Resonant transmission of acoustic phonons in multisuperlattice structures

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We study theoretically the resonant transmission of acoustic phonons in multisuperlattice (SL) systems. As a typical structure, the stack ABA of the periodic SL's A and B is considered. A periodic SL is known to exhibit a strong filtering action on the phonons in the stop bands. Hence, in this multi-SL system, A SL's act as barriers for phonons in their stop bands (A stop bands). However, we show that those phonons in A stop bands are transmitted very efficiently through this system when they satisfy the resonance condition determined by the structure of the sandwiched B SL. This is analogous to the resonant tunneling of electrons in multiple-quantum-well structures. We demonstrate these results by calculating both the frequency and angular dependences of phonon transmission.

Recent developments in the technology for fabricating multilayered thin-film systems enable us to obtain various kinds of superlattice (SL) with periodic, quasiperiodic, and random orders in the growth direction.[1] In addition to electronic and optical properties, the vibrational (acoustical) properties in these systems are studied both experimentally and theoretically.[2] Recently intense interest in the multilayered-semiconducting systems concerns resonant tunneling of electrons in the multiple quantum wells consisting of GaAs and AlAs layers.[3] In such a system, if the double barriers (AlAs) are thin enough, there is a possibility for electrons to tunnel through them into and out of the quantum wells (GaAs). The transmissivity of electrons through the barriers shows resonant enhancement when the kinetic energy of the incident electrons equals an energy level in the quantum well. In the present work we shall study the possibility of resonant transmission of phonons analogous to electron tunneling.

For acoustic phonons propagating in elastic media, it is usually difficult to realize barriers for phonons that reflect phonons effectively. However, for phonons in a certain range of frequency, a periodic array of alternating elastic layers, i.e., a periodic SL, is known to exhibit a strong phonon-filtering action.[4] This is attributed to the Bragg reflection of phonons due to a periodicity much longer than the atomic spacing.[5] Accordingly, we can consider a stack ABA of two kinds of SL, A and B, as a typical system where the resonant transmission of phonons may occur. The A-type SL should be a periodic one, whereas the B-type SL can be replaced by a single-layer substance but a SL consisting of a periodic multilayered structure exhibits more striking effects.

The triple SL system we consider is depicted in Fig. 1, where the A- (B-) type SL consists of m (n) repetitions of the unit block A_0 (B_0) with a period $D_A=d_1^A+d_2^A$ ($D_B=d_1^B+d_2^B$), where d_1^A (d_1^B) and d_2^A (d_2^B) are the thicknesses of the first and second layers. For both A_0 and B_0, we assume that the first layers (also the second layers) consist of the same substances. The simplest way of understanding what happens for the phonon transmission spectrum of this system is to calculate the structure factor S_{2m+n}, which describes the interference effect of phonons reflected from layer interfaces. Neglecting the multiple reflections from interfaces, we find for phonons impinging perpendicularly on the interfaces,

$$S_{2m+n}=[r_{21}+r_{12}\exp(i\alpha_1)](1+e^{i(ma+nb)})\frac{1-e^{ima}}{1-e^{ia}}+[r_{21}+r_{12}\exp(i\beta_1)]e^{ima}\frac{1-e^{inb}}{1-e^{ib}}\ , \qquad (1)$$

where $\alpha_1=2k_1d_1^A$ ($\beta_1=2k_1d_1^B$), $\alpha_2=2k_2d_2^A$ ($\beta_2=2k_2d_2^B$), $a=\alpha_1+\alpha_2$ ($b=\beta_1+\beta_2$), and k_1 and k_2 are wave numbers of phonons in the first and second layers, respectively. The amplitude-reflection coefficients r_{ij} (for phonons impinging on the interface from the ith layer to the jth layer) depend on acoustic impedances of adjacent layers and the incident angle of phonons.[6] For normal incidence, they satisfy $r_{ij}=-r_{ji}$. However, no explicit expression for r_{ij} is needed for the present study.

Bragg conditions in the pure A- and pure B-type SL's are $a=2\pi l$ and $b=2\pi l'$ (l and l' are positive integers), respectively. At frequencies satisfying these conditions, the structure factor increases linearly in proportion to the number of periodicity m or n in the system, as long as the prefactors of $(1-e^{ima})/(1-e^{ia})$ or $(1-e^{inb})/(1-e^{ib})$ in Eq. (1) do not vanish. Incidentally, we note that the factors $[r_{21}+r_{12}\exp(i\alpha_1)]$ and $[r_{21}+r_{12}\exp(i\beta_1)]$ are due to extra reflections of phonons from the interfaces within the unit periods A_0 and B_0.

