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Quantum-Confined Stark Effect Analysis of GeSn/SiGeSn Quantum wells for Mid-Infrared Si-Based Electroabsorption Devices Based on Many-Body Theory

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Abstract—Quantum-confined Stark effect (QCSE) of group IV Ge(Sn)/SiGe(Sn) quantum wells (QWs) on Si substrate is analyzed by microscopic many-body theory for mid-infrared Si-based electroabsorption devices. To show the validity of the theory, QCSE of Ge/SiGe QW is investigated and very good agreement between theory and reported measured results is obtained. Next, QCSE of GeSn/SiGeSn QWs is analyzed and the QW design for electroabsorption modulators to obtain large extinction ratio in mid-infrared region is presented. It is shown that compressive and tensile strained well and barrier layers is preferable to obtain large extinction ratio due to its large conduction band offset.

Index Terms — Si photonics, GeSn quantum wells, electroabsorption modulators, quantum-confined Stark effect, and many-body theory.

I. INTRODUCTION

Optical devices for mid-infrared (mid-IR) wavelength region (2 to 5 μm) have attracted great attention for an environmental gas sensing and new wavelength optical communication system using photonic bandgap fiber and this research field is called “mid-IR photonics” [1]. Along with optical devices using conventional III-V material [2], Si-based mid-IR devices are also intensively studied since they can be fabricated in a CMOS environment, leading to lower cost of devices. In [3], some passive components using Si waveguides such as arrayed waveguide grating and concave grating, GeSn photo-diodes, and hybridly integrated GaSb light sources fabricated on Si substrate were demonstrated. Among them, light sources are crucial components for both sensing and communication applications. For light sources that can be monolithically integrated with other components on Si substrate, GeSn is one of the promising material. By introducing small amount of Sn into Ge, it is known that the bandgap (of GeSn) is drastically reduced and reaches mid-IR region [4]. Also, GeSn with Sn fraction exceeds about 10%, the Γ–bandgap becomes smaller than that of L-bandgap, and GeSn becomes direct bandgap material. Direct transition photoluminescence and p-doping were experimentally observed and realized on GeSn buffer layer grown on Si substrate [5]. Therefore, GeSn is a promising material for Si-based mid-IR light sources.

Recently, the first GeSn laser was successfully demonstrated by using bulk GeSn and optical pumping [6]. Various designs of GeSn quantum wells (QWs) were also investigated for obtaining large material gain and differential gain [7,8].

Other important active component is an electroabsorption (EA) device. EA devices can be used for various optical signal processing, such as switching, and one of the most important application is an EA modulator (EAM) due to its high-speed characteristics [9,10]. EAMs and lasers can be (in principle) monolithically integrated, and therefore, an on-chip optical communication may be possible by integrating all the components (lasers, modulators, and detectors) on the same Si chip by using mid-IR wavelengths.

Recently, strong EA based on quantum-confined Stark effect (QCSE) was observed in Ge/SiGe QWs on Si substrate around 1.45 μm [11]. By incorporating small amount of Sn into the well, it can be used as mid-IR EAM. However, the QCSE of group IV QWs in mid-IR region has not been investigated so far.

In this paper, the QCSE of group IV QWs, Ge(Sn)/SiGe(Sn), is investigated by using microscopic many-body theory (MBT) [8,12-16]. MBT does not need any fitting parameters used in conventional analysis method to obtain optical properties of QWs, except for the effect of inhomogeneous broadening [13] originating from fabrication imperfection. First, the QCSE of Ge/SiGe QWs [11] is analyzed and the results show that calculated absorption spectra are in very good agreement with the experiment, showing the validity of MBT. It should be noted that this is the first time that MBT is applied to the analysis of QCSE of group IV QWs. Next, the effect of Sn-incorporation is investigated for mid-IR EAM applications. It is shown that QWs with compressively strained well and tensile strained barrier is effective to obtain large contrast between ON and OFF states.

