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Improved Finite-Element Formulation in Terms of the Magnetic Field Vector for Dielectric Waveguides

MASANORI KOSHIBA, SENIOR MEMBER, IEEE, KAZUYA HAYATA, AND MICHIKO SUZUKI, SENIOR MEMBER, IEEE

Abstract — An improved finite-element method for the analysis of dielectric waveguides is formulated in terms of all three components of the magnetic field $H$. In this approach, the spurious, nonphysical solutions do not appear anywhere above the "air-line," and therefore the present formulation is very useful for the analysis of the surface-wave modes of dielectric waveguides. The application of this improved finite-element method to the dielectric waveguides with perfect electric and magnetic conductors is also discussed. In particular, the discussion is how to use the conditions on a boundary surface of a perfect electric or magnetic conductor whose normal direction is not coincident with the direction of a coordinate axis. Application of these boundary conditions for perfect conductors to the dielectric waveguides with planes of symmetry reduces the matrix size. The strength of this approach to boundary conditions is not just the economical use of computer memory but the elimination of spurious solutions through rigorous enforcement of boundary conditions as well.

I. INTRODUCTION

SEVERAL METHODS for the analysis of dielectric waveguides in Fig. 1 have been proposed, and the vectorial finite-element formulation in terms of the longitudinal electric ($E_x$) and magnetic ($H_z$) field components, which enables one to compute accurately the mode spectrum of a waveguide with arbitrary cross section, is widely used [1]-[14]. The most serious difficulty in using the finite-element analysis, for inhomogeneous dielectric waveguides, is the appearance of the so-called spurious, nonphysical solutions [1]-[14]. The longitudinal $E_x - H_z$ formulation contains mathematical singularities [2],[3]. Recently, Mabaya, Lagasse, and Vandebulcke [12] found that by explicitly enforcing the continuity of the tangential components of the transversal fields, at the interface, by means of Lagrange multipliers, most of the spurious solutions disappear. The disadvantage of this method lies in the greatly increased complexity of the program and of the numerical operators that have to be used to enforce those continuity conditions [12]. Konrad [15] proposed the vectorial finite-element formulation in terms of all three components ($H_x$, $H_y$, and $H_z$) of the magnetic field $H$. The three-component formulation does not contain mathematical singularities as is the case with the $E_x - H_z$ formulation, but the spurious solutions do appear [15]-[17]. As noted by Davies, Fernandez, and Philippou [16], the spurious solutions in the three-component formulation do not satisfy the divergence relation for $H$, $\nabla \cdot H = 0$.

In this paper, an improved finite-element method for the analysis of dielectric waveguides is formulated in terms of all three components of $H$. For an abrupt discontinuity in the permittivity in an inhomogeneous medium, there is an abrupt change in the electric field $E$. In such cases, it is advantageous to solve for the values of $H$ at the nodal points. In this approach, the spurious solutions do not appear anywhere above the "air-line" corresponding to $\beta/k_0 = 1$ in a $\beta/k_0$ versus $k_0$ diagram (a plot of $\beta/k_0$ on the vertical axis against $k_0$ on the horizontal axis), where $k_0$ is the wavenumber of free space and $\beta$ is the phase constant in the $z$-direction. Therefore, the present formulation is very useful for the analysis of the surface-wave modes of dielectric waveguides which correspond to the solutions above the "air-line."

The application of this improved finite-element method to the dielectric waveguides with perfect electric and magnetic conductors is also discussed. In particular, the discussion is how to use the conditions on a boundary surface of a perfect electric or magnetic conductor whose normal direction is not coincident with the direction of a coordinate axis. In the analysis of dielectric waveguides with planes of symmetry, these boundary conditions for perfect conductors are used on each plane of symmetry. Application of these conditions reduces the matrix size. The strength of this approach to boundary conditions is not just the economical use of computer memory but the elimination of spurious solutions through rigorous enforcement of boundary conditions as well.

