Parity reversal of $^{12}_Λ$Be

H. Homma, M. Isaka, and M. Kimura

Department of Physics, Hokkaido University, Sapporo 060-0810, Japan

RIKEN Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan

(Received 14 October 2014; revised manuscript received 9 December 2014; published 16 January 2015)

We study the spectrum of $^{12}_Λ$Be by using an extended version of antlsymmetrized molecular dynamics for hypernuclei. Our result indicates that the $Λ$-particle impurity effect causes the positive-parity ground state of $^{12}_Λ$Be to revert to the normal state, i.e., negative parity. This parity reversion is attributed to the difference in the $Λ$ binding energy between the positive- and negative-parity states, which in turn originates in the difference in $α$ clustering and deformation.

DOI: 10.1103/PhysRevC.91.014314

PACS number(s): 21.80.+a, 21.10.Dr, 27.20.+n

I. INTRODUCTION

Despite their short lifetime, hypernuclei have been a subject of particular interest in nuclear physics because they provide an almost unique opportunity to investigate underlying baryon-baryon interactions. In particular, knowledge of the interaction between a $Λ$ hyperon and nucleons has greatly increased in recent decades [1–7], which strongly promotes the physics of $Λ$-baryon interactions. In particular, knowledge of the interaction of particular interest in nuclear physics because they provide the breakdown of the Fermi momentum $k_F$ and has successfully described them as the breakdown of the $N = 8$ and $N = 20$ magic numbers [24–29]. In the present study, we use an extended version of AMD for hypernuclei (HyperAMD) to investigate $^{12}_Λ$Be. HyperAMD has already been applied to $p$-shell hypernuclei [19,20], and its detailed formulation is given there.

II. FORMALISM

The Hamiltonian used in this study is

$$H = H_N + H_Λ - T_g,$$

where $H_N = T_N + V_{NN} + V_C$, $H_Λ = T_Λ + V_{ΛN}$, and $T_g$ are the kinetic energies of the nucleons, $Λ$ particle, and center-of-mass motion, respectively. We use the Gogny D1S interaction [30] as effective nucleon-nucleon interaction $V_{NN}$, and the Coulomb interaction $V_C$ is approximated by the sum of seven Gaussians. To see the interaction dependence of the result, we tested various $ΛN$ interactions for the central part of $V_{ΛN}$ [31–33], and in this paper we present the results obtained by the $YN G$-matrix interactions derived from the Nijmegen potentials named model D [7,34], NSC97f [35], and ESC08c [36], which we denote ND, NSC97f and ESC08c, respectively. Their strengths depend on the nuclear Fermi momentum $k_F$, and we adopt the value of $k_F = 1.06$ fm$^{-1}$, which is obtained by applying averaged density approximation [37]. As the spin-orbit interaction part of $V_{ΛN}$, we always used that of the ESC08c.

The variational HyperAMD wave function is a parity eigenstate. The intrinsic wave function $Ψ_{\text{int}}$ is represented by the direct product of the $Λ$ single-particle wave function $Ψ_Λ$ and the wave function $Ψ_N$ of $N$ nucleons, which is a Slater determinant of the nucleon wave packets $ψ_i$,

$$\Psi_± = P_± Ψ_{\text{int}}, \quad Ψ_N = Ψ_N ⊗ Ψ_Λ,$$

where $P_±$ is the parity projector. The nucleon single-particle wave packets $φ_i$ are represented by a Gaussian:

$$ψ_i(r) = \phi_i(r) |^1_4 \sqrt{\frac{2ν_0}{π}} \exp \{-ν_0 (r - Z_i)^2 \},$$

$$φ_i(r) = \prod_{x,y,z} \left( 2\nu_0 \right)^{1/4} \exp \{ -ν_0 (r - Z_i)^2 \},$$

where $P_±$ is the parity projector. The nucleon single-particle wave packets $φ_i$ are represented by a Gaussian:

$$\phi_i(r) = \prod_{x,y,z} \left( 2\nu_0 \right)^{1/4} \exp \{ -ν_0 (r - Z_i)^2 \},$$

and

$$χ_i = a_i χ_↑ + b_i χ_↓, \quad τ_i = p \text{ or } n.$$

0556-2813/2015/91(1)/014314(5) 014314-1 ©2015 American Physical Society
The Λ single-particle wave function is represented by a sum of Gaussians,

\[ \varphi_\Lambda(r) = \sum_{m=1}^{M} c_m \varphi_m(r), \quad \varphi_m(r) = \varphi_m(r) \chi_m, \quad (8) \]

\[ \varphi_m(r) = \prod_{\sigma=x,y,z} \left( \frac{2 \nu_\sigma}{\pi} \right)^{1/4} \exp \left\{ -\nu_\sigma (r - \xi_m)^2 \right\}, \quad (9) \]

\[ \chi_m = \alpha_m \chi_\uparrow + \beta_m \chi_\downarrow, \quad (10) \]

where the number of superposed Gaussians is chosen to be sufficiently large to achieve energy convergence. The variational parameters are centroids of the Gaussian wave packets \( Z_i \) and \( \xi_m \), the Gaussian width \( \nu_\sigma \), the coefficients \( c_m \), and spin directions \( \alpha_i, \beta_i, \alpha_m, \) and \( \beta_m \). These parameters are set to minimize the total energy under the constraint on the matter quadrupole deformation parameter \( \beta \) [24]. The value of \( \beta \) is constrained to range from 0 to 1.2 in intervals of 0.025.

After the variation, we project out the eigenstate of the total angular momentum \( J \) for each value of \( \beta \),

\[ \psi_{J,\Lambda K}^\pm(\beta) = \frac{2J + 1}{8\pi^2} \int d\Omega D_{MK}^{J_\Lambda}(\Omega) R(\Omega) \psi^{\pm}(\beta). \quad (11) \]

The integrals over the three Euler angles \( \Omega \) are performed numerically. Then we perform the generator coordinate method (GCM) [38]. The wave functions with differing values of \( K \) and \( \beta \) are superposed:

\[ \psi_{nJ}^\pm = \sum_{K=-J}^{J} c_{nJK} \psi_{J,\Lambda K}^\pm(\beta). \quad (12) \]

The coefficients \( c_{nJK} \) are determined by solving the Griffin-Hill-Wheeler equation.

### III. RESULTS AND DISCUSSIONS

Before discussing \( ^{12}_\Lambda \text{Be} \), it is helpful to review the structure of \( ^{11}_\Lambda \text{Be} \). Figure 1(a) shows the observed spectra of the \( N = 7 \) isotones, \( ^{13}_\text{C} \) and \( ^{11}_\text{Be} \). The large gap between the \( p_{1/2} \) and \( sd \) shells in \( ^{13}_\text{C} \) collapses in \( ^{11}_\text{Be} \). The \( ^{11}_\text{Be} \) ground state has positive parity, and the order of the \( p_{1/2} \) and \( sd \) shells looks inverted, which is called parity inversion [21,22]. Using the original Gogny D1S parameter set, our calculation reproduces the spin-parity of the ground state and gives a binding energy of 65.32 MeV; the first excited state \( 1/2^- \) is at 540 keV (the observed values are 65.48 MeV and 320 keV, respectively [23]). For more quantitative discussion of \( ^{12}_\Lambda \text{Be} \), we weakened the spin-orbit interaction of Gogny D1S by 5% to exactly reproduce the observed \( 1/2^- \) excitation energy. With this modification, we calculate the binding energy of \( ^{11}_\Lambda \text{Be} \) to be 64.77 MeV, and the resulting spectrum is shown in Fig. 1(b). Here the excited unbound states are calculated within the bound-state approximation.