II. ANALYSIS METHODS

A. Band structure and absorption spectrum

Figure 1 shows the QW structure analyzed here. (Si)Ge(Sn) buffer is placed on Si substrate and QW structures are grown on the buffer layer. The well is Ge(Sn) and the barrier is SiGe(Sn).
and their thickness and strain are $L_w$ and $v_{sn}, L_B$ and $e_B$. The band structure of QWs is calculated by k·p theory [8,14]. To treat QCSE, the effect of applied electric field, $F$, is taken into account. We assume that the bias electric field is effective only in the well region and the band offset is constant under the electric field.

By solving $k \cdot p$ equation, we obtain subband energies of the conduction, heavy-hole, light-hole, and split-off bands together with their wavefunctions for given transverse wavenumber and $F$. From the wavefunctions, the dipole matrix element for the transition between conduction and valence subbands is obtained by calculating the overlap of the two wavefunctions. These subband energies and matrix elements are the input for calculating the material absorption by MBT as explained in the following.

For the absorption spectra of QWs, although conventional free-carrier theory (FCT) based on Fermi’s golden rule gives a simple explicit formula, it is well-known that the magnitude and position of absorption spectra are not consistent with the experimental results [12-14]. For EAM design, in addition to the position of absorption spectra, it is well-known that the magnitude and position of absorption spectra are not consistent with the simple explicit formula. Although in FCT, one needs extra treatment to include exciton effect [17], it is naturally incorporated in the framework of MBT. Therefore, we used microscopic MBT for the calculation of absorption spectra. By formulating the problem on the basis of second quantization, the so-called semiconductor Bloch equation can be derived as

$$\frac{dp_k}{dt} = -i\omega_k p_k - i\Omega_k \left( f_+ + f_- - 1 \right) + \frac{\partial P_{\text{loc}}}{\partial t}$$  \hspace{1cm} (1)

where $k$ is the transverse wavenumber parallel to the QW plane, $p_k$ is the microscopic polarization, $\hbar \omega_k$ is the renormalized transition energy, and $\Omega_k$ contains the interaction term between the light and the carrier. $f_+$ and $f_-$ are Fermi-Dirac distributions for electrons and holes. The final term in (1) is the scattering contribution and derived from the commutation of Hamiltonian and the operator for $p_k$. The detailed formulation and numerical treatment can be found in [8,12,18].

By discretizing (1) for the steady-state condition (the left-hand side of (1) is zero), we obtain a simultaneous equation and by solving it, the microscopic polarization is calculated for discretized $k$, and a macroscopic polarization, $P_{\text{macro}}$, is obtained by summing the microscopic polarization over all the states. The macroscopic polarization is related to material absorption through Maxwell’s equations as

$$\alpha(\omega) = -\text{Im} \left[ \frac{P_{\text{macro}}}{\epsilon_0 n_B^2 E_0} \right] = -\text{Im} \left[ \frac{2}{\epsilon_0 n_B^2 E_0} \sum \mu^*_{\alpha} P_{\alpha} \right]$$  \hspace{1cm} (2)

where $\epsilon_0$ is the permittivity in vacuum, $n_B$ is the background refractive index, $V$ is the volume, $\mu_{\alpha}$ is the dipole matrix element, and $E_0$ is the electric field of light. The theory enables us to account for various important physical phenomena, such as the absorption peak due to excitons, bandgap renormalization, and the collapse of the absorption peak due to Coulomb screening effects, which cannot be considered in FCT.

Here, the absorption between L- conduction and valence bands is neglected due to the small absorption, especially at the band edge, originating from momentum mismatch [19]. As shown later, the experimental results are in good agreement with the calculated ones, showing the validity of the assumption.

Room temperature (298K) is assumed for all the results.

