Rational Design Optimization Method for Reducing Cost and Improving Performance of Commonalized IPM Motors

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In this paper, we propose an economically-efficient optimal design method of different-size interior permanent magnet (IPM) motors to maximize their efficiency. Since the homothetic shape is used, the optimization design is, at once, performed. The use of the commonalized design can reduce the productive costs. Game theory is employed as an optimization method. The game theory can simultaneously increase the efficiency of all the interior permanent magnet motors with the common and homothetic shape. So far, there are no previous reports considering both the "economical efficiency" and the motor performance simultaneously. Therefore, the proposed optimal design method of the commonalized different-size interior permanent magnet motors using the game theory method achieves a high electrical efficiency in addition to the economical efficiency due to design optimization.

***Index Terms*—Efficiency design, game theory, motor design, optimal design.**

I. Introduction

MORE DEVELOPMENT of permanent magnet (PM) motors is indispensable for global energy saving. In addition, it is required to reduce the product cost of PM motors. Many researchers have studied motor design methods [1], [2], [3], [4], and, over recent years, an optimal design method has attracted attention. It was reported that the shape of a PM motor, ased on such parameters as teeth width, teeth length, magnet size, *etc*., was optimally designed for the purpose of increasing the efficiency under a torque [1]. Takahashi *et al.* proposed an ON/OFF optimization method for an interior permanent magnet (IPM) motor design to reduce a torque ripple and enhance a driving torque [2]. The paper [3] proposed a robust optimization method of averaging a material characteristic variation for PM motor design. It usually consists of a finite element method (FEM) and an optimization method such as Genetic Algorithm (GA), Simulated Annealing (SA), Particle Swam Optimization (PSO), *etc*. There is an obvious need to largely develop optimal design methods for PM motors.

In recent years, new kinds of optimization methods have been investigated. Essentially, they are not categorized into an optimization method in an engineering field but into a decision-making method in economics. Kim *et al.* proposed to use the Taguchi method for multi-objective optimal design of an IPM motor [5]. The game theory method was also used for a multi-objective optimal design problem of an electromagnetic apparatus [6]. For more practical use of an optimization shape design method, it is necessary to optimize multi-objective problems, and a result to be obtained has to be rational from the viewpoints of engineering, productivity, and economy.

This paper focuses not only on the motor specifications but also the economical efficiency of the IPM motor design. Usually every IPM motor is designed individually depending on the applications in order to attain a high efficiency and a low torque ripple. However, the IPM motor design costs a lot since the labor cost is very expensive. Therefore, we propose a new design concept for an IPM motor in order to enhance the economical efficiency as well as the motor specifications. In the proposed method, utilizing the game theory method [6], three homothetic motors are, at once, designed to save the motor design cost. The motor shape designed by the proposed method can simply and commonly be adjusted for different-size motors by homothetic transformation. The proposed method produces a basic design using a common architecture; therefore the obtained motor shape is easier to be used for subsequent designs. The proposed method can provide a commonalized shape of versatile motors by one design trial; *i.e.*, it enables to design many similar motors at the minimum design cost.

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II. Optimal Design of IPM Motor Shape

A. Commonalized IPM Motor Model

This paper proposes a way of designing a versatile IPM motor. The commonalized design realizes the reduction of the design cost, because it is unnecessary to redesign the shape of even a different-size IPM motor or a different-output one. Fig. 1 shows the common shape of an IPM motor model of three different sizes with individual rated outputs. Table I lists the specifications of the homothetic IPM motors #1, #2, and #3, and Table II shows the design variables for the optimal design. All the IPM motors have the air gap of 0.8 mm commonly.

The variables x_1–x_6 are treated as a common design variable, *i.e.*, x_1–x_6 are scaled according to the motor size ratio. On the other hand, the variables x_7–x_{15} are optimally designed individually to the IPM motor models #1, #2, and #3. The common objective is the efficiency of the IPM motor. Using the commonalized shape of the IPM motor reduces the design cost.

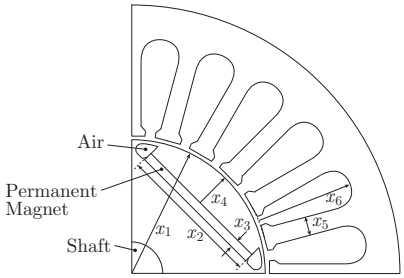


Fig. 1. Commonalized IPM motor model and design variables.