II. FUNCTIONAL FORMULATION

We consider a dielectric waveguide with arbitrary cross-section $\Omega$ in the $xy$-plane as shown in Fig. 1. With a time dependence of the form $\exp(j\omega t)$ being implied, Maxwell's equations are

$$\nabla \times E = -j\omega \mu_0 H$$  \hspace{1cm} (1)
$$\nabla \times H = j\omega \varepsilon_0 |k| E$$  \hspace{1cm} (2)

where $\omega$ is the angular frequency, $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability of free space, respectively.
is derived:

\[ \nabla \times ( [ K ]^{-1} \nabla \times H ) - k_0^2 H = 0 \quad (3) \]

where

\[ k_0^2 = \omega^2 \varepsilon_0 \mu_0. \quad (4) \]

The functional [15] for (3) is known to be

\[ F = \iint_{\Omega} \left( (\nabla \times H)^* \cdot ([K]^{-1} \nabla \times H) - k_0^2 H^* \cdot H \right) d\Omega \quad (5) \]

where the asterisk denotes a complex conjugate. The formulation of (5) does not contain mathematical singularities as is the case with the \( E_s - H \) formulation, but the spurious solutions do appear [15]–[17]. These spurious solutions fall into two fairly clear categories [16]. The first one (S1) can be characterized as follows:

\[ \nabla \times H = 0, \quad \nabla \cdot H \neq 0 \quad \text{for } k_0^2 = 0. \quad (6) \]

The second group (S2) can be characterized as follows:

\[ \nabla \times H \neq 0, \quad \nabla \cdot H \neq 0 \quad \text{for } k_0^2 > 0. \quad (7) \]

These spurious solutions do not satisfy the relation \( \nabla \cdot H = 0 \) [16], [17].

Now, we consider the following functional [18]–[20]:

\[ \tilde{F} = F + \iint_{\Omega} (\nabla \cdot H)^* (\nabla \cdot H) d\Omega. \quad (8) \]

For the functional (8), the first variation \( \delta \tilde{F} \) is given by

\[ \delta \tilde{F} = \iint_{\Omega} \delta H^* \cdot \left[ \nabla \times ([K]^{-1} \nabla \times H) - (\nabla \cdot H) - k_0^2 H \right] d\Omega \]

\[ - \int_{\Gamma} \delta H^* \cdot \left[ n \times ([K]^{-1} \nabla \times H) - n \cdot (\nabla \cdot H) \right] d\Gamma \quad (9) \]

where \( \Gamma \) represents the contour of the region \( \Omega \), \( n \) is the outward unit normal vector to \( \Gamma \), and the term \( n \times ([K]^{-1} \nabla \times H) \) corresponds to the tangential components of the electric field \( E \) on \( \Gamma \). The stationarity requirement \( \delta \tilde{F} = 0 \) shows that

\[ \nabla \times ([K]^{-1} \nabla \times H) - (\nabla \cdot H) - k_0^2 H = 0 \quad (10a) \]

as the Euler equation and

\[ n \times ([K]^{-1} \nabla \times H) = 0 \quad \text{on perfect electric conductor} \]

\[ n(\nabla \cdot H) = 0 \quad \text{on perfect magnetic conductor} \]

as natural boundary conditions, since \( \delta H^* \) in (9) is arbitrary.

Multiplying (10a) by \( H^* \) and integrating over the region \( \Omega \), the following equation is obtained using Green's formula and the boundary conditions on \( \Gamma \):

\[ \iint_{\Omega} \left( (\nabla \times H)^* \cdot ([K]^{-1} \nabla \times H) + (\nabla \cdot H)^* (\nabla \cdot H) - k_0^2 H^* \cdot H \right) d\Omega = 0. \quad (11) \]

In (11), if \([K]^{-1}\) is a positive definite matrix, then \( \nabla \times H = 0 \) and \( \nabla \cdot H = 0 \) are satisfied for \( k_0^2 = 0 \). Therefore, the spurious solutions \( S_1 \) are eliminated.