A possible resonant transmission of phonons at frequencies within the stop bands of the A-type SL arises from the factor $(1+e^{i(ma+nb)})$ in Eq. (1). Let ν_0 be the frequency for which the Bragg condition $e^{ia}=1$ for the A-type SL is satisfied. If the condition $e^{inb}=-1$ is satisfied simultaneously at ν_0, the intensity of reflected phonons does not grow with the system size, leading to the enhancement in transmission at this frequency. This resonance condition is equivalent to

$$nD_B=\left[\frac{l''}{2}+\frac{1}{4}\right]\lambda_B,\quad l''=0,1,2,\ldots\ , \qquad (2)$$

where $\lambda_B = 2\pi/q_B$ and q_B is the wave number of phonons in the B-type SL defined by $2q_B D_B = b$. These equations are valid in the limit of small acoustic mismatch between the constituent layers. Equation (2) implies that the resonance occurs at a frequency for which the thickness of the B-type SL becomes a quarter-wave layer for phonons. This corresponds to the well-known fact that the least reflection is observed for acoustic and electromagnetic waves incident on a plane layer when the thickness of the layer is equal to a quarter of a wavelength.[6]

Actually, the phonon stop bands have finite widths in frequencies that are determined by the magnitude of the acoustic mismatch between the constituent layers in a unit period. Hence, the resonance will be observed at several frequencies within the stop band of an A-type SL. Thus, the resonance condition should be $e^{i(ma+nb)} = -1$ or

$$ma + nb = (2N+1)\pi, \quad N = 0, 1, 2, \ldots , \tag{3}$$

rather than Eq. (2). Resonant enhancement in transmission at frequencies satisfying Eq. (3) will be found in any stop bands of a pure A-type SL as long as they do not possess frequency ranges common to the stop bands in a pure B-type SL. The absence of resonance in the common frequency gaps of both A-type and B-type SL's is a consequence of the Saxon-Hutner theorem[7] which claims that any frequency region in a spectral gap for both A- and B-type SL's is necessarily a gap for any mixed SL consisting of A- and B-type unit blocks, i.e., A_0 and B_0. This can be seen explicitly from Eq. (1). If a Bragg reflection occurs at a given frequency in both A- and B-type SL's, then both terms of the right-hand side of Eq. (1) grow in proportion to the sizes of A and B SL's which constitute the multi-SL structure we consider. Accordingly, even if the resonance condition (3) suppresses the growth of the first term of Eq. (1), the second term is still as large as the size of the B SL.

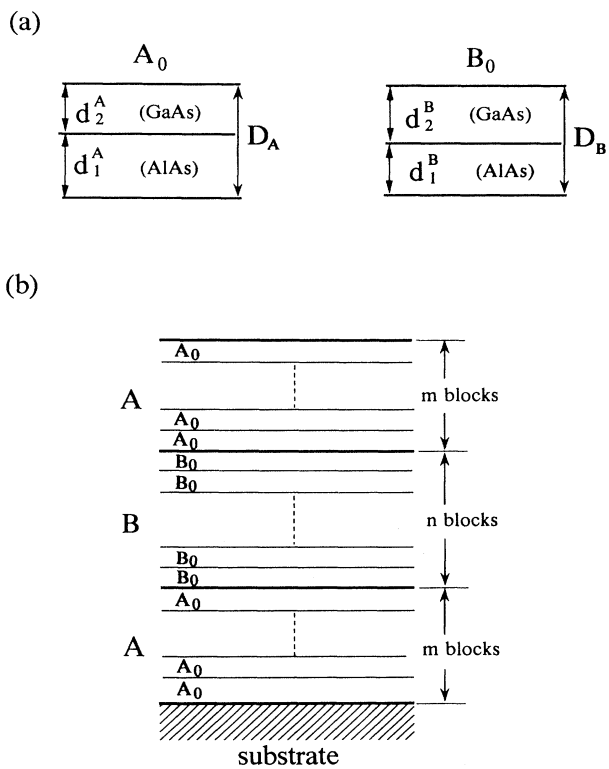

FIG. 1. (a) Schematic of unit blocks A_0 and B_0 for the periodic A and B SL's. In the numerical calculations AlAs and GaAs are assumed for the first and second layers of both A_0 and B_0 blocks. (b) Schematic ABA multi-SL structure consisting of periodic SL's A and B grown on a substrate. The numbers of periods in A and B SL's are m and n, respectively.