### B. Band parameters of SiGeSn

The basic band parameters of Ge, Si, and Sn are taken from [7,20-26]. The bandgap of Si$_x$Ge$_{1-x}$Sn$_z$ alloy is given by

$$E_{g, \text{SiGeSn}}(x, y) = xE_{g, \text{Ge}} + yE_{g, \text{Si}} + zE_{g, \text{Sn}}$$

$$-xyb_{\text{SiGe}} - xzb_{\text{SiSn}} - yzb_{\text{GeSn}}$$  \hspace{1cm} (3)

where $z = 1-x-y$. $E_{g, \text{Ge}}$, $E_{g, \text{Si}}$, and $E_{g, \text{Sn}}$ are the bulk bandgap of Si, Ge, and Sn. $b_{\text{SiGe}} = 0.21$ eV, $b_{\text{GeSn}} = 1.94$ eV, and $b_{\text{SiSn}} = 13.2$ eV are bowing parameters. Other band parameters are given by simple linear interpolations, namely,

$$P_{\text{SiGeSn}}(x, y) = xP_{\text{Ge}} + yP_{\text{Si}} + zP_{\text{Sn}}$$  \hspace{1cm} (4)

where $P$ is the band parameters of interest (lattice constant, Luttinger parameters, etc) and they are summarized in Table I.

The band lineup of QWs is determined by model-solid theory [27]. The top of valence band of the alloy is given by
Here, we consider Ge/SiGe QW on SiGe buffer on Si substrate [11] to show the validity of MBT. $L_w$ and $\varepsilon_w$ are 10 nm and -0.2%. $L_B$ and $\varepsilon_B$ are 20 nm and 0.4%. The minus sign for the strain means compressive strain. Although the experimental composition and the thickness of the buffer layer is Si 0.9Ge0.1 and 500 nm, it was pointed out in [19] that the tensile strain posed by Si substrate cannot be fully relaxed. To take into account this correction, we use Si 0.95Ge0.05 buffer [19] in our simulation.

Figure 2 (a) shows the absorption spectra of Ge/SiGe QWs obtained by MBT for the carrier density from $3 \times 10^{16}$ cm$^{-3}$ to $9 \times 10^{16}$ cm$^{-3}$ (low carrier density region). For low carrier density, the exciton absorption peak is very sharp and it is rapidly collapsed for large carrier density due to Coulomb screening effect. Since there is always “inhomogeneous broadening” in the experimental optical spectra originating from, for example, a fabrication imperfection, we take into account the effect by following equation.

\[
E_{\text{v,up}} = E_{\text{v,ave}} + \frac{\Delta}{3}
\]  

III. QCSE OF GE/SIGE QUANTUM WELLS

Since there is always “inhomogeneous broadening” in the experimental optical spectra originating from, for example, a fabrication imperfection, we take into account the effect by following equation.

\[
\alpha_{\text{broaden}}(\hbar \omega_0) = \int d\omega \alpha_{\text{MBT}}(\hbar \omega) \frac{\gamma}{\pi \gamma^2 + (\hbar \omega - \hbar \omega_0)^2}
\]

where $\alpha_{\text{MBT}}$ is the absorption coefficient calculated by MBT. $L$ is called broadening function and various types of functions have been proposed. Here, we use Lorentzian function given by

\[
L(h \omega_b - \hbar \omega) = \frac{1}{\pi \gamma^2 + (h \omega_b - h \omega)^2}
\]

where $\gamma$ is the broadening energy.

Figure 2 (b) shows the same absorption spectra shown in Fig. 2 (a) but with Lorentzian broadening of $\gamma = 7.5$ meV. The sharp exciton peak almost vanishes and carrier density dependence of the spectra becomes very small. Figure 3 (a) shows the calculated absorption spectra without electric field for different $\gamma$ values, at the carrier density of $5 \times 10^{16}$ cm$^{-3}$. Measured spectrum at 0 V is also plotted (taken from [11]). Calculated and
measured results are in very good agreement, and therefore, we use Lorentzian broadening with $\gamma = 7.5$ meV and the carrier density of $5 \times 10^{16}$ cm$^{-3}$ for the theoretical results hereafter.