TABLE I
SPECIFICATIONS OF HOMOTHETIC IPM MOTOR MODEL

Model	#1	#2	#3
Rated speed (rpm)	4000	4000	4000
Rated torque (Nm)	1.2	1.7	2.0
Outside diameter of stator (mm)	80	100	120
Diameter of shaft (mm)	12	16	20
Motor size ratio (%)	80	100	120

B. Individual and Simultaneous Optimal Designs

So far, the motor shape is separately designed for each application. First, in this paper, we have optimally designed #1, #2, and #3 motors conventionally using the simple genetic algorithm (GA) [1] for each individual design. Each single objective function is as follows:

$$\text{maximize} \qquad \eta_{\#1,\#2,or\#3} \tag{1}$$

where η is the motor efficiency. In the individual design, the design variables x_1–x_6 are independent for each motor model. Table II shows the search space of the design variables, and the GA has finished with 1000 generations. As the GA parameters, the number of population, the cross-over rate, and the mutation rate are 20, 60%, and 5%, respectively. Table III shows the motor efficiency when the motors are optimally and individually designed.

Secondly, all the motor models are optimally and simultaneously designed with the common design variables x_1–x_6. The use of the homothetic shape design can reduce the design cost. The objective function in the simultaneous design is as follows:

$$\text{maximize} \qquad \eta_{\#1} + \eta_{\#2} + \eta_{\#3}. \tag{2}$$

The simple GA method is employed as the optimization method. Table III shows the motor efficiency when simultaneously designing. Each efficiency is lower than that in the individual design. Table IV shows the specifications of the motor models #1, #2, and #3, and Fig. 2 their shapes with the magnetic field map. In the result, the wire diameter is small due to the wide teeth width. That results in the large amount of copper loss of the smallest IPM motor model #1.

Although we could obtain a good efficiency even in the simultaneous design, the objective function (2) has a well known problem: that is, an extremely low efficiency of one motor model would be allowed if the other took an extremely high efficiency. An optimization algorithm simply seeks the greatest summation of the efficiencies $\eta_{\#1}$, $\eta_{\#2}$, and $\eta_{\#3}$. Such a result is undesirable for a commonalized optimal design.

TABLE II
DESIGN VARIABLES

Design variable		Search space
Rotor radius	x_1 (mm)	20 – 30
PM width	x_2 (mm)	5 – 15
PM thickness	x_3 (mm)	1 – 5
PM depth	x_4 (mm)	5 – 10
Teeth width	x_5 (mm)	2 – 5
Teeth height	x_6 (mm)	10 – 30
Motor length	$x_{7\#1}$, $x_{8\#2}$, and $x_{9\#3}$ (mm)	50 – 100
Wire diameter	$x_{10\#1}$, $x_{11\#2}$, and $x_{12\#3}$ (mm)	0.1 – 2.0
Supply voltage	$x_{13\#1}$, $x_{14\#2}$, and $x_{15\#3}$ (deg.)	0 – 45

TABLE III
EFFICIENCY OF MOTOR OPTIMIZATION DESIGN

Model	#1	#2	#3
Individual design (%)	96.55	97.25	97.36
Simultaneous design (%)	96.09	96.98	96.63

TABLE IV
SPECIFICATIONS OF SIMULTANEOUSLY OPTIMIZED MOTORS

Model	#1	#2	#3
Output P_{out} (W)	498.2	707.1	830.7
Copper loss P_{c} (W)	13.23	10.08	6.81
Iron loss P_{i} (W)	7.04	11.97	22.15
Torque Ripple T_{r} (Nm)	0.12	0.24	0.41
Efficiency η (%)	96.09	96.98	96.63

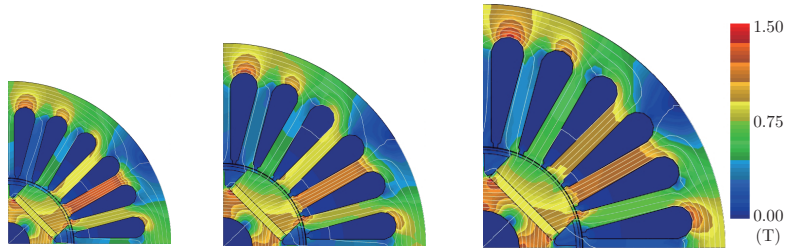
III. OPTIMAL DESIGN OF COMMONALIZED IPM MOTOR SHAPE USING GAME THEORY

If the simple summation of motor efficiencies is employed as a objective function, a one-sided and irrational solution would be produced. In this paper, we propose a method utilizing the game theory [6], [7], [8] for obtaining the rational commonalized shape of the IPM motor. Since the detailed procedure of the game theory is shown in [6], the optimization method using the game theory is omitted due to the lack of space in this paper.

A. Game Settings and Conditions

The object of the game is set to obtain a commonalized and homothetic IPM motor shape which gives rational and reasonable efficiency simultaneously in all the models #1, #2, and #3.

