Finite-Element Analysis of H-Plane Waveguide Junction with Arbitrarily Shaped Ferrite Post

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Abstract — A numerical approach for solving the problem of H-plane waveguide junctions with lossy ferrite posts of arbitrary shape is proposed. The junctions are allowed to have arbitrary cross section. The approach is a combination of the finite-element method and the analytical method. To show the validity and usefulness of the method, Y-junction circulators with a circular ferrite post are considered. Our results agree well with earlier experimental and theoretical results. The performances of Y-junction circulators with a triangular equilateral ferrite post or a triangular ferrite post having depressed sides are investigated. The influences of the ferrite losses on the performance are examined.

I. INTRODUCTION

APPLICATIONS OF waveguide junctions with ferrite posts have been of wide-ranging use in microwave devices and circuits, and research on them has been continued steadily. Davies [1] presented the theoretical treatment for a symmetrical waveguide junction circulator with a circular ferrite post. This method was extended to junctions with coaxial composite ferrite posts which produced much larger bandwidths [2]–[4]. This analysis, however, is limited for junctions that have geometrical symmetry. Recently, the point-matching method was extended to the asymmetrical junctions [5] and was applied to the junctions with a triangular ferrite post [6]. The point-matching technique is powerful for the waveguide junctions with arbitrarily shaped ferrite posts, but the ferrite losses are neglected. Okamoto [7] presented a method based on the integral equations for solving the problem of waveguide junctions with lossy ferrite posts. In his approach, the junctions are allowed to have an arbitrary cross section and arbitrary number of ports, but only the ferrite posts with smooth boundaries such as a circular ferrite post and a triangular ferrite post having rounded angles are studied.

In this paper, a finite-element method for the analysis of the H-plane waveguide junctions with lossy ferrite posts of arbitrary shape is described.

For the analysis of planar circulators, Lyon and Helszajn [8] presented a method based on circuit theory and the finite-element method. In their approach, the system is assumed to be free of any losses and the finite-element method is used for the computation of the eigenvalues and eigenvectors of the normal modes of a magnetized ferrite resonator, and then the circuit parameters are determined by using these data of the normal modes. Their approach is very useful for the planar circulators using arbitrarily shaped resonators and it can be applied to H-plane waveguide junctions with arbitrarily shaped ferrite posts. However, the scattering coefficients are quite sensitive to the values of the circuit parameters, so it is necessary to ensure that sufficient significant digits are carried through in the computation of the normal modes. In our approach, on the other hand, it is not necessary to compute the eigenvalues and eigenvectors of the normal modes. Making use of the method, we first treat Y-junction circulators with a circular ferrite post for comparison with the previously published experimental and theoretical results [3], [5]–[7]. The performance of Y-junction circulators with a triangular equilateral ferrite post or a triangular ferrite post having depressed sides are next investigated. The influences of the ferrite losses on the performance are examined.

II. BASIC EQUATIONS

Fig. 1 shows the H-plane waveguide junction with a full-height ferrite post of arbitrary shape. The dc magnetic field is applied in parallel with the z axis. The boundaries $\Gamma_i (i' = 1', 2', 3')$ lie in the region $\Omega$ with $\Gamma_i$ ($i = 1, 2, 3$) and the short-circuit boundary $\Gamma$, and the region surrounded by $\Gamma_i$ and $\Gamma$ completely encloses the waveguide discontinuities. In general, the waveguides need not be symmetrically
located around the junction. Although the number of ports is arbitrary, for simplicity, three-port junctions are considered. The waveguides propagate only the dominant TE\textsubscript{10} mode, while all higher modes are cutoff. However, this does not mean that the higher modes are neglected.

With a time dependence of the form \( \exp(j\omega t) \) being implied, the permeability tensor \([\mu]\) is \([9]\)

\[
[\mu] = \begin{bmatrix}
\mu - j\kappa & 0 \\
j\kappa & \mu \\
0 & 0 & \mu_0
\end{bmatrix}
\]  

(1)

where

\[
\mu = \mu_0 \left\{ 1 + \frac{(\omega_0 + j\omega_\alpha)\omega_m}{(\omega_0 + j\omega_\alpha)^2 - \omega^2} \right\}
\]

(2)

\[
\kappa = -\frac{\omega_m}{(\omega_0 + j\omega_\alpha)^2 - \omega^2}
\]

(3)

\[
\omega_0 = \gamma H_0
\]

(4)

\[
\omega_m = \frac{\gamma M_s}{\mu_0}
\]

(5)

\[
\alpha = \frac{\gamma \Delta H}{2\omega}
\]

(6)

Here \(\omega\) is the angular frequency, \(\mu_0\) is the permeability of free space, \(H_0\) is the internal dc magnetic field, \(M_s\) is the saturation magnetization, \(\Delta H\) is the resonance linewidth, \(\gamma\) is the gyromagnetic ratio, and \([\cdot]\) denotes a matrix.