Figure 3 (b) shows the absorption spectra of Ge/SiGe QW for both TE (solid) and TM (dashed) polarizations. Since the well has large compressive strain, the first and second valence subbands are heavy- and light-hole bands. Therefore, the photon energy of the absorption edge for TM mode is larger than that of TE mode.

Solid and dashed lines in Fig. 4 show measured [11] and calculated absorption spectra of Ge/SiGe QW for different values of voltage and electric field. According to [11], total thickness of the intrinsic layer is 576 nm, and the electric field corresponding to experimental 1 V is about 20 kV/cm. The built-in electric field is also about 20 kV/cm [19]. From the figure, excellent agreement between theory and experiment can be seen for 0, 1, and 2 V in terms of spectral position and shape including exciton absorption peak, showing the validity of MBT used here. For 3 and 4 V, although the theoretical electric field matching to the position of experimental spectra is larger than expected, the bandedge spectral position and shape are well reproduced. The discrepancy between the theoretical electric field and experimental voltage may come from unintentional background doping in the intrinsic region. Background doping results in non-uniform applied electric field for multiple quantum well structure even with very small doping level of $10^{15}$ cm$^{-3}$ [28]. Also, especially for 3 and 4 V, although the bulk absorption increase for larger photon energy can be seen for experimental spectra, it is not taken into account in our simulation.

IV. QCSE OF GeSn/SiGeSn QUANTUM WELLS

In this section, the QCSE of GeSn/SiGeSn QWs on GeSn buffer is investigated for mid-IR EAM based on Si photonics. The buffer layer is assumed to be lattice matched to the buffer layer and the bandgap wavelength of the barrier layer is $\lambda_B = 2.1$ $\mu$m. Since SiGeSn has three elements, the composition is fully determined by its strain and bulk bandgap. Therefore, we specify the composition of SiGeSn by its strain and bulk bandgap instead of the composition. $\lambda_{C1V1}$ ranges from 2.3 to 3.4 $\mu$m and useful for mid-IR applications. To enhance the QCSE, it is important to use deep well (large band offset), especially for conduction band. A deep well makes the confinement of electron and hole stronger, leading to larger absorption even when the electric field is applied [15]. Therefore, to design the GeSn/SiGeSn QWs for EAM, the QW structure having large band offset has to be investigated first to use QCSE effectively. And then, absorption and extinction characteristics are discussed based on the results of MBT.

Figure 6 (a) shows the conduction band offset, $\Delta E_c$, and the valence band offset, $\Delta E_v$, as a function of $\varepsilon_w$. The bulk bandgap and the strain of the barrier layer are $\lambda_B = 2.3$ $\mu$m and $\varepsilon_B = 0\%$. For the well, although compressive strain is better to obtain large $\Delta E_c$, $\Delta E_v$ is also increased. Figure 6 (b) shows $\Delta E_c$ as a function of $\varepsilon_B$ for different values of $\lambda_B$. The strain of the well layer is set to zero. From the Figure, we can see that tensile strain and small $\lambda_B$ for the barrier layer make $\Delta E_c$ larger. Therefore, to obtain large $\Delta E_c$, compressive and tensile strained well and barrier layers are preferable to obtain large extinction ratio.

Figure 7 (a) shows wavefunctions of the first conduction subbands for $F = 0, 50, 100$ kV/cm of GeSn/SiGeSn QW for $L_w = 10$ nm, $\varepsilon_w = 0\%$, $\varepsilon_B = 0\%$, and $\lambda_B = 2.3$ $\mu$m ($\Delta E_c = 56$ meV). Under large electric field, the wavefunction leaks to the barrier layer, leading to smaller overlap with valence band wavefunctions. Figure 7 (b) shows the same wavefunctions of QWs but with $\varepsilon_w = -1\%$ ($\Delta E_c = 78$ meV). Due to the stronger confinement of electrons in the well, the leak to the barrier layer is suppressed and overlap between conduction and valence wavefunctions becomes large, leading to larger contrast between ON and OFF states of EAM.

Figures 8 (a), (b), and (c) show absorption spectra of
GeSn/SiGeSn QWs. The structural parameters for each Figure are (a) $\varepsilon_w = 0\%$, $\varepsilon_B = 0\%$, $\lambda_B = 2.2$ $\mu$m, $\Delta E_c = 72$ meV, $\Delta E_v = 46$ meV, (b) $\varepsilon_w = -0.4\%$, $\varepsilon_B = 0.2\%$, $\lambda_B = 2.1$ $\mu$m, $\Delta E_c = 100$ meV, $\Delta E_v = 103$ meV, and (c) $\varepsilon_w = -1\%$, $\varepsilon_B = 0.5\%$, $\lambda_B = 2.1$ $\mu$m, $\Delta E_c = 130$ meV, $\Delta E_v = 171$ meV. $L_w = 10$ nm. Although the broadening parameter for Sn-based alloy is not known well, we use the same parameters as in Ge/SiGe QW (Lorentzian broadening with $\gamma = 7.5$ meV). $\varepsilon_B$ is determined by strain compensation condition given by

$$\varepsilon_w L_w + \varepsilon_B L_B = 0$$  \hspace{1cm} (8)$$

The absorption spectra are plotted from 0 kV/cm with 20 kV/cm interval. For lattice matched QW (Fig. 4 (a)), the bandedge absorption change is so small due to small $\Delta E_c$ that the structure is not suitable for EAM application. For larger compressive strain in the well, clear QCSE red-shift can be seen for larger values of electric fields. The magnitude of absorption coefficient is larger for larger values of compressive strain in the well. Therefore, large compressive strained well and tensile strained barrier is effective to enhance the absorption under the electric field, resulting in EAMs with large extinction ratio.

Next, we discuss the extinction ratio (ER) of GeSn/SiGeSn EAMs. We consider silica ridge structure proposed in [7] with the optical confinement factor of 7%, which is reasonable value for most multiple QW waveguides. The length of the waveguide is set to 200 $\mu$m, which is typical value for high-speed EAMs. The ER is calculated by

$$ER = 10 \log_{10} \left[ \frac{\Gamma}{\Gamma_{\text{broader}} \left( \hbar \omega_0, F \right)} L_{EAM} \right]$$  \hspace{1cm} (9)$$

where $\Gamma$ is the optical confinement factor, $\lambda$ is the incident wavelength, $L_{EAM}$ is the length of EAMs. Figure 9 shows the ER of GeSn/SiGeSn QW waveguide. The QW structural parameters for blue and red lines are identical to those of Fig. 8 (b) and (c). The incident wavelengths for blue and red lines are 3.1 and 3.625 $\mu$m (0.4 and 0.342 eV). Static ERs of more than 10 and 20 dB can be obtained for the structures.

For modulator applications, the depth of the well, especially for the valence band, affects the high-speed characteristics due to the hole-pile-up effect. Therefore, not only $\Delta E_v$ but also the valence band offset, $\Delta E_{v,\text{offset}}$, is important for the EAM design. As shown in Fig. 6 (a), for GeSn/SiGeSn QWs, if the value of $\varepsilon_w$ is...
large, both the conduction and valence band offsets become large. Therefore, there is a tradeoff between available extinction ratio and high-speed characteristics. Since the state of the art 40-Gbit/s EAM for 1.55 μm using InAlGaAs-based QWs employ the valence band offset around 120 meV [29], similar valence band offset is available for GeSn/SiGeSn QWs.

**CONCLUSION**

We have investigated QCSE of group IV QWs by using MBT. For Ge/SiGe QW, calculated and measured results are in very good agreement, showing the validity of MBT for the QCSE analysis of group IV QWs. The QCSE analysis of GeSn/SiGeSn QWs were also performed for mid-IR Si-based EAM. To enhance the absorption under the electric field, and hence, large extinction ratio, it was found that the strain compensation structure (compressive strained well and tensile strained barrier) with large strain is useful due to the large $\Delta E_c$.

**REFERENCES**


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