_a(T_{eq}, h_{eq})$, is obtained as follows:

$$S_a(T_{eq}, h_{eq}) = F_h \cdot S_a(T_{eq}, h = 0.05)\,, \tag{16}$$

where S_a (T_{eq}, h = 0.05) is the acceleration response spectra of the input motion computed by earthquake response analysis when $T = T_{eq}$ and $h = 0.05$, and F_h is the damping modification coefficient considering the response reduction effect due to the increase in h_{eq} and is given as follows:

$$F_h = \frac{1.5}{1+10h_{eq}}\,. \tag{17}$$

In the present paper, the estimated maximum response is obtained as the intersection point of $S_a(T_{eq}, h_{eq})$ and the acceleration skeleton curve of the restoring force characteristics shown in **Fig. 7** and **Table 2**. The estimation results are illustrated below. Here, the structural properties and the input motions are the same as those used in the estimation by the proposed method (Section 3.2).

Figure 12 shows the estimated results of the existing method using Eqs. (14), (15a), (16), and (17) compared to the exact value obtained by the non-linear time history response analyses, as is the case in **Fig. 11**. **Figure 13** also shows the estimated results of the existing method using Eqs. (14), (15b), (16), and (17).

Moreover, in order to quantitatively compare the estimation accuracy of the proposed method with the accuracy of existing methods, **Table 3** shows the average value (AV), the standard deviation (SD), and the coefficient of variation (CV (= SD/AV)) of the ratio of the estimated maximum response ductility factor μ_{est} to the exact ductility factor μ_{exa}. Each value in **Table 3** is an average value for the initial period T_0 (between 0.2 and 1.5 s) and the exact ductility factor μ_{exa} (= 1.5, 2, 3, 4, 5, and 6). **Table 3** shows that the proposed method has relatively high accuracy compared with the two existing methods, because each AV of the proposed method is closer to unity than that for the existing method using Eq. (15a) and each CV of the proposed method is smaller than that for the existing method using Eq. (15b).

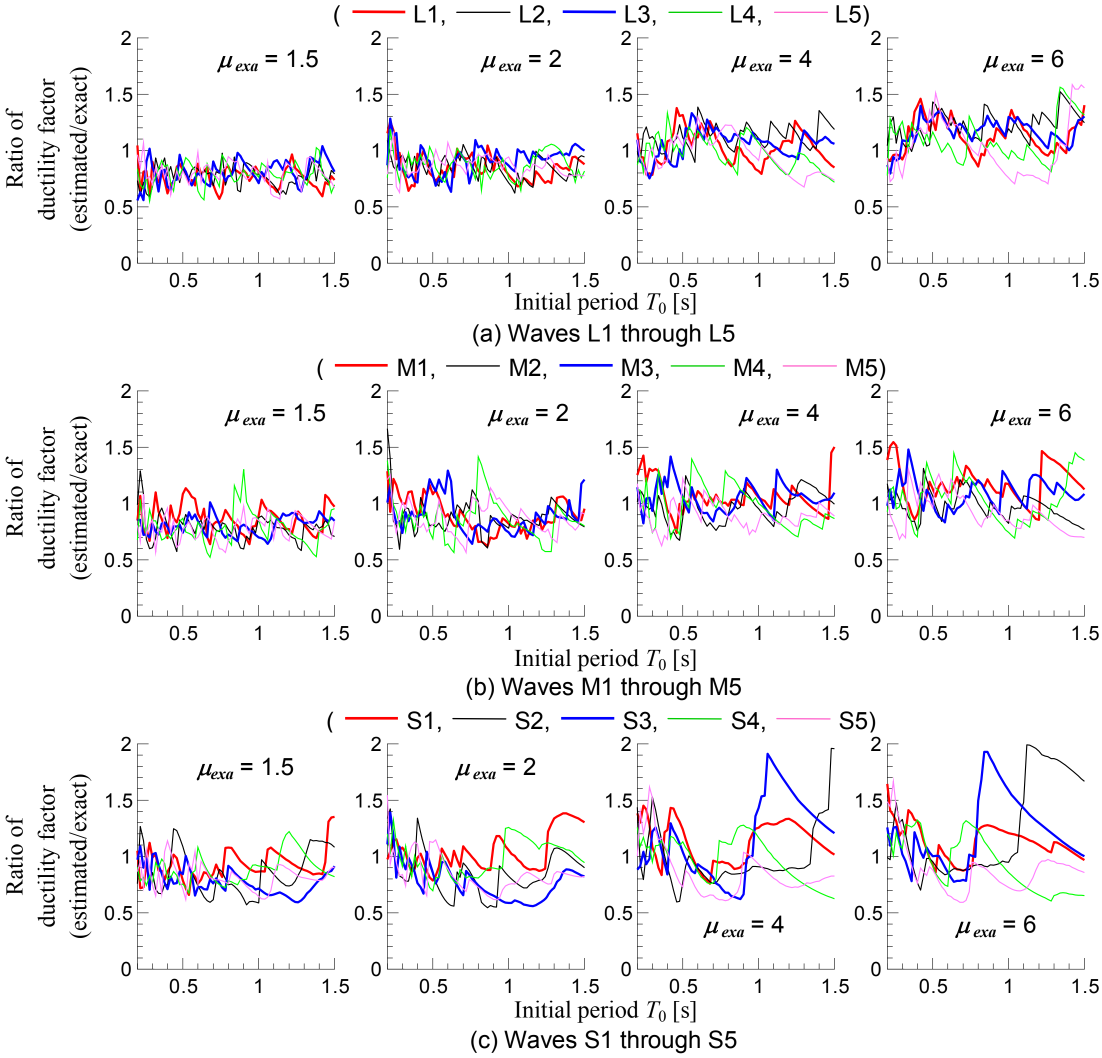

Fig. 13 Ratio of estimated ductility factor μ_{est} to exact ductility factor μ_{exa} (existing method using Eq. (15b)).