Three players P_1, P_2, and P_3 are considered in the game. The players P_1, P_2, and P_3 govern the motors #1, #2, and #3, respectively. Their objective functions f_1, f_2, and f_3 to be simultaneously maximized are $\eta_{\#1}$, $\eta_{\#2}$, and $\eta_{\#3}$, respectively. The players P_1, P_2, and P_3 predominate the teeth height x_6, the rotor radius x_1, and the teeth width x_5, respectively. Whereas the design variables x_1, x_5, and x_6 takes the discrete values shown in Table V, the other design variables are decided by the GA for each combination of x_1, x_5, and x_6.



(a) Model #1 (b) Model #2 (c) Model #3

Fig. 2. Simultaneously optimized versatile IPM motors with magnetic field map.

TABLE V
PLAYERS AND DESIGN VARIABLES IN GAME THEORY

Player	Model	Dominant variable	Values (in scale of #2)
P_1	#1	x_6 (mm)	{18.5, 20.5, 22.5, 24.5}
P_2	#2	x_1 (mm)	{21.8, 22.9, 24.0}
P_3	#3	x_5 (mm)	{2.8, 3.5, 4.2}

B. Rational solution by game theory

Table VI shows the pay-off tables used in the game. The equilibrium selection probabilities obtained by the noncooperative game [6] are

$$\begin{aligned} \boldsymbol{p}_1 &= \{0.02, 0.00, 0.25, 0.73\} \\ \boldsymbol{p}_2 &= \{0.96, 0.01, 0.03\} \\ \boldsymbol{p}_3 &= \{0.00, 0.21, 0.79\}. \end{aligned} \tag{3}$$

Consequently, the equilibrium strategy is

$$(x_6, x_1, x_5) = (24.5, 21.8, 4.2) \tag{4}$$

with the objective functions

$$(f_1, f_2, f_3) = (96.27, 97.10, 97.02). \tag{5}$$

Subsequently, the rational (Nash) solution is obtained by the cooperative game [6]. The objective functions of the rational solution are as follows:

$$(f_1, f_2, f_3) = (96.31, 97.20, 97.11) \tag{6}$$

at

$$(x_6, x_1, x_5) = (22.5, 21.8, 3.5) \tag{7}$$

in the scale of the motor model #2, and it is necessary to scale these values to the other models #1 and #3. Table VII shows the rationally optimized values of the design variables, and Table VIII provides the specifications of the optimized motors. Fig. 3 represents the commonalized shape and magnetic field map of the motor models #1, #2, and #3 optimized by the cooperative game theory.

Fig. 4 shows the candidates, the equilibrium strategy, and the Nash solution. The Nash solution increases in all the efficiencies, compared with the equilibrium strategy. The number of better solutions than the equilibrium strategy is 5 in Fig. 4 (a), 1 in (b), and 9 in (c), respectively. Consequently, the better solution than the equilibrium strategy in Fig. 4 (b) is selected as the Nash solution, That is, the Nash solution is decided under the strong influence of the motors #1 and #3.

TABLE VII
DESIGN VARIABLES OPTIMIZED BY COOPERATIVE GAME THEORY

Model		#1	#2	#3
Rotor radius	x_1 (mm)	17.44	21.80	26.16
PM width	x_2 (mm)	7.98	9.97	11.96
PM thickness	x_3 (mm)	1.72	2.15	2.58
PM depth	x_4 (mm)	5.98	7.47	8.96
Teeth width	x_5 (mm)	2.80	3.50	4.20
Teeth height	x_6 (mm)	18.0	22.5	27.0
Motor length	$x_{7,8,9}$ (mm)	55.87	67.01	60.12
Wire diameter	$x_{10,11,12}$ (mm)	0.674	1.108	0.553
Voltage phase	$x_{13,14,15}$ (deg.)	33.92	28.90	27.79

TABLE VIII
SPECIFICATIONS OF MOTORS OPTIMIZED BY COOPERATIVE GAME THEORY

Model	#1	#2	#3
Output P_{out} (W)	498.8	707.2	830.8
Copper loss P_{c} (W)	12.86	8.50	7.16
Iron loss P_{i} (W)	6.23	11.91	17.56
Torque Ripple T_{r} (Nm)	0.24	0.54	0.74
Efficiency η (%)	96.31	97.20	97.11

The Nash solution exists on the Pareto front of Figs. 4 (a) and (b). From these facts, it is obvious that the obtained Nash solution is rational to the optimization problem.

C. Discussions

From Tables IV and VIII, it is obvious that the efficiency of the versatile IPM motors obtained by the game theory is higher than that of the simultaneously optimized motors. In the simultaneous optimization design, the search area is too wide to obtain the best solution because of a large number of design variables.