The low-lying states of Be isotopes are known to have a \( 2\alpha \) cluster core and valence neutrons occupying the molecular orbits around the core, which are called \( \pi \) and \( \sigma \) orbits [26]. The formation of the \( 2\alpha \) cluster core in each state is confirmed by the proton density shown in Fig. 2. The ground state is a member of the \( K^\pi = 1/2^- \) band in which two of the three valence neutrons occupy the \( \pi \) orbit, and the third valence

FIG. 2. (Color online) Matter (\( \rho_\text{m} \)) and proton (\( \rho_\text{p} \)) density distributions for the bandhead states of \( ^{11}_\text{Be} \) and \( ^{12}_\Lambda \text{Be} \). Color plots show the density distribution of the \( \Lambda \) particle.
neutron occupies the $\sigma$ orbit. In terms of the spherical shell model, a neutron is promoted to $sd$ shell across the $N = 8$ shell gap (breakdown of magic number $N = 8$). The first excited state has negative parity and belongs to the $K^\pi = 1/2^-$ band. All valence neutrons occupy the $\pi$ orbit or $\rho$ shell, which corresponds to the normal shell order. As we can see from Fig. 2 and Table I, the ground state has a more pronounced $2\pi$ clustering and a larger quadrupole deformation $\beta$ than the first excited state. Here the quadrupole deformation $\beta$ of each state is defined as the deformation of the basis wave function $\Psi_{ME}^\pm(\beta)$ that has maximum overlap with the GCM wave function [Eq. (12)]. Among the bandhead states, the $3/2^-_1$ state (bandhead of $K^\pi = 3/2^-$) with two valence neutrons in a $\sigma$ orbit is the most deformed. The assignment of the unbound excited states, including the $3/2^-_1$ state, is still under discussion, so we devote further discussions to Refs. [40–42].

Here we simply note that the positive-parity ground state is more deformed than the first excited state with negative parity. This point makes a difference when a $\Lambda$ particle is injected.

We found that all of the $\Lambda N$ interactions tested in the present study yield the same qualitative result for $^{12}\Lambda$Be and predict the negative-parity ground state of $^{12}\Lambda$Be. In other words, the inverted ground-state parity of $^{11}\Lambda$Be will be reverted in $^{12}\Lambda$Be by injecting a $\Lambda$ particle. The spectra of $^{12}\Lambda$Be obtained by using ESC08c, ND, and NSC97f are shown in Fig. 1(c). Among the tested $\Lambda N$ interactions, the ND gives the largest energy gap between the negative-parity ground state and the positive-parity excited state, while the NSC97f gives the smallest. For the moment, we focus on the result obtained by using the ESC08c, and later we discuss the quantitative difference and $\Lambda N$ interaction dependence of the results.

All states shown in Fig. 1(c) have a $\Lambda$ in an $s$ orbit and are classified into three bands. They are generated by the coupling of the $K^\pi = 1/2^+, 1/2^-$, and $3/2^-$ bands of $^{11}\Lambda$Be with an $\Lambda$ particle in an $s$ orbit; thus, there are always doublet states of $^{12}\Lambda$Be for each corresponding state of $^{11}\Lambda$Be. These bands are denoted as $K^\pi = 1/2^+ \otimes \Lambda_s, 1/2^- \otimes \Lambda_s$, and $3/2^- \otimes \Lambda_s$, respectively. In the case of the ESC08c, the ground doublet has a $^{11}\Lambda$Be$(1/2^-) \otimes \Lambda_s$ configuration and consists of the $0^-_1$ and $1^-_1$ states with binding energies of 75.29 and 75.27 MeV, while the first excited doublet with a $^{11}\Lambda$Be$(1/2^+) \otimes \Lambda_s$ configuration consists of the $0^+_1$ state at 180 keV and the $1^+_1$ state at 220 keV. Thus, the ground-state parity has reverted to negative parity in $^{12}\Lambda$Be, as if adding a $\Lambda$ particle has restored the $N = 8$ shell gap. This parity reversion of $^{12}\Lambda$Be is due to the difference in the $\Lambda$ particle binding energy $B_{\Lambda}$ for the ground and first excited doublets. Here $B_{\Lambda}$ is defined as the difference in binding energies between a given $^{12}\Lambda$Be and the corresponding $^{11}\Lambda$Be state,