Table 3 Accuracy comparison for the estimation results.

	Proposed method			Existing method using Eq. (15a)			Existing method using Eq. (15b)		
	AV	SD	CV	AV	SD	CV	AV	SD	CV
Average value for Waves L1 through L5	1.011	0.131	13.0%	0.878	0.133	15.1%	0.978	0.182	18.6%
Average value for Waves M1 through M5	0.996	0.156	15.8%	0.854	0.122	14.3%	0.949	0.166	17.4%
Average value for Waves S1 through S5	1.026	0.233	22.8%	0.878	0.195	22.2%	0.981	0.248	25.2%
Overall average	1.011	0.174	17.2%	0.870	0.150	17.2%	0.970	0.198	20.4%

AV: average value of ratio of ductility factor (estimated/exact)
SD: standard deviation of ratio of ductility factor (estimated/exact)
CV: coefficient of variation (= SD/AV) [%]
Eq. (15a): $h_{eq} = 0.05 + 0.25\{1-1/(\mu^{0.5})\}$
Eq. (15b): $h_{eq} = 0.05 + 0.20\{1-1/(\mu^{0.5})\}$

4. Conclusion

In the present paper, a non-linear seismic response estimation method for RC structures based on random vibration theory and the equivalent linearization technique has been proposed. In contrast to previous estimation methods using response spectra, the proposed method enables individual evaluation of the power spectral densities and duration times of input ground motions. Equivalent linear SDOF systems, which have complex stiffness, are adopted in order to approximate the displacement-dependent hysteretic characteristics of RC structures. The proposed method estimates the maximum response displacement ductility factor using the transfer function of the equivalent linear system, the RMS response, and the peak factor.

The estimated responses obtained by the proposed method were verified through comparisons with the results of non-linear time history analyses when subjected to 15 simulated earthquake motions, which have the same target response spectra and different duration times. The estimation results of the maximum response ductility factors, the RMS responses, and the peak factors were found to approximately correspond to the exact analytical results. Moreover, the estimation results obtained using the proposed method have relatively high accuracy compared with the results of two existing methods based on the capacity spectrum method in CRLS. Although there is room for improvement of the estimate accuracy, the proposed method has been demonstrated to be useful for maximum non-linear response estimation of RC structures considering the frequency spectral characteristics and the duration effects separately.

Applications of the proposed method will hopefully allow a simple and simultaneous evaluation of the effects of site amplifications, soil-structure interactions, and responses of RC upper structures. Investigation of the estimation accuracy of the proposed method for structures that have other hysteresis loop characteristics (e.g., non-linear elastic and elastic perfectly plastic) and other input motions (e.g., long-period earthquake ground motions) are issues for future study.

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List of Symbols

a_1, a_2, a_3: parameters of the time envelope function of the input motion
A_c: amplification coefficient of the input motion
C_y: yield shear force coefficient
F_c: crack shear force
F_h: damping modification coefficient
F_y: yield shear force
g: acceleration of gravity
$G_I(\omega)$: power spectral density of the input motion
h: viscous damping ratio
h_{eq}: equivalent viscous damping ratio
$|H(\omega)|$: amplitude of the transfer function of the linear system
$|H_{eq}(\omega)|$: amplitude of the transfer function of the equivalent linear system
i: imaginary unit
k: stiffness
k': imaginary part of complex stiffness
k_0: initial stiffness
k_{eq}: equivalent stiffness
k_u: stiffness after yielding
k_y: secant stiffness at yield point
m: mass
p_0: exceedance probability
P_{est}: estimated peak factor of response displacement obtained using the proposed method
P_{exa}: exact peak factor of response displacement obtained through non-linear time history analysis
S_a: acceleration response spectra
t_1, t_2: starting time and ending time of the duration time of the input motion
t_d: duration time of the input motion
T: natural period
T_0: initial period
T_{eq}: equivalent period
x: response displacement
$|x_{exa}(t)|$: exact maximum response displacement by non-linear time history analysis
x_g: input displacement
α_y: yield stiffness ratio
β: complex damping ratio
β_0: initial complex damping ratio
β_{eq}: equivalent complex damping ratio
γ: reduction coefficient of equivalent damping
δ: peak displacement in stationary response
δ_y: yield displacement
ΔW: dissipated energy per cycle in stationary response
ε: acceptable error
η: modification factor considering transient response
μ: assumed maximum response displacement ductility factor
μ_{est}: estimated maximum response displacement ductility factor
μ_{exa}: exact maximum response displacement ductility factor obtained through non-linear time history analysis
σ_{est}: estimated RMS of response displacement obtained using the proposed method
σ_{exa}: exact RMS of response displacement obtained through non-linear time history analysis
ω: circular frequency