Considering the excitation by the dominant TE\textsubscript{10} mode, the fields \(E_z\), \(H_x\), and \(H_y\) satisfy the following relations:

\[
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \epsilon E_z
\]

(7)

\[
H_x = \frac{1}{j\omega (\mu_0 - \kappa)} \left\{ -\frac{\partial E_z}{\partial y} + j\kappa \frac{\partial E_z}{\partial x} \right\}
\]

(8)

\[
H_y = \frac{1}{j\omega (\mu_0 - \kappa)^2} \left\{ \frac{\partial E_z}{\partial x} + j\kappa \frac{\partial E_z}{\partial y} \right\}
\]

(9)

where

\[
\epsilon = \epsilon_0 \epsilon_r (1 - j\tan\delta).
\]

(10)

Here \(\epsilon_0\) is the permittivity of free space, \(\epsilon_r\) is the relative permittivity, and \(\delta\) is the dielectric loss angle.

III. MATHEMATICAL FORMULATION

A. Finite-Element Approach

Dividing the region \(\Omega\) into a number of second-order triangular elements in Fig. 2, the electric field \(E_z\) within each element is defined in terms of the electric field \(E_z\) at the corner and midside nodal points

\[
E_z = (N)^T (E_z)_{e}
\]

(11)

where \((E_z)_{e}\) is the electric field vector corresponding to the nodal points within each element, \((N)\) is the shape function vector [10], and \(T\), \(\cdot\), and \((\cdot)^T\) denote a transpose, a column vector, and a row vector, respectively.

Using a Galerkin procedure on (7), we obtain

\[
\iint_{\Omega} \left\{ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - j\omega \epsilon E_z \right\} d\Omega = \{0\}
\]

(12)

where the integration is carried over the element subdomain \(\Omega_e\) and \(\{0\}\) is a null vector.

Integrating by parts, (12) becomes

\[
\iint_{\Gamma_e} \left\{ \frac{\partial (N)}{\partial x} H_y - \frac{\partial (N)}{\partial y} H_x - j\omega \epsilon E_z \right\} d\Gamma
\]

\[
- \iint_{\Omega} (N) H_z d\Omega = \{0\}
\]

(13)

where the second integration on the left-hand side is carried over the contour \(\Gamma_e\) of the region \(\Omega_e\), and \(H_z\) is the transverse component of the magnetic field on \(\Gamma_e\).

Substituting (8) and (9) into (13), considering \(\mu = \mu_0\) and \(\kappa = 0\) on \(\Gamma_e\) and \(E_z = 0\) on \(\Gamma\), using (11), and assembling
the complete matrix for the region $\Omega$ by adding the contributions of all different elements, we obtain

$$ [A] \{ E_z \} = - \sum_{i=1}^{n} \int_{e} \frac{\partial E_z}{\partial x(i)} \, dy(i) = \{ 0 \} \quad (14a) $$

where

$$ k_0^2 = \omega^2 \epsilon_0 \mu_0. \quad (15) $$

Here the components of the $\{ E_z \}$ vector are the values of the only nonzero components of the electric field $E_z$ at all nodal points in the region $\Omega$ except the short-circuit boundary $\Gamma$, $\Sigma_e$ and $\Sigma_e$ extend over all different elements and the elements related to $\Gamma$, respectively, and $[A]$ is a complex matrix. For loss-free materials, namely $\Delta H = 0$ and $\tan \delta = 0$, $[A]$ becomes Hermitian. For $H$-plane waveguide junctions without ferrite posts, namely $\mu = \mu_0$ and $\kappa = 0$, (14) is reduced to the equation derived by Koshiba, Sato, and Suzuki [11, 12].

