Finite-Element Analysis of Magnetostatic Wave Propagation in a YIG Film of Finite Dimensions

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Abstract — A unified numerical approach based on the finite-element method is described for the magnetostatic wave propagation in a YIG film of finite dimensions. Both magnetostatic wave volume and magnetostatic surface wave modes are treated. The validity of the method is confirmed by calculating the magnetostatic wave modes in a YIG-loaded rectangular waveguide and in a YIG film of finite width. The numerical results of a YIG film with nonuniform bias field along the film width are also presented, and the effects of bias field distributions on the delay characteristics and potential profiles are examined.

I. INTRODUCTION

MAGNETOSTATIC WAVE (MSW) propagation in a YIG film of finite dimensions has been reported in previous papers [1]–[11]. O'Keeffe et al. [2] and Bajpai et al. [4] have investigated the effects of finite sample widths on MSW propagation. Recently, interest in three-dimensional inhomogeneous MSW waveguides is increasing because high-precision dispersion control is making MSW device application possible [1], [3], [5]–[11]. Morgan- thaler et al. [1], [3], [6], [8], [9] have discussed the control of MSW propagation by means of a spatially nonuniform bias field. It is also found that control of important features of MSW modes is afforded through the use of bias field gradients and that magnetostatic forward volume waves can be forced to have strong field-displacement characteristics that are either nearly reciprocal or very strongly nonreciprocal. Such control may provide the basis for new forms of microwave signal processors [6]–[9]. In order to analyze these inhomogeneous MSW waveguides, the variational method [12], [13] and the finite-element method [14] have been introduced. These methods are valid for the solution of inhomogeneous waveguide structures. In the former approach, however, great care is necessary in choosing the trial functions. The latter approach, on the other hand, is applied only to planar structures of infinite width.

In this paper, a unified approach based on the finite-element method is described for the MSW propagation in a YIG film of finite dimensions. Both magnetostatic forward volume wave (MSFVW) and magnetostatic surface wave (MSSW) modes are treated. In this finite-element approach, the cross section of the waveguide is divided into triangular elements [15], [16] and it is easy to consider the inhomogeneities in the bias field and/or the magnetization. The validity of the method is confirmed by calculating the MSW modes in a YIG-loaded rectangular waveguide [10] and in a YIG film of finite width [4]. Numerical results on the delay characteristics and potential profiles of a YIG film with nonuniform bias field distributions are examined.

II. BASIC EQUATIONS

Fig. 1 shows a MSW waveguide, where the boundaries \( \Gamma_1 \) and \( \Gamma_2 \) are assumed to be perfect electric conductors (PEC's) or perfect magnetic conductors (PMC's), and \( \Gamma_3 \) and \( \Gamma_4 \) are assumed to be PEC's.

The constitutive relations are

\[
B = \mu_0 \left[ \mu_r \right] H \quad \text{for ferrite} \tag{1a}
\]
\[
B = \mu_0 H \quad \text{for dielectric} \tag{1b}
\]

where \( B \) is the magnetic flux density, \( H \) is the magnetic field, \( \mu_0 \) is the permeability in free space, and \( [\mu_r] \) is the relative permeability tensor [14].

For the MSW propagating along the \( y \) direction, the governing equations can be written by

\[
\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial z} - js\beta B_y = 0 \tag{2}
\]

\[
H_x = -\frac{\partial \phi}{\partial x} \tag{3a}
\]

\[
H_y = js\beta \phi \tag{3b}
\]

\[
H_z = -\frac{\partial \phi}{\partial z} \tag{3c}
\]
where $\beta$ is the phase constant in the $y$ direction, $s = \pm 1$ is a directional parameter [14], and $\phi$ is the magnetostatic potential.

### III. Mathematical Formulations

Dividing the region enclosed by the boundaries $\Gamma_1$ to $\Gamma_4$ into a number of second-order triangular elements [15], [16] as shown in Fig. 2, the magnetic potential $\phi$ within each element is defined in terms of the magnetic potential $\phi_k$ at the nodal point $k$ ($k = 1, 2, \ldots, 6$):

$$\phi = \{N\}^T \{\phi\}_e \exp(-j s \beta y)$$

where

$$\{\phi\}_e = [\phi_1 \phi_2 \phi_3 \phi_4 \phi_5 \phi_6]^T$$

$$\{N\} = [N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_6]^T$$

Here shape functions $N_1$ to $N_6$ are given by

$$N_1 = L_1 (2L_1 - 1)$$

$$N_2 = L_2 (2L_2 - 1)$$

$$N_3 = L_3 (2L_3 - 1)$$

$$N_4 = 4L_1 L_2$$

$$N_5 = 4L_2 L_3$$

$$N_6 = 4L_3 L_1$$

with the area coordinates $L_1$, $L_2$, and $L_3$ [15], [16]. The relation between the area coordinates and the Cartesian coordinates is expressed in the following form:

$$\begin{bmatrix} x \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ z_1 & z_2 & z_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

where $(x_j, z_j)$ are the Cartesian coordinates of the vertex $j$ ($j = 1, 2, 3$) of the triangle. Also, the diagonal component $\mu$ and the off-diagonal component $\kappa$ of the relative permeability tensor within each element are approximated by

$$\mu = \{N\}^T \{\mu\}_e$$

$$\kappa = \{N\}^T \{\kappa\}_e$$

and $\mu_k$ and $\kappa_k$ ($k = 1, 2, \ldots, 6$) are, respectively, the $\mu$ and $\kappa$ values at the nodal point $k$. Using a Galerkin procedure on (2), we obtain

$$\int_{\Omega_e} \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} - js \beta B_y \right) d\Omega = \{0\}$$

where the integration is carried out over the element subdomain $\Omega_e$, and $\{0\}$ is a null vector.

Integrating by parts, we obtain for (10)

$$\int_{\Omega_e} \left( \{N\} B_x + \{N\} B_z + js \beta \{N\} B_y \right) d\Omega$$

$$- \int_{\Omega_e} \{N\} B_n d\Gamma = \{0\}$$

where

$$B_n = B_n x + B_n z.$$
extend over all elements and the elements related to $\Gamma_i$ ($i=1,2$), respectively; $B_{x,i}$ is the magnetic flux density on $\Gamma_i$; and $\{N_i\}$ is the shape function vector on $\Gamma_i$, namely, $\{N_i\} = \{N(x_i, z_i)\}$.

Combining the boundary conditions on the planes $\Gamma_1$ and $\Gamma_2$, (13) becomes

\[ [A]\{\phi\} = \{0\} \tag{15} \]

where

\[
[A] = \begin{bmatrix}
[A]_{00} & [A]_{01} & [A]_{02} \\
[A]_{01} & [A]_{11} & [A]_{12} \\
[A]_{02} & [A]_{12} & [A]_{22}
\end{bmatrix}
\]

\[
\{\phi\} = \begin{bmatrix}
\{\phi\}_0 \\
\{\phi\}_1 \\
\{\phi\}_2
\end{bmatrix}
\]

Here $[A]_{00}, [A]_{01}, \ldots, [A]_{22}$ are the submatrices of $[A]$, and $\{\phi\}_i$, is the nodal magnetic potential on $\Gamma_i$ ($i=1,2$). Using (15), one can determine the dispersion characteristics of MSW waveguides.

IV. COMPUTED RESULTS

First, we consider a YIG-loaded rectangular waveguide [10] as shown in Fig. 3, where the planes $x = 0$, $x = a$, $z = 0$, and $z = b$ are assumed to be PEC's, the bias field is along the $x$ axis, and there exist MSSW modes. In this case, the potential profile is symmetric with respect to the plane $x = a/2$. Taking advantage of this symmetry, we subdivided one half of the cross section into second-order triangular elements, where the plane $x = a/2$ is assumed to be a PMC. Table I exhibits the computed results, where $a = 20$ mm, $b = 10$ mm, $t = 1$ mm, the saturation magnetization $4\pi M_s = 1750$ G, the bias field $H_0 = 1800$ Oe, $\beta_{\text{exact}}$ is the exact solution [10], and $\beta_{\text{FEM}}$ is the finite-element solution. It is apparent from Table I that the finite-element solutions are in good agreement with the exact solutions. The convergence behavior of the phase constant $\beta$ is shown in Fig. 4. With an increasing number of elements, the phase constant $\beta$ converges monotonically. Also, we find from Table I and Fig. 4 that the accuracy of the finite-element solutions falls off as the film position $z_0$ is lowered or the frequency $F$ increases.

Next, we consider a YIG film of finite width [4] as shown in Fig. 3, where the planes $x = 0$ and $x = a$ are assumed to be PMC's, the planes $z = 0$ and $z = b$ are assumed to be PEC's [2], [4], the bias field is along the $z$ axis, and there exist MSFVW modes. Table II exhibits the computed results, where $a = 3$ mm, $d_1 = 50$ mm, $d_2 = 178$ $\mu$m, $t = 9.1$ mm, $4\pi M_s = 1750$ G, and the effective static field $H_s = 3000$ Oe ($H_s = H_0 - 4\pi M_s$ for MSFVW modes). The finite-element solutions agree well with the exact solutions [4]. Also, larger errors are observed as the frequency increases.