$$B_{\Lambda} = B(12^{\Lambda}\text{Be}(J^\pi)) - B(11^{\Lambda}\text{Be}(J^\pi)).$$

and as shown in Table I, $B_{\Lambda}$ of the $0^-_1$ state is 500 keV larger than that of the $0^+_1$ state. The change of the core nucleus energy $H_{\Lambda}$ caused by injection of a $\Lambda$ particle is rather small; therefore, the difference in $B_{\Lambda}$ overweights the energy difference between the $1/2^+_1$ states of $^{11}\Lambda$Be, leading to the parity reversion in $^{12}\Lambda$Be. This difference in $B_{\Lambda}$ mainly arises from the $\Lambda N$ potential $V_{\Lambda N}$ as shown in Table I. In turn, the difference in the $\Lambda N$ potential $V_{\Lambda N}$ originates in the difference in quadrupole deformation. Figure 3 shows the $\Lambda$ particle binding energies for the $0^+_1$ states as a function of quadrupole deformation, which is defined as the expectation value of $H_{\Lambda}$ by the angular-momentum-projected wave functions for each deformation parameter $\beta$,

$$b_{\Lambda}^\pm(\beta) = -\langle \Psi_{ME}^\pm(\beta) | H_{\Lambda} | \Psi_{ME}^\pm(\beta) \rangle.$$  

The $b_{\Lambda}$ of the $0^-_1$ and $0^+_1$ states rapidly decrease as the deformation becomes larger, which is mostly due to the reduction of $V_{\Lambda N}$ caused by the decrease of the overlap between the $\Lambda$ particle and the nucleons wave functions, as discussed in the case of $p-sd-pf$ shell nuclei [19,20,43]. The results shown in Fig. 3 allow us to confirm that the 500-keV difference in $B_{\Lambda}$ originates from the different deformation of the $0^-_1$ states. Because the $3/2^-_1$ state is the most deformed among the bandhead states of $^{11}\Lambda$Be, the $1^-_1$ state ($^{11}\Lambda$Be$(3/2^-_1) \otimes \Lambda_s$) has the smallest $B_{\Lambda}$ and its excitation energy is considerably shifted up.