We may rewrite (14) as follows:

$$ \begin{bmatrix} [A]_{II} & [A]_{IB} & [A]_{IB} \\ [A]_{B1} & [A]_{B1} & [A]_{B1} \\ [A]_{B1} & [A]_{B2} & [A]_{B2} \end{bmatrix} \begin{bmatrix} \{ E_z \}_1 \\ \{ E_z \}_2 \\ \{ E_z \}_3 \end{bmatrix} = \begin{bmatrix} \{ 0 \} \\ \{ 0 \} \end{bmatrix} + \sum' \int_{e} \frac{\partial E_z}{\partial x(i)} \, dy(i) \quad (16) $$

where

$$ \{ E_z \}_B = \begin{bmatrix} \{ E_z \}_1 \\ \{ E_z \}_2 \\ \{ E_z \}_3 \end{bmatrix}, \quad \{ E_z \}_B' = \begin{bmatrix} \{ E_z \}'_1 \\ \{ E_z \}'_2 \\ \{ E_z \}'_3 \end{bmatrix}. \quad (17, 18) $$

Here the components of the $\{ E_z \}_1$, and $\{ E_z \}_B$ vectors are the values of the electric field $E_z$ at nodal points on the boundaries $\Gamma_i (i = 1, 2, 3)$ and $\Gamma_r (i' = 1, 2, 3')$, respectively, the components of the $\{ E_z \}_2$ vector are the values of $E_z$ at nodal points in the interior region except the boundaries $\Gamma$, $\Gamma_r$, and $\Gamma_r'$, and $[A]_{II}, [A]_{IB}, \ldots, [A]_{BB}$ are the submatrices of $[A]$.

### B. Analytical Approach

Assuming that the dominant $TE_{10}$ mode of unit amplitude is incident from the waveguide $j (j = 1, 2, 3)$ in Fig. 1, $E_z$ on $\Gamma$ may be expressed analytically as

$$ E_z(x(i) = d_i, y(i)) = \delta_{ij} 2 j \sin \beta_j d_i f_i(y(i)) $$

$$ + \sum_{m=1}^{\infty} \int_{0}^{W} \exp(-j\beta_m d_i) f_m(y(i)) f_i(y(i)) \, dy(i) \quad (19) $$

where

$$ f_m(y(i)) = \frac{\sqrt{2/W}}{\sin(m\pi/W)} \beta_m^2 = \frac{\sqrt{k_0^2 - (m\pi/W)^2}}{\sin(m\pi/W)} \quad (20, 21) $$

Here $\delta_{ij}$ is the Kronecker $\delta$.

Using (11), (19) can be discretized as follows:

$$ \{ E_z \}_i = \delta_{ij} \{ f \}_j + [Z] \{ E_z \}_r \quad (22) $$

where

$$ \{ f \}_j = 2 j \sin \beta_j d_i \{ f \}_j \quad (23) $$

and

$$ [Z] = \sum_{m=1}^{\infty} \int_{0}^{W} \exp(-j\beta_m d_i) f_m(y(i)) \sum' f_{m}(y(i)) \, dy(i) \quad (24) $$

Here the components of the $\{ f_m \}$ vector are the values of $f_{m}(y(i))$ at the nodal points on $\Gamma_r$ and $\Sigma_e$ extends over the elements related to $\Gamma_r$.

### C. Combination of Finite-Element and Analytical Relations

Using (22), from (16) we obtain the following final matrix equation:

$$ \begin{bmatrix} [A]_{II} & [A]_{IB} & [A]_{IB} \\ [A]_{B1} & [A]_{B1} & [A]_{B1} \\ [A]_{B1} & [A]_{B2} & [A]_{B2} \end{bmatrix} \begin{bmatrix} \{ E_z \}_1 \\ \{ E_z \}_2 \\ \{ E_z \}_3 \end{bmatrix} = \begin{bmatrix} \{ 0 \} \\ \{ 0 \} \end{bmatrix} + \sum' \int_{e} \frac{\partial E_z}{\partial x(i)} \, dy(i) \quad (16) $$

where

$$ \begin{bmatrix} \{ E_z \}_B \\ \{ E_z \}_B' \end{bmatrix} = \begin{bmatrix} \{ 0 \} \\ \{ 0 \} \end{bmatrix} + \sum' \int_{e} \frac{\partial E_z}{\partial x(i)} \, dy(i) \quad (16) $$

Here [1] is a unit matrix and [0] is a null matrix.