Fig. 5 shows the efficiencies of the commonalized models, which are normalized to the individual design, in the case of the simultaneous and game theory designs. The IPM motor model #3 in the simultaneous designs is the worst, whereas the sum of the efficiencies is maximized. On the other hand, in the game theory design, the efficiencies of the models #1 and #3 are minimum and almost the same. Approximately 2% improvement in the motors #1 and #2 is observed, but approximately 5% in the motor #3. For rationality, the efficiency of only the motor #3 is improved largely. That is, the game theory achieves the rational design of the commonalized IPM motors, and only one of them is not deteriorated.

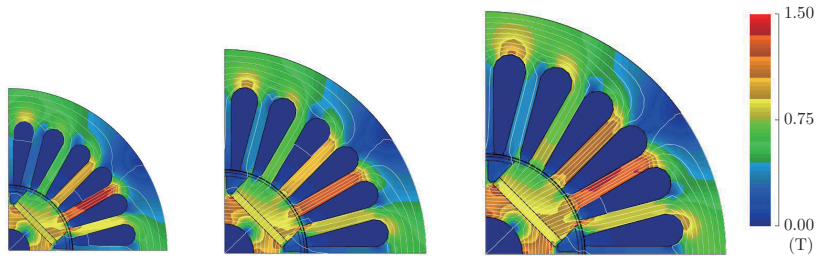
TABLE VI
PAY-OFF TABLES IN GAME THEORY

$x_5 = 2.8$		x_6			
		18.5	20.5	22.5	24.5
x_1	21.8	96.01	96.25	96.40	96.49
		97.21	97.21	97.18	97.11
		97.12	97.05	96.88	96.74
	22.9	96.11	96.32	96.44	96.48
		97.14	97.11	97.06	96.96
		96.90	96.86	96.50	96.37
	24.0	96.20	96.37	96.46	96.49
		97.11	97.06	96.98	96.81
		96.87	96.69	96.49	96.18

$x_5 = 3.5$		x_6			
		18.5	20.5	22.5	24.5
x_1	21.8	95.74	96.09	96.31	96.44
		97.21	97.23	97.20	97.12
		97.27	97.12	97.11	96.84
	22.9	95.88	96.19	96.36	96.46
		97.13	97.11	97.08	97.01
		97.17	97.01	96.68	96.69
	24.0	96.08	96.27	96.39	96.46
		97.11	97.10	97.00	96.84
		97.07	96.88	96.65	96.38

$x_5 = 4.2$		x_6			
		18.5	20.5	22.5	24.5
x_1	21.8	95.39	95.73	95.99	96.27
		97.10	97.16	97.16	97.10
		97.28	97.24	97.08	97.02
	22.9	95.61	95.90	96.18	96.33
		97.08	97.10	97.07	96.96
		97.22	97.09	97.01	96.70
	24.0	95.82	96.09	96.28	96.37
		97.08	97.04	96.99	96.87
		97.10	97.03	96.81	96.54

upper: f_1, middle: f_2, lower: f_3



(a) Model #1 (b) Model #2 (c) Model #3

Fig. 3. Versatile IPM motors rationally optimized by game theory and their magnetic field map.

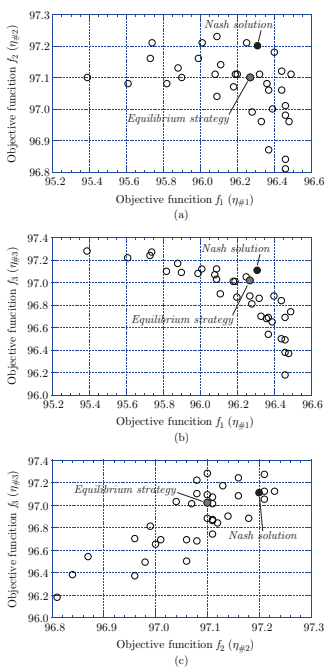


Fig. 4. Relation between objective functions f_1, f_2, and f_3.

To simultaneously design the rational shape of the commonalized IPM motors reduces the design cost. It can facilitate the economically-efficient design for the versatile IPM motors.

IV. CONCLUSION

We propose a new concept for an optimal design method of an IPM motor. In the concept, from the viewpoint of economical efficiency, multiple different-size IPM motors are simultaneously and optimally designed by using the game theory method. Game theory can rationally maximize the efficiency of the different-size IPM motors whose shape is homothetic. The optimal design saves the motor design cost; that is, the economically-efficient design is done for the enhancement of the motor efficiency.

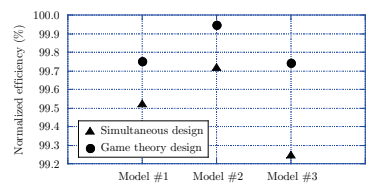


Fig. 5. Normalized efficiencies of motor models #1, #2, and #3 in simultaneous and game-theory designs.

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