Adding a $\Lambda$ particle slightly modifies the structure of the $^{11}\Lambda$Be core nucleus. The attraction of the $\Lambda$ particle sitting at the center of the system causes the intercluster distance between...
the 2σ clusters to decrease (Fig. 2), and it also leads to the a decrease in the deformation and radius (Table I). In spite of the neutron-halo structure of $^{11}\text{Be}$, this reduction is rather small compared with what is observed for $^7\Lambda\text{Li}$ [12] and cannot be clearly seen in the density distribution (Fig. 2). We attribute this to the presence of valence neutrons occupying molecular orbits in $^{12}\Lambda\text{Be}$. A decrease in the distance between the 2σ clusters would result in the valence neutrons in $\pi$ or $\sigma$ orbits loosing their binding energy [26]. Consequently, the valence neutrons effectively serve to prevent a drastic reduction of the 2σ distance. Note that this also explains why the change in the expectation value of the nuclear part is not large compared to the difference in $B_\Lambda$. Finally, we examine how the properties and uncertainties of the $\Lambda N$ interactions affect the quantitative results and the parity reversion in $^{12}\Lambda\text{Be}$. The first point we focus on is the spin-independent odd-parity part of the $\Lambda N$ central force that acts between the $\Lambda$ particle in $s$ orbit and nucleons in $p$ orbits. Recall that the $1/2^-_1$ state of $^{11}\text{Be}$ has 7 nucleons in the $p$ orbits, whereas the $1/2^+_1$ state has 6 nucleons. Therefore, roughly speaking, the $0^+_1$-$1^-_1$ doublet has greater number of spin-independent odd-parity interactions between the $\Lambda$ particle and nucleons (7 interactions) than the $0^+_1$-$1^-_1$ doublet (6 interactions). Consequently, we can imagine if the spin-independent odd-parity interaction is too strongly repulsive, the centroid energy of the $0^+_1$ will be pushed up relative to the $0^+_1$-$1^-_1$ doublet and the parity reversion will not occur. It is known that this interaction is more repulsive in NSC97f than in ESC08c, while it is attractive in ND. This difference can be seen in Fig. 1(c), i.e., NSC97f gives smaller centroid energy difference (0.14 MeV) than ESC08c (0.19 MeV), while ND gives the largest difference (0.34 MeV). The second point is the $\sigma_\Lambda$-$\sigma_N$ dependent part of the central force and the spin-orbit force, which apparently affects the energy splitting within a doublet. It is also known that NSC97f has stronger $\sigma_\Lambda$-$\sigma_N$-dependent term than ESC08c and ND and reproduces the observed doublet splitting in $^3\text{H}$ and $^3\Lambda\text{Li}$ [44]. This characteristic is also confirmed in Fig. 1, in which NSC97f gives the largest doublet splitting. As a result of the spin-independent odd-parity and $\sigma_\Lambda$-$\sigma_N$-dependent parts, the energy gap between the $1^-_1$ and $0^+_1$ is only 40 keV in NSC97f result. It is noted that the sign of the positive- and odd-parity $\sigma_\Lambda$-$\sigma_N$-dependent part of $\Lambda$ is opposite to those of NSC97f and ESC08c. In Fig. 1(c), this difference leads to the opposite level ordering of the the $0^-_1$-$1^-_1$ and $0^+_1$-$1^-_1$ doublets between the ND and NSC97f results, while these doublets are almost degenerated in the ESC08c result. It is found that the contribution from the spin-orbit interaction of ESC08c is less than 100 keV for $0^+_1$ and $1^-_1$ doublets, which is consistent with the observations in $^8\Lambda\text{Be}$ [45,46] and $^9\Lambda\text{C}$ [47,48]. Therefore, we consider that the uncertainty of the spin-orbit interaction has minor influence to the parity reversion compared to other uncertainties discussed above. Although all the $\Lambda N$ interactions tested in this study suggest that the reversion will occur, several properties of the $\Lambda N$ interactions have influence to the parity reversion in $^{12}\Lambda\text{Be}$, and more detailed knowledge on $\Lambda N$ interaction such as tensor term will be needed for further quantitative studies.

IV. CONCLUSION

In summary, we have investigated the low-lying states of $^{12}\Lambda\text{Be}$ by using the HyperAMD. We predict a parity reversion of $^{12}\Lambda\text{Be}$; that is the inverted ground-state parity of $^{11}\text{Be}$ reverts to "normal" (i.e., negative) parity in $^{12}\Lambda\text{Be}$. The parity reversion is caused by the different deformations of the ground and first excited states of $^{11}\text{Be}$, which lead to a difference in the $\Lambda$ particle binding energy $B_\Lambda$. In addition, the parity reversion suggests that it may be possible to probe the different deformations of the ground and first excited states of $^{12}\Lambda\text{Be}$ by adding a $\Lambda$-particle impurity. Therefore, it is quite interesting and important to investigate the excitation spectrum of $^{12}\Lambda\text{Be}$ by the $^{12}\text{C}(\pi^-,K^+)$ reaction with high-energy resolution.

ACKNOWLEDGMENTS

Part of the numerical calculations was performed on the HITACHI SR16000 at KEK. One of the authors (M.K.) acknowledges support by the Grants-in-Aid for Scientific Research on Innovative Areas from MEXT (Grant No. 2404; 24105008) and JSPS KAKENHI Grant 563 No. 25400240. M.I. is supported by the Special Postdoctoral Researcher Program of RIKEN.


014314-5