Taking divergence of (10a), we obtain

\[ (\nabla^2 + k_0^2)(\nabla \cdot H) = 0. \quad (12) \]

If the curl of \( H \) is not zero for \( k_0^2 > 0 \), the eigenvectors of (10) cannot satisfy (12). Therefore, the eigenvectors of (10) should obey \( \nabla \cdot H = 0 \) and the spurious solutions \( S_1 \) are eliminated.

When \( \nabla \times H = 0 \) for \( k_0^2 > 0 \), (8) may have the solutions other than those of (3) [19]. This new group \( (S_2) \) characterized by

\[ \nabla \times H = 0, \quad \nabla \cdot H \neq 0 \quad \text{for } k_0^2 > 0 \quad (13) \]

obeys the following equations:

\[ H = \nabla \phi \quad (14a) \]

\[ (\nabla^2 + k_0^2)\phi = 0 \quad \text{in region } \Omega \quad (14b) \]

\[ \partial \phi / \partial n = 0 \quad \text{on perfect electric conductor} \quad (14c) \]

\[ \phi = 0 \quad \text{on perfect magnetic conductor} \quad (14d) \]

where \( \phi \) is the scalar field. The magnetic field \( H \) of (14) satisfies the stationarity requirement \( \delta \tilde{F} = 0 \), but the divergence of \( H \) is not zero. Therefore, in the finite-element analysis using (8), the spurious solutions \( S_2 \) which are not included in (5) do appear. Fortunately, the solutions \( S_2 \) are equivalent to the TE modes of “hollow” waveguides (replace \( \phi \) in (14b)–(14d) with \( H_t \)) and the appearance is limited to the region \( \beta / k_0 < 1 \). They do not appear anywhere above the “air-line.” Therefore, if one is interested only in the solutions in the region \( \beta / k_0 > 1 \) which correspond to the surface-wave modes of dielectric waveguides [4]–[7], [10]–[13], [17], [21], [22], the appearance of the solutions \( S_2 \) are not a serious problem. The value of (14) is that it enables us to evaluate the behavior of the spurious solutions \( S_2 \) of the finite-element method based on (8). On the other hand, the spurious solutions \( S_1 \) and \( S_2 \) of the finite-element method based on (5) are unpredictable.

### III. FINITE-ELEMENT DISCRETIZATION

Dividing the cross-section \( \Omega \) of the waveguide with a diagonal permittivity tensor into a number of second-order
triangular elements in Fig. 1, the magnetic fields within
each element are defined in terms of the magnetic fields at
the corner and midside nodal points:
\[
H = [N]^T \{ H \}_e \exp(-j\beta z)
\]
(15)
where
\[
\{ H \}_e = \begin{bmatrix}
\{ H_x \}_e \\
\{ H_y \}_e \\
\{ H_z \}_e
\end{bmatrix}
\]
(16)
\[
[N] = \begin{bmatrix}
\{ N \} & \{ 0 \} & \{ 0 \} \\
\{ 0 \} & \{ N \} & \{ 0 \} \\
\{ 0 \} & \{ 0 \} & j\{ N \}
\end{bmatrix}
\]
(17)
\[
\{ N \} = [N_1, N_2, N_3, N_4, N_5, N_6]^T.
\]
(18)
Here \{ H_x \}_e, \{ H_y \}_e, and \{ H_z \}_e are magnetic field vectors
 corresponding to the nodal points within each element, \{ 0 \}
is a null vector, \{ \cdot \}, and \{ \cdot \}^T denote a transpose, a
column vector, and a row vector, respectively, and the
shape functions \( N_1 \) to \( N_6 \) are given by
\[
\begin{align*}
N_1 &= L_1 (2L_1 - 1) \\
N_2 &= L_2 (2L_2 - 1) \\
N_3 &= L_3 (2L_3 - 1) \\
N_4 &= 4L_1 L_2 \\
N_5 &= 4L_2 L_3 \\
N_6 &= 4L_3 L_1
\end{align*}
\]
(19a-19f)
with the area coordinates \( L_1, L_2, \) and \( L_3 \) \([4], [10]\). The
relation equation between the area coordinates and Carte­sian
coordinates is given by
\[
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix} = \begin{bmatrix}
x_1 & x_2 & x_3 \\
y_1 & y_2 & y_3 \\
1 & 1 & 1
\end{bmatrix} \begin{bmatrix}
L_1 \\
L_2 \\
L_3
\end{bmatrix}
\]
(20)

The functional for the whole region \( \Omega \) is given by
\[
\tilde{F} = \{ H \}_{\Omega}^T \{ \cdot \}^T \{ H \}_{\Omega} \cdot \{ \cdot \}_{\Omega}^T \{ H \}_{\Omega} - k_0^2 \{ T \}_{\Omega}^T \{ H \}_{\Omega} = \{ 0 \}
\]
(27)
where \{ \cdot \}_{\Omega} is the nodal magnetic field vector and, for
loss-free media, \{ S \}, \{ T \}, and \{ U \} are real, symmetric
matrices. Variation of (27) with respect to the nodal vari­ables leads to the following eigenvalue problem:
\[
([S] + [U]) \{ H \} - k_0^2 [T] \{ H \} = \{ 0 \}
\]
(28)

Using the functional (5), we obtain the following eigen­value problem:
\[
[S] \{ H \} - k_0^2 [T] \{ H \} = \{ 0 \}
\]
(29)

IV. BOUNDARY CONDITIONS

In (27), the nodal magnetic field vector \{ H \} should be
forced to satisfy the boundary conditions on \( \Gamma \) in Fig. 1,
where the unit vector \( \mathbf{n} \) normal to \( \Gamma \) lies at an angle \( \theta \)
from the x-axis in the xy-plane. The functional (27) can be
rewritten as
\[
\tilde{F} = \{ H \}_{\Gamma}^T \{ S \}_{\Gamma} \{ H \}_{\Gamma} + \{ H \}_{\Gamma}^T \{ U \}_{\Gamma} \{ H \}_{\Gamma} - k_0^2 \{ T \}_{\Gamma} \{ H \}_{\Gamma} = \{ 0 \}
\]
(30)
where the components of the \{ H_i \} vector are the values of
the magnetic field \( H_i \) at all nodal points in \( \Omega \) except \( \Gamma \), the
components of the \{ H_{\Gamma} \} vector are the values of \( H_i \) at all
nodal points on \( \Gamma \), and \{ A_{xx}, A_{xy}, A_{yz}, \} and \{ A_{xz}, A_{zy}, A_{yz} \}
are the submatrices of \{ A \} = \{ S \} + \{ U \} - k_0^2 \{ T \}.

Using the boundary condition for the perfect electric
conductor \( \mathbf{n} \cdot \mathbf{H} = 0 \), namely
\[
\{ H_{\Gamma} \} = -\tan \theta \{ H_r \}
\]
(31)
on \( \Gamma \) and minimizing (30), we obtain
\[
\begin{bmatrix}
[A_{xx}] & [A_{xy}] & [A_{xz}] & [A_{xy}'] & [A_{xz}'] \\
[A_{yx}] & [A_{yy}] & [A_{yz}] & [A_{yx}'] & [A_{yz}'] \\
[A_{zx}] & [A_{zy}] & [A_{zz}] & [A_{zx}'] & [A_{zy}'] \\
[A_{xy}']' & [A_{yz}']' & [A_{xz}']' & [A,.] & [A,'] \\
[A_{xz}']' & [A_{zy}']' & [A_{yz}']' & [A_{.y}] & [A_{.y}'] \\
\end{bmatrix}
\begin{bmatrix}
\{H_x\} \\
\{H_y\} \\
\{H_z\} \\
\{H_y'\} \\
\{H_z'\} \\
\end{bmatrix}
= \begin{bmatrix} 0 \end{bmatrix}
\tag{32}
\]
where
\[
\begin{align*}
[A_{y'y}] &= [A_{y'y}] - \tan \theta ([A_{x'y}] + [A_{y'x}]) + \tan^2 \theta [A_{x'x}] \\
[A_{y'y}] &= [A_{y'y}] - \tan \theta [A_{x'y}], \quad j = x, y, z, z' \quad \tag{33a} \\
[A_{y'y}] &= [A_{y'y}] - \tan \theta [A_{x'y}], \quad j = x, y, z, z'. \quad \tag{33b} \\
[A_{y'y}] &= [A_{y'y}] - \tan \theta [A_{x'y}], \quad j = x, y, z, z'. \quad \tag{33c}
\end{align*}
\]
Using the boundary condition for the perfect magnetic conductor \( \mathbf{n} \times \mathbf{H} = 0 \), namely
\[
\begin{align*}
\{H_x\} &= \cot \theta \{H_y\} \quad \tag{34a} \\
\{H_y\} &= \{H_y'\} = \{0\} \quad \tag{34b}
\end{align*}
\]
on \( \Gamma \) and minimizing (30), we obtain
\[
\begin{align*}
\begin{bmatrix}
[A_{xx}] & [A_{xy}] & [A_{xz}] & [A_{xy}'] & [A_{xz}'] \\
[A_{yx}] & [A_{yy}] & [A_{yz}] & [A_{yx}'] & [A_{yz}'] \\
[A_{zx}] & [A_{zy}] & [A_{zz}] & [A_{zx}'] & [A_{zy}'] \\
[A_{xy}']' & [A_{yz}']' & [A_{xz}']' & [A,.] & [A,'] \\
[A_{xz}']' & [A_{zy}']' & [A_{yz}']' & [A_{.y}] & [A_{.y}'] \\
\end{bmatrix}
\begin{bmatrix}
\{H_x\} \\
\{H_y\} \\
\{H_z\} \\
\{H_y'\} \\
\{H_z'\} \\
\end{bmatrix}
= \begin{bmatrix} 0 \end{bmatrix}
\tag{35}
\]
where
\[
\begin{align*}
[A_{y'y}] &= [A_{y'y}] + \cot \theta ([A_{x'y}] + [A_{y'x}]) + \cot^2 \theta [A_{x'x}'] \\
[A_{y'y}] &= [A_{y'y}] + \cot \theta [A_{x'y}], \quad j = x, y, z \quad \tag{36a} \\
[A_{y'y}] &= [A_{y'y}] + \cot \theta [A_{x'y}], \quad j = x, y, z. \quad \tag{36b} \\
[A_{y'y}] &= [A_{y'y}] + \cot \theta [A_{x'y}], \quad j = x, y, z. \quad \tag{36c}
\end{align*}
\]
When \( \tan \theta \to \infty \) in (33) and \( \cot \theta \to \infty \) in (36), \( \{H_y'\}, [A_{y'y'}], [A_{.y'}], \) and \( [A_{y'y}] \) should be replaced by \( \{H_x\}, [A_{x'y'}], [A_{.y'}], \) and \( [A_{x'y}] \), respectively. It should be noted that (32) and (35) can be used to obtain the dispersion characteristics of dielectric waveguides with planes of symmetry.

V. NUMERICAL RESULTS

First, let us consider a half-filled dielectric waveguide as shown in Fig. 2, where \( n \) is the refractive index. We subdivide one half of the cross section into second-order triangular elements. The solid and dashed lines in Fig. 2 represent the solutions of the improved finite-element program in (28), while the solutions of the earlier finite-element program in (29) are indicated by the dots, where the plane of symmetry is assumed to be the perfect magnetic conductor. Computed results (solid lines) for the \( LSM_1 \) and \( LSE_{pq} \) modes [23] agree well with the exact results [23]. Spurious solutions \( S_3 \) (dashed lines) corresponding to the solutions of (14) appear only in the region \( \beta/k_0 < 1 \). The solutions \( S_3 \) with cutoff frequencies \( k_0W = \pi, \sqrt{2}\pi, \) and \( \sqrt{3}\pi \) are equivalent to the \( TE_{10} \), \( TE_{11} \), and \( TE_{12} \) modes of a “hollow” waveguide of square cross section, respectively. It is found that when (29) is used the spurious solutions are scattered all over the propagation diagram.

Next, let us consider an embedded rectangular waveguide [4, 5, 10, 12] and an embossed rectangular waveguide [4, 10, 12]. We subdivide one half of the cross section into second-order triangular elements as shown in Fig. 3, where \( W \) and \( t \) are the width and the thickness of a rectangular core, respectively, and boundaries \( BC, CD, \) and \( DA \) are assumed to be perfect electric conductors. Fig. 4 shows the dispersion characteristics for the \( E_{11} \) mode [21] of these waveguides, where \( v = k_0t\sqrt{n_1^2 - n_2^2}/\pi \) and \( b = \).
For the $E_{11}^y$ mode, the boundary $AB$ in Fig. 3 (plane of symmetry) becomes the perfect magnetic conductor. Our results agree well with the finite-element solutions [10] in the $E_z - H_y$ formulation. Fig. 5 shows the dispersion characteristics for the $E_{11}^y$ mode of an anisotropic embedded rectangular waveguide. Our results agree well with the finite-element solutions [5] in the $E_x - H_z$ formulation. Note that the spurious solutions are included in the solutions of the finite-element method in the $E_x - H_z$ formulation and they cannot be eliminated mathematically [1]-[14]. The $E_x - H_z$ formulation contains mathematical singularities, and the actual solutions are plotted as a continuous interpolated curve between points sufficiently removed from the singularity to be unaffected by it [2], [3], [8], [9], [11]. In order to avoid confusion, such spurious solutions in the $E_x - H_z$ formulation are not shown in Figs. 4 and 5.

Lastly, let us consider a dielectric square waveguide [4], [21], [22] with four planes of symmetry. We subdivide one quarter or one eighth of the cross section into second-order triangular elements as shown in Fig. 6, where boundaries $CD$ and $DA$ are assumed to be perfect electric conductors and the conditions on boundaries $AB$, $BC$, and $DB$ (planes of symmetry) are given in Table I. Fig. 7 shows the dispersion characteristics for the $E_{12}^{x,y}$ modes [21] of this waveguide. Our results agree well with the results of the collocation method [22]. For the $E_{21}^{x,y}$ and $E_{12}^{x,y}$ modes whose fields satisfy the boundary condition for the perfect electric or magnetic conductor on the boundary $DB$, the results of Fig. 6(b) are identical to those of Fig. 6(a). This fact proves the validity of (32) and (35). The strength of this approach to boundary conditions is not just the economical use of computer memory but the elimination of spurious solutions through rigorous enforcement of boundary conditions as well. The dots in Fig. 8 represent the solutions of the earlier finite-element program in (29), while the results of the improved finite-element program in
Fig. 7. Dispersion characteristics of a dielectric square waveguide.

Fig. 8. Solutions of (29) for the waveguide configuration in Fig. 7. Boundary conditions are \( H_x = 0 \) and \( H_z = H_y = 0 \) on \( AB \) and \( BC \) in Fig. 6(a), respectively.

(28) are indicated by a solid line (the \( E_{11}^0 \) mode in Fig. 7), where the conditions on boundaries \( AB \) and \( BC \) in Fig. 6(a) are \( H_x = 0 \) and \( H_z = H_y = 0 \), respectively. It is found that when (29) is used, numerous spurious solutions appear.

In Figs. 4, 5, and 7, the spurious solutions do not appear because the spurious solutions \((S)\) appear only below the "air-line," and the surface-wave modes (the \( E_{pq}^x \) modes) of dielectric waveguides of Figs. 4, 5, and 7 correspond to the solutions above the "air-line."

VI. CONCLUSION

An improved finite-element method for the analysis of dielectric waveguides with a diagonal permittivity tensor was formulated in terms of all three components of the magnetic field \( H \). In this approach, the spurious, nonphysical solutions do not appear anywhere above the "air-line," and therefore the present formulation is very useful for the analysis of the surface-wave modes of dielectric waveguides. The application of this improved finite-element method to the dielectric waveguides with perfect electric and magnetic conductors was also discussed.

This approach can be applied easily to the anisotropic waveguides having a permittivity tensor with nonzero off-diagonal elements.

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REFERENCES

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