Lastly, we consider the MSW waveguide with a bias field of parabolic distribution where the planes $x = 0$ and $x = a$ in Fig. 3 are assumed to be PMC's, the planes $z = 0$ and $z = b$ are assumed to be PEC's, and the bias field is...
TABLE I  
COMPARISON BETWEEN EXACT SOLUTIONS AND FEM SOLUTIONS FOR MSSW MODES

<table>
<thead>
<tr>
<th>( z_0 ) (mm)</th>
<th>( F ) (GHz)</th>
<th>Phase constant ( \beta ) (1/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \beta_{\text{exact}} )</td>
</tr>
<tr>
<td>7.1</td>
<td>0.0419056</td>
<td>0.0419052</td>
</tr>
<tr>
<td>7.3</td>
<td>0.1193626</td>
<td>0.1193620</td>
</tr>
<tr>
<td>7.4</td>
<td>0.1633299</td>
<td>0.1633295</td>
</tr>
<tr>
<td>7.5</td>
<td>0.2235310</td>
<td>0.2235304</td>
</tr>
<tr>
<td>7.6</td>
<td>0.3563842</td>
<td>0.3563856</td>
</tr>
<tr>
<td>7.1</td>
<td>0.1104845</td>
<td>0.1104839</td>
</tr>
<tr>
<td>7.2</td>
<td>0.2253471</td>
<td>0.2253469</td>
</tr>
<tr>
<td>7.3</td>
<td>0.4832040</td>
<td>0.4832315</td>
</tr>
<tr>
<td>7.4</td>
<td>0.9011406</td>
<td>0.9019049</td>
</tr>
</tbody>
</table>

For \( a = 20 \text{ mm}, b = 10 \text{ mm}, t = 1 \text{ mm}, 4\pi M_s = 1750 \text{ G}, H_0 = 1800 \text{ Oe} \).

TABLE II  
COMPARISON BETWEEN EXACT SOLUTIONS AND FEM SOLUTIONS FOR MSFVW MODES

<table>
<thead>
<tr>
<th>( F ) (GHz)</th>
<th>Phase constant ( \beta ) (1/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_{\text{exact}} )</td>
</tr>
<tr>
<td>8.81</td>
<td>3.200592</td>
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<tr>
<td>8.91</td>
<td>16.25167</td>
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<td>9.01</td>
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<td>9.21</td>
<td>53.96416</td>
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<td>9.31</td>
<td>67.73479</td>
</tr>
<tr>
<td>9.41</td>
<td>82.69378</td>
</tr>
<tr>
<td>9.51</td>
<td>99.12766</td>
</tr>
</tbody>
</table>

For \( d_1 = 50 \mu \text{ m}, d_2 = 178 \mu \text{ m}, t = 9.1 \mu \text{ m}, a = 3 \text{ mm}, 4\pi M_s = 1750 \text{ G}, \) and \( H_i = 3000 \text{ Oe} \).

Fig. 5. Bias field distribution along the width of a YIG film.

The time delay characteristics are illustrated in Fig. 6(a) and (b) for cases A and B, respectively, where the dotted line represents the delay curve for the homogeneous case [4]. These figures show that the nondispersive bandwidth is extended in the case of the parabolic distribution of the bias field. Comparing case A with case B, it is also found that the projecting parabolic bias field offers a wider nondispersive bandwidth.

The effect of parabolic bias field on the potential profile is investigated, Fig. 7(a), (b), and (c) shows the potential profiles for the uniform, hollowed parabolic, and projecting parabolic bias fields, respectively. The phase constant \( \beta \) is independent of the direction of propagation. In the case of parabolic bias fields, the potential has a stronger localization. Comparing case A with case B, it is found that the projecting parabolic bias field creates stronger potential–displacement nonreciprocity.

A similar potential–displacement nonreciprocity was reported in [9].

V. CONCLUSIONS

A finite-element method is developed for the analysis of the MSW propagation in a YIG film of finite dimensions. Both MSVW and MSSW modes are treated in a unified
Fig. 7. Potential profiles. (a) Homogeneous distribution. (b) Hollowed parabolic distribution. (c) Projecting parabolic distribution.

manner. In order to examine the validity of the present method, MSW modes in a YIG-loaded rectangular waveguide and in a YIG film of finite width are calculated. We also present the application of this approach by analyzing the MSFVW propagation in a YIG film with the bias field of parabolic distribution along the film width. It is found that the parabolic bias field offers an improvement in the nondispersive bandwidth.

REFERENCES