The transmission rate of phonons propagating through multilayered systems is calculated by employing the transfer-matrix method. For the propagation of phonons oblique to the growth direction, the continuity of the lattice displacement and stress at layer interfaces requires 6×6 components for the transfer matrix.[8] Provided that the interfaces are mirror-symmetry planes, the transfer matrix is reduced to 2×2 for the propagation parallel to the growth direction. This is because all three modes of phonons are decoupled from one another in this propagation configuration and hence they can be treated independently. As a numerical example, we assume a block consisting of six monolayers (ML) of AlAs and 6 ML of GaAs as the unit block A_0 ($D_A = 34$ Å) of the A-type SL, and that consisting of 9 ML of AlAs and 9 ML of GaAs as the unit block B_0 ($D_B = 51$ Å) of the B-type SL.[9] The layer interfaces are the (100) planes.

In Fig. 2(a), we plot the transmission versus frequency of the longitudinal (L) phonons propagating normal to the interfaces in the AB SL system, where the B-type SL of 15 periods is stacked on the A-type SL of 20 periods (see the inset). [For transverse (T) phonons only the scale of frequency should be modified by taking account of the difference of sound velocities between L and T phonons.] The transmission dips found in this system are due to Bragg reflections occurring in the pure A- and pure B-type SL's as indicated in the figure. The frequencies at the centers of these dips predicted by the first-order Bragg condition for L phonons are 504 GHz for the B-type SL and 756 GHz for the A-type SL. Note that the periodicity of the B-type SL is longer than that of the A-type SL.

Figure 2(b) shows the frequency dependence on the L-phonon transmission rate in the ABA SL system, where the assumed numbers of the periods in the A- and B-type SL's are 10 and 15, respectively. We see several sharp enhancements in transmission in the dip originated from the Bragg reflection in the A-type SL. However, there are no enhancements in the transmission dip caused by the Bragg reflection in the B-type SL. The observed enhancements in transmission are attributed exactly to the resonance of L phonons prediced by Eq. (3). We have indicated in Fig. 2(b) the orders of resonance N deduced from Eq. (3). It should be remarked that these enhancements are, however, reduced as the number of periods of A-type SL's is increased. This is because for phonons inside the stop bands of an A-type SL the associated lattice displacement decays exponentially in the A SL's away from the boundaries between A and B SL's.

The angular dependence of phonon transmission spectra at a fixed frequency provides complementary information on the resonant characteristics of phonons in the multi-SL systems. Figure 3(a) plots the transmission versus incident angle θ (in the GaAs layer) of 750-GHz L phonons in the AB SL system assumed for Fig. 2(a), where θ indicates the angle of the group-velocity vector measured from the growth direction. The propagation plane is 22.5° rotated away from both the (100) and (110) planes in the real space. The overall structure of Fig. 3(a) arises from the transmission of phonons through the A-type SL. At this frequency the transmission rate of L phonons in the pure B-type SL is close to unity for $\tan\theta = 0-1.5$ and exhibits no characteristic behaviors such as dips. Here, we note that for phonon propagations oblique to the layer interfaces the intermode Bragg reflection occurs in addition to the ordinary intramode Bragg reflection.[10] The intermode Bragg reflection yields a frequency gap of the spectrum inside the folded Brillouin zone of the periodic SL. The calculation of the phonon dispersion relations in the pure A-type SL tells us that the large dip in transmission at small angles ($\tan\theta = 0-0.35$) is due to L-to-L intramode Bragg reflection and that at large angles ($\tan\theta = 1.0-1.5$) is due to the intermode Bragg reflection from L to slow transverse (ST) modes.

In Fig. 3(b) we plot the angular dependence of the transmission of 750-GHz L phonons in the ABA SL system assumed for Fig. 2(b). Comparing with Fig. 3(a), we recognize several sharp enhancements in transmission in the dip regions, indicating the existence of resonances also in the angular dependence of transmission. For a small angle of incidence relative to the growth direction, the couplings among three different phonon modes are small and Eq. (3) is still useful to identify the origin of the