The values of $E_z$ at nodal points on $\Gamma_r$, namely $\{ E_z \}_{r}$, are computed from (25), and then the electric field $E_z(x(i) = 0, y(i))$ on $\Gamma_r$ can be calculated from (11). The solutions on $\Gamma_r$ allow the determination of the power reflection coefficient $|R_r|^2$ and the power transmission coefficient...
The dissipative loss $P_d$ is given by

$$P_d = 1 - (R_{11}^2 + T_{21}^2 + T_{31}^2).$$

A. Y-Junction with a Central Circular Ferrite Post

For comparison with previously published experimental and theoretical results, we first treat Y-junction circulators with a central circular ferrite post. There is some difference between the earlier theoretical results [3], [5]–[7]. The circulator performances using two different ferrite samples, that is, TTI-109 and G-1002, have been calculated and are shown in Figs. 3 and 4, respectively, where the magnetic losses are considered and the dielectric losses are neglected, namely $\Delta H \neq 0$ and $\tan \delta = 0$. Material parameters are given in [3], [5]–[7]. The results for $\Delta H = 0$ (lossless) are represented by the solid lines, while the results for $\Delta H \neq 0$ (lossy) are represented by the dashed lines. The experimental results of Castillo and Davis [3] are also represented by the dashed lines. In lossless cases, the condition of power conservation $|R_{11}|^2 + |T_{21}|^2 + |T_{31}|^2 = 1$ is satisfied to an accuracy of $\pm 10^{-4}$. In the lossy cases, the isolation slightly deteriorates in the neighborhood of the respective maxima of the performance curves in comparison with the lossless cases [7]. For the lossy case in Fig. 3, the numerical results (dots) agree approximately with the experimental results.
TABLE I
Dissipative Losses of Y-Junctions with a Circular Ferrite Post

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>TT1-109</th>
<th>G-1002</th>
</tr>
</thead>
<tbody>
<tr>
<td>tan δ = 0</td>
<td>tan δ = 0.0005</td>
<td>tan δ = 0.001</td>
</tr>
<tr>
<td>8.0</td>
<td>0.044</td>
<td>0.046</td>
</tr>
<tr>
<td>9.0</td>
<td>0.051</td>
<td>0.054</td>
</tr>
<tr>
<td>10.0</td>
<td>0.064</td>
<td>0.058</td>
</tr>
<tr>
<td>11.0</td>
<td>0.082</td>
<td>0.072</td>
</tr>
<tr>
<td>12.0</td>
<td>0.116</td>
<td>0.120</td>
</tr>
</tbody>
</table>

In Fig. 4, the agreement with the experimental results is not as good as in Fig. 3. However, for the lossy cases both in Figs. 3 and 4, the numerical results (dots) agree well with the results of the integral equation method [7]. In comparison with the other theoretical results [3], [5], [6], the integral equation method and the present method are found to give fairly good results close to the experimental results on the whole [7].

Table I shows the dissipative losses. For a TT1-109 ferrite sample, the results obtained by considering both the magnetic and dielectric losses are also shown. It is found that dielectric losses do not add much to the dissipative losses. Therefore, we neglect the dielectric losses in the following numerical results.

B. Y-Junction with a Triangular Ferrite Post

Consider a Y-junction with a triangular equilateral ferrite post. Two specific cases [6] are considered. In the first case, the points of the triangle are in the centers of the waveguides, whereas in the second case, the sides of the triangle are in the centers of the waveguides. Numerical
results are obtained for a TTI-109 ferrite sample. The circulator performances for the first and the second arrangements are shown in Figs. 5 and 6, respectively, where $a$ is the radius of an inscribed circle of the triangle. It is found that as the value of $a$ increases, the circulation frequency decreases. In the lossy cases, the isolation slightly deteriorates and the reflection slightly deteriorates in the neighborhood of the respective maxima of the performance curves in comparison with the lossless cases. The values of the maximum isolation for the first arrangement (Fig. 5) are larger than those for the second arrangement (Fig. 6).

Table II shows the dissipative losses due to the magnetic losses. The dissipative losses for the first arrangement are smaller than those for the second arrangement in the neighborhood of the circulation frequency.

C. V-Junction with a Triangular Ferrite Post Having Depressed Sides

We propose a Y-junction with a triangular ferrite post having depressed sides as shown in Figs. 7 and 8. Two specific cases are considered. In the first case (Fig. 7), the points of the triangle are in the centers of the waveguides, whereas in the second case (Fig. 8), the sides of the triangle are in the centers of the waveguides. Numerical results are obtained for a TTI-109 ferrite sample. The circulator performances for the first and the second arrangements are shown in Figs. 7 and 8, respectively, where the magnetic losses are considered. The frequency of the best isolation for the triangular ferrite post having depressed sides in Figs. 7 and 8 is higher than that for the triangular ferrite post in Figs. 5 and 6. The values of the maximum isolation for the first arrangement (Fig. 7) are smaller than those for the second arrangement (Fig. 8).

In the first arrangement (Fig. 7), the points of the triangle, which are in the centers of the waveguides, act as dielectric tapers [13]. However, each side of the triangle is bent abruptly. Therefore, it seems that the performances obtained with the triangular ferrite post having depressed sides (Fig. 7) are inferior to those obtained with the triangular ferrite post having straight sides (Fig. 5). In the second arrangement (Fig. 8), the sides of the triangle, which are in the centers of the waveguides, are trimmed [14] and the dissipative losses due to the magnetic losses may be reduced. Therefore, it seems that the performances obtained with the triangular ferrite post having depressed sides (Fig. 8) are better than those obtained with the triangular ferrite post having straight sides (Fig. 6).

Table III shows the dissipative losses due to the magnetic losses. The dissipative losses obtained with the triangular ferrite post having depressed sides in Table III are smaller than those obtained with the triangular ferrite post in Table II for the same value of $a$.
is possible to optimize the form of the cross section of the posts of arbitrary shape. The validity of the method was confirmed by comparing numerical results for circular ferrite posts with previously published experimental and theoretical results. The performances of Y-junction circulators with a triangular equilateral ferrite post or a triangular ferrite post having depressed sides were also investigated. The influences of the ferrite losses on the performance were examined.

This method can be easily extended to the planar circulators using arbitrarily shaped resonators [8]. The problem of how to deal with waveguide junctions with partial-height ferrite posts [15]-[17] hereafter still remains.

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REFERENCES


TABLE III
DISSIPATIVE LOSSES OF Y-JUNCTIONS WITH A TRIANGULAR FERRITE POST HAVING DEPRESSED SIDES

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Arrangement in Fig.7</th>
<th>Arrangement in Fig.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 2.6 mm</td>
<td>α = 2.8 mm</td>
<td>α = 2.6 mm</td>
</tr>
<tr>
<td>8.0</td>
<td>0.013</td>
<td>0.021</td>
</tr>
<tr>
<td>9.0</td>
<td>0.023</td>
<td>0.035</td>
</tr>
<tr>
<td>9.5</td>
<td>0.029</td>
<td>0.041</td>
</tr>
<tr>
<td>10.0</td>
<td>0.034</td>
<td>0.044</td>
</tr>
<tr>
<td>10.31</td>
<td>0.037</td>
<td>0.045</td>
</tr>
<tr>
<td>11.0</td>
<td>0.037</td>
<td>0.043</td>
</tr>
<tr>
<td>11.4</td>
<td>0.036</td>
<td>0.042</td>
</tr>
<tr>
<td>12.0</td>
<td>0.036</td>
<td>0.022</td>
</tr>
</tbody>
</table>

From Figs. 5-8 and Tables II and III, it is found that it is possible to optimize the form of the cross section of the ferrite to find the best possible circulator structure, namely higher isolation and reflection losses, smaller insertion loss, and smaller dissipative loss at the circulation frequency.

V. CONCLUSION

A method of analysis, based on the finite-element approach and the analytical approach, was developed for the solution of H-plane waveguide junctions with lossy ferrite posts of arbitrary shape. The validity of the method was confirmed by comparing numerical results for circular ferrite post circulators with previously published experimental and theoretical results. The performances of Y-junction circulators with a triangular equilateral ferrite post or a triangular ferrite post having depressed sides were also investigated. The influences of the ferrite losses on the performance were examined.

This method can be easily extended to the planar circulators using arbitrarily shaped resonators [8]. The problem of how to deal with waveguide junctions with partial-height ferrite posts [15]-[17] hereafter still remains.

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