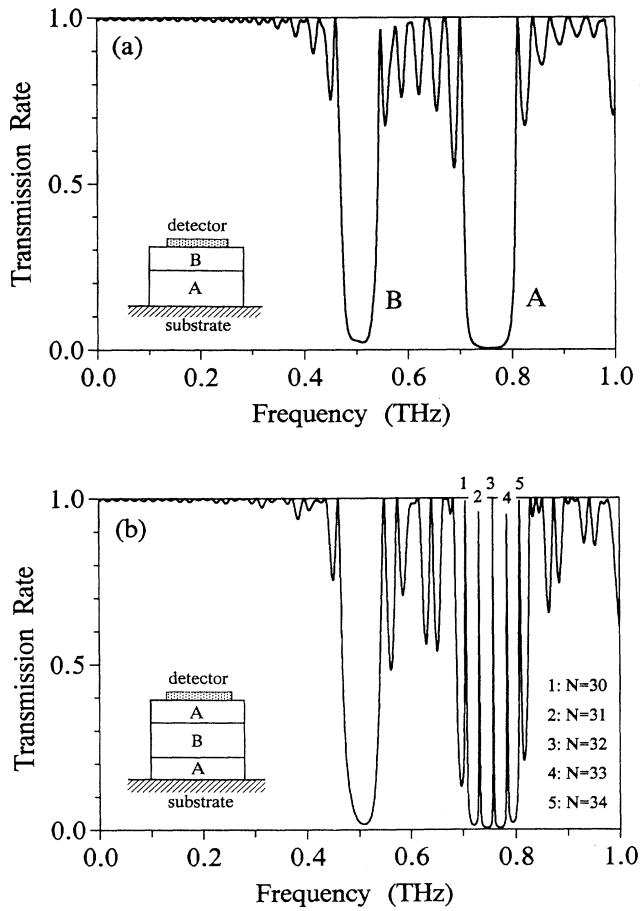

FIG. 2. (a) Frequency dependence of L-phonon transmission rate in the AB structure of SL's, where phonons are incident perpendicularly on the SL's from the substrate side. The unit periods A_0 and B_0 of A and B SL's are composed of (6-ML AlAs) (6-ML GaAs) and (9-ML AlAs) (9-ML GaAs), and the numbers of the periods are 20 and 15, respectively. The dips are indicated by the label of the SL from which they are originated. (b) Frequency dependence of L-phonon transmission rate in the ABA structure of SL's for the normal incidence. The numbers of periods, i.e., the number of A_0 and B_0 blocks, are 10 and 15 for A and B SL's, respectively. Sharp spikes in transmission are labeled by the order of resonance N defined by Eq. (3).

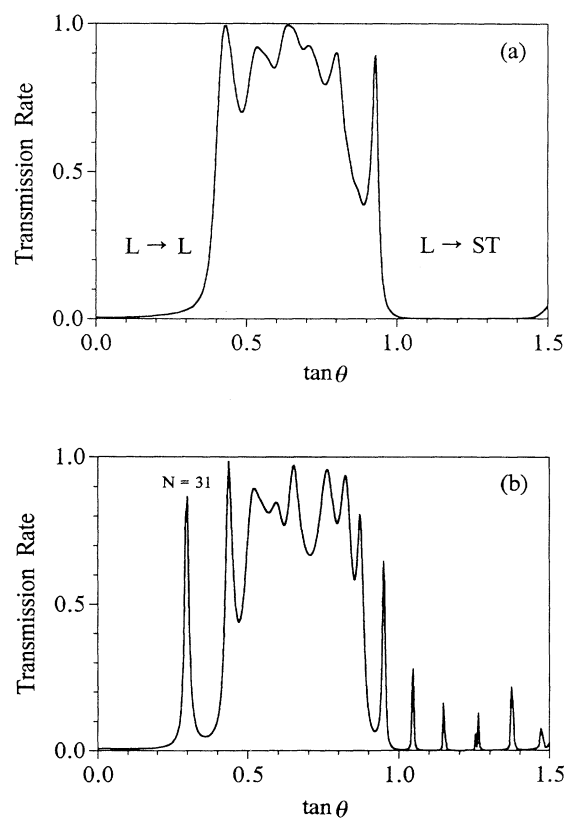

FIG. 3. (a) Angular dependence of L-phonon transmission in the AB structure of SL's assumed for Fig. 2(a). The angle θ is the group-velocity direction measured from the normal of interfaces, i.e., the [001] axis. The frequency is 750 GHz. (b) Angular dependence of L-phonon transmission in the ABA structure of SL's assumed for Fig. 2(b). The first peak is labeled by the order of resonance N. The frequency is 750 GHz.

resonance by identifying k_1 and k_2 as the components of wave vectors normal to the interfaces. The order of resonance $N=31$ for the first peak in Fig. 3(b) is determined in this manner. However, Eq. (3) derived for the resonance of a single mode of phonons is not applicable to the resonance at larger angles of incidence where the couplings among different phonon modes are important.

To summarize, we have examined resonant transmission of phonons in an ABA structure consisting of periodic A and B SL's. The resonant transmission in a similar system, where the B SL is replaced by a single or double layers of materials, has already been found by the author in a study of the vibrational modes localized in the vicinity of an impurity layer embedded in a perfect SL.[11] However, as we have shown here, the system with the multilayered B block shows much richer structures of resonance, which should be relevant to the experimental study.

So far, the phonon filtering action of periodic and quasiperiodic SL's has been proven experimentally[4,12] and the possibility of designing a practical phonon filter is suggested. The result of the present work similarly suggests the potential of designing a phonon resonator by combining periodic SL's. The use of such a resonator which selectively transmits phonons in narrow ranges of frequency and propagation direction will make it possible to generate and detect monochromatic, high-frequency acoustic phonons in the 0.1–1-